International Journal of Multidisciplinary Research and Growth Evaluation.

Two Parametric SEE transformation and its Properties

Muhammad Imran [1], Muhammad Suleman [2], Shahbaz Zafar [3], Usman Amjed [4*]
[1-4] University of Agriculture Faisalabad, Pakistan
[2] Minhaj University Lahore, Pakistan
[3] University of Lahore, Pakistan

* Corresponding Author: **Usman Amjed**

Article Info

ISSN (online): 2582-7138
Volume: 03
Issue: 05
September-October 2022
Received: 20-07-2022
Accepted: 22-08-2022
Page No: 222-235

Abstract
Transformation plays much important role in every sciences. In this research article, two parametric form of SEE transformation has been explored and the fundamental properties of two parametric SEE transformation has been shown. Furthermore the transformed function of some fundamental function and its time derivative rule has been shown. The application of two parametric SEE transformation in solving differential equation has been shown. At the end of research article, result are drawn.

Keywords: SEE transformation, differential equations, time derivative rule

1. Introduction

The integral transformations map a complicated function from a function space into a simple function in transformed space. Where the function being characterized easily and manipulated through integration in transformed function space. To convert the transformed function into its original space often inverse transform tin technique arc used. The information on the topic is not lacking but the derivation of various research is not reliable and show contrasts answered to the material utilized and area of analysis. A few studies from writing are assessed here which are related to the subject. The theory of Fourier and Laplace transformations was presented by many mathematicians, but no one has compared k-Fourier and k-Laplace transforms. The work on modern theory such as Integral transformation, Laplace transformation, Fourier transformation, Mahgoub transformation, Mohand transformation, Aboodh transformation is still in progress. The authors have done comparative study of these transforms and proved that they are integral for solving many advanced problems for engineering and sciences [1]. Have explored that the algorithm for numerically Laplace transform and designed it if Probability cumulative distribution [2]. Have described that the topological space and its properties, Laplace transformation, topological iso-morphism and applied to differential equations of non- Archimedean also have described that the Laplace transform, and inversion method achieve real valued function $f(t)$ and Trapezoidal to numerical evaluation to Fourier series [3]. Have described Laplace transform method considered to solve a diffuse convection problem analytically across a finite domain. A simple diffusion problem over a finite domain has had two different analytical solutions obtained by the method of Laplace transform. In contrast to the other solution, which is disjointed at the origin, one exact analytical is uninterrupted both at ends of the region of interest. It is explained why the two solutions differ from each other as well [4]. Introduction The conventional Fourier integral is where all the Kamal Transform takes its title based on the fundamental characteristics and simplicity of math of the Kamal transform. Abdelilah Kamal developed the Kamal transform to make it easier to solve ordinary and partial differential equations in the time domain. The commonly used mathematical techniques for solving differential equations are the Fourier, Laplace, Sumudu and Mahgoub transforms. Kamal transform and some of its basic features are also employed [5]. Attempted to define the double Kamal transform of constrained support in two dimensions using the Zeman Ian approach. Using Mikasuki's theory and some of the features, the integral equations Bohemian of the double Kamal transform is created.

Ordinary and partial differential equations have a crucial role in the field of engineering. Integral transformations are effectively utilised to simplify and conveniently acquire the solutions to the differential equations. While the Kamal transform for single variable functions is discussed, the double Natural transform is researched in deals with applications [6]. Have learned The Kamal transform is applied to the second set of linear Volterra integral-differential equations. The technique is only briefly described and shown using a few numerical examples. The findings demonstrate that this approach offers precise returns with little computing effort. Functional derivatives, delay system of equations, solving differential equations, integral equations, integral-differential equations, probability equations, etc. are the mathematical solutions to real-world situations. A variety of processes and events in numerous branches of research and engineering depend heavily on integral-differential equations [7]. Have Solutions that additionally include error and a complementary error function have been researched for a number of complicated engineering issues, including those required Fick's second law, mass and heat, and vibrating beams. When employing any integral transform to tackle problems of this nature, it is crucial to comprehend the integral transformation of the error function. In this section, the complementary error function and the Kamal transform of errors are presented [8]. Have demonstrated that functional responses can make predator-prey systems themselves inherently oscillating. Cushing, 1987 pointed out that it is necessary and important to consider models with periodic ecological parameters or perturbations which might be quite naturally exposed (for example, those due to seasonal effects of weather, food supply, mating habits, hunting, or harvesting seasons. In fact, almost without exception biological communities are visited by perturbations that occur in a more-or-less periodic fashion. Thus, an interesting problem rises from forcing predator-prey systems periodically [9]. Demonstrated that functional responses can make predator-prey systems themselves inherently oscillating. Cushing, 1987 pointed out that it is necessary and important to consider models with periodic ecological parameters or perturbations which might be quite naturally exposed (for example, those due to seasonal effects of weather, food supply, mating habits, hunting, or harvesting seasons. In fact, almost without exception biological communities are visited by perturbations that occur in a more-or-less periodic fashion. Thus, an interesting problem rises from forcing predator-prey systems periodically.

Def: SEE Transform of Two Parameters

SEE integral transform of two parameters of the function *(t)* for all $t \geq 0$ is defined as,

$$S_{a,b}\{f(t)\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty f(t)e^{-(av)t}dt = T_{a,b}(v),\ n \in Z \tag{1}$$

Where $S\{.\}$ is the SEE transform of two parameter operator.

Def: Inverse of Two Parametric Extensions of SEE Transformation.

The inverse of SEE Transform over a set of functions,

$$S_{a,b}^{-1}\{T_{a,b}(v)\} = f(t) \tag{2}$$

Results

SEE Transform of Some Functions

Discuss some function by using SEE transform

Theorem

The SEE transformation of polynomial of nth power can expressed as $S_{a,b}\{t^m\} = \dfrac{m!.a^n e^{-vb}}{(av)^{m+n+1}}$.

Proof:

If $f(t) = k$ where k is a constant function.

Using $f(t) = k$ it follows from (1)

$$S_{a,b}\{k\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty ke^{-(av)t}dt = T_{a,b}(v) \tag{3}$$

Substituting in equation (3) $av = s$ and $v = \dfrac{s}{a}$ we get,

$$= a^n e^{-vb}\left[\frac{1}{s^n}\int_0^t e^{-st}kdt\right]$$

$$= a^n e^{-vb}\left[\frac{1}{s^n}.\frac{-k}{s}\int_0^t (-s)e^{-st}kdt\right]$$

$$= a^n e^{-vb}.\left|\frac{-k}{s^{n+1}}e^{-st}\right|_0^\infty$$

$$= a^n e^{-vb}.\frac{k}{s^{n+1}}$$

For $av = s$ it follows from the equation that

$$T_{a,b}\{k\} = a^n e^{-vb}.\frac{k}{(av)^{n+1}}$$

Thus, finally we obtain

$$S_{a,b}\{k\} = a^n e^{-vb}\frac{k}{(av)^{n+1}}$$

SEE Transformation of a function when $f(t) = t$

Using $f(t) = t$ it follows from the equation (1)

$$S_{a,b}\{t\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty te^{-(av)t}dt = T_{a,b}(v) \tag{5}$$

Substitute in equation (5) $av = s$ and $v = \frac{s}{a}$ we get

$$= a^n e^{-vb}\left[\frac{1}{s^n}\int_0^\infty te^{-st}dt\right]$$

Integrating by parts

$$= \frac{a^n e^{-vb}}{s^n}\left[t.\frac{e^{-st}}{-s}\bigg|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s}dt\right]$$

$$= \frac{a^n e^{-vb}}{s^n}\left[\frac{1}{s}\left\{\frac{e^{-st}}{-s}\right\}_0^\infty\right]$$

$$= \frac{a^n e^{-vb}}{s^n}\left[\frac{-1}{s^2}.e^{-st}\right]_0^\infty$$

Applying limit

$$= \frac{a^n e^{-vb}}{s^n}.\frac{1}{s^2}$$

For $av = s$ it follows from the equation that

$$= \frac{a^n e^{-vb}}{(av)^{n+2}}$$

Thus, finally we obtain

$$S_{a,b}\{t\} = \frac{a^n e^{-vb}}{(av)^{n+2}} \qquad (6)$$

SEE transform of two parameters when $f(t) = t^2$.

Using $f(t) = t^2$ it follows from the equation (1)

$$S_{a,b}\{t^2\} = \frac{a^n e^{-bv}}{(av)^n} \int_0^\infty t^2 e^{-(av)t} dt = T_{a,b}(v) \qquad (7)$$

Substituting $av = s$ and $v = \frac{s}{a}$ in equation (7) becomes

$$= a^n e^{-vb} \left[\frac{1}{s^n} \int_0^\infty t^2 e^{-st} dt \right]$$

$$= a^n e^{-vb} \left[\frac{1}{s^n} \int_0^\infty t^2 . e^{-st} dt \right]$$

Integrating by parts

$$= \frac{a^n e^{-vb}}{s^n} \left[t^2 . \frac{e^{-st}}{-s} \Big|_0^\infty - \int_0^\infty 2t \frac{e^{-st}}{-s} dt \right]$$

$$= \frac{a^n e^{-vb}}{s^n} \left[\frac{2}{s} \int_0^\infty t.e^{-st} dt \right]$$

Again, integrating by parts

$$= \frac{a^n e^{-vb}}{s^n} . \frac{2}{s} \left[t. \frac{e^{-st}}{-s} \Big|_0^\infty - \int_0^\infty t \frac{e^{-st}}{-s} dt \right]$$

$$= \frac{a^n e^{-vb}}{s^n} . \frac{2}{s} \left[\frac{1}{s} \int_0^\infty e^{-st} dt \right]$$

$$= \frac{a^n e^{-vb}}{s^n} \frac{2}{s} \left[\frac{1}{s} \left\{ \frac{e^{-st}}{-s} \right\}_0^\infty \right]$$

$$= \frac{a^n e^{-vb}}{s^n} \left[\frac{-2}{s^3} . e^{-st} \right]_0^\infty$$

$$= \frac{a^n e^{-vb}}{s^n} \cdot \frac{2}{s^3}$$

$$= \frac{2.a^n e^{-vb}}{s^{n+3}}$$

For $s = av$ it follows from the equation (7) that

$$= \frac{2.a^n e^{-vb}}{(av)^{n+3}}$$

Thus, finally we obtain

$$S_{a,b}\{t^2\} = \frac{2.a^n e^{-vb}}{(av)^{n+3}} \quad (8)$$

SEE Transform of two parameters when $f(t) = t^3$ and $f(t) = t^m$

From equation (6) and (8) we obtain the result when $f(t) = t^3$

$$S_{a,b}\{t^3\} = \frac{3!.a^n e^{-vb}}{(av)^{n+4}} \quad (9)$$

From equation (6), (8) and (9) we generalize, when *m* is a positive number

$$S_{a,b}\{t^m\} = \frac{m!.a^n e^{-vb}}{(av)^{m+n+1}}.$$

Theorem

SEE transform of two parameters when $f(t) = e^{ut}$ where u is a constant.

Proof

Using $f(t) = e^{ut}$ follows from the equation (1)

$$S_{a,b}\{e^{ut}\} = \frac{a^n e^{-bv}}{(av)^n} \int_0^\infty e^{ut} e^{-(av)t} dt = T_{a,b}(v) \quad (10)$$

Substituting in equation (10) $av = s$ and $v = \frac{s}{a}$, then

$$= a^n e^{-vb} \left[\frac{1}{s^n} \int_0^\infty e^{ut} e^{-st} dt \right]$$

$$= a^n e^{-vb} \left[\frac{1}{s^n} \int_0^\infty e^{-(s-u)t} dt \right]$$

$$= a^n e^{-vb} \frac{-1}{s^n (s-u)} \left\{ e^{-(s-u)t} \right\}_0^\infty$$

Apply limit

$$= a^n e^{-vb} \frac{1}{s^n (s-u)}$$

Using $s = av$ it follows from the equation (10)

$$= a^n e^{-vb} \frac{1}{(av)^n (av-u)}$$

Thus, finally we obtain

$$S_{a,b}\{e^{ut}\} == a^n e^{-vb} \frac{1}{(av)^n (av-u)}$$

Theorem

SEE transform of two parameters when $f(t) = \sin(ut)$ where a is a constant

Proof

Using $f(t) = \sin(ut)$ it follows from the equation (1)

$$S_{a,b}\{\sin(ut)\} = \frac{a^n e^{-bv}}{(av)^n} \int_0^\infty \sin(ut) e^{-(av)t} dt = T_{a,b}(v) \quad (11)$$

Substituting $av = s$ and $v = \frac{s}{a}$ in equation (10) , thus

$$= a^n e^{-vb} \left[\frac{1}{s^n} \int_0^\infty \sin(ut).e^{-st} dt \right]$$

$$= a^n e^{-vb} \left[\frac{1}{s^n} \int_0^\infty e^{-st} \left[\frac{e^{iut} - e^{-iut}}{2i} \right] dt \right]$$

$$= a^n e^{-vb} . \frac{1}{2is^n} \left[\frac{-1}{(s-ui)} \int_0^\infty e^{-(s-ui)t} dt - \frac{1}{-(s+ui)} \int_0^\infty e^{-(s+ui)} dt \right]$$

$$= a^n e^{-vb} . \frac{1}{2is^n} \left[\frac{-1}{s-ui} e^{-(s-ui)t} \Big|_0^\infty + \frac{1}{(s+ui)} e^{-(s+ui)} \Big|_0^\infty \right]$$

$$= a^n e^{-vb} . \frac{1}{2is^n} \left[\frac{1}{s-ui} - \frac{1}{s+ui} \right]$$

$$= a^n e^{-vb} . \frac{1}{2is^n} \left[\frac{2ui}{s^2 + u^2} \right]$$

$$= a^n e^{-vb} . \frac{1}{s^n} \left[\frac{u}{s^2 + u^2} \right]$$

For $s = av$ follows from the equation (11)

$$= \frac{a^n e^{-vb}}{(av)^n}\left[\frac{u}{(av)^2 + u^2}\right]$$

Thus, finally we obtain

$$S_{a,b}\{\sin(ut)\} == \frac{a^n e^{-vb}}{(av)^n}\left[\frac{u}{(av)^2 + u^2}\right]$$

Theorem
SEE transform of two parameters when $f(t) = \cos(ut)$

Proof
Using $f(t) = \cos(ut)$ follows from the equation (1)

$$S_{a,b}\{\cos(ut)\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty \cos(ut)e^{-(av)t}dt = T_{a,b}(v) \quad (12)$$

Substituting in equation (10) $av = s$ and $v = \frac{s}{a}$. Thus

$$= a^n e^{-vb}\left[\frac{1}{s^n}\int_0^\infty \cos(ut).e^{-st}dt\right]$$

$$= a^n e^{-vb}\left[\frac{1}{s^n}\int_0^\infty e^{-st}\left[\frac{e^{iut} + e^{-iut}}{2}\right]dt\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\int_0^\infty e^{-(s-ui)t}dt - \int_0^\infty e^{-(s+ui)t}dt\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{-1}{s-ui}e^{-(s-ui)t}\Big|_0^\infty - \frac{1}{(s+ui)}e^{-(s+ui)}\Big|_0^\infty\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{1}{s-ui} + \frac{1}{s+ui}\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{2s}{s^2 + u^2}\right]$$

$$= a^n e^{-vb}.\frac{1}{s^{n-1}}\left[\frac{1}{s^2 + u^2}\right]$$

For $s = av$ follow from equation (11)

$$= \frac{a^n e^{-vb}}{(av)^{n-1}}\left[\frac{1}{(av)^2 + u^2}\right]$$

Thus, finally we obtain

$$S_{a,b}\{\cos(ut)\} = \frac{a^n e^{-vb}}{(av)^{n-1}}\left[\frac{1}{(av)^2 + u^2}\right]$$

Theorem

SEE transform of two parameters when $f(t) = \cosh(ut)$

Proof

Using $f(t) = \cosh(ut)$ follow from equation (1)

$$S_{a,b}\{\cosh(ut)\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^{\infty}\cosh(ut)e^{-(av)t}dt = T_{a,b}(v) \tag{12}$$

Substituting $av = s$ and $v = \dfrac{s}{a}$ in equation (12), thus

$$= a^n e^{-vb}\left[\frac{1}{s^n}\int_0^{\infty}\cosh(ut).e^{-st}dt\right]$$

$$= a^n e^{-vb}\left[\frac{1}{s^n}\int_0^{\infty}e^{-st}\left[\frac{e^{ut} + e^{-ut}}{2}\right]dt\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\int_0^{\infty}e^{-(s-u)t}dt + \int_0^{\infty}e^{-(s+u)}dt\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{-1}{s-ui}e^{-(s-u)t}\Big|_0^{\infty} - \frac{1}{(s+ui)}e^{-(s+u)}\Big|_0^{\infty}\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{1}{s-u} + \frac{1}{s+u}\right]$$

$$= a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{2s}{s^2 - u^2}\right]$$

$$= a^n e^{-vb}.\frac{1}{s^{n-1}}\left[\frac{1}{s^2 - u^2}\right]$$

Using $av = s$ follow from equation (12)

$$= \frac{a^n e^{-vb}}{(av)^{n-1}}\left[\frac{1}{(av)^2 - u^2}\right]$$

Thus, finally we obtain

$$S_{a,b}\{\cosh(ut)\} = \frac{a^n e^{-vb}}{(av)^{n-1}}\left[\frac{1}{(av)^2 - u^2}\right]$$

Theorem

SEE transform of two parameters when $f(t)=\sinh(ut)$

Proof

Using $f(t)=\sinh(ut)$ follow from equation (1)

$$S_{a,b}\{\sinh(ut)\}=\frac{a^n e^{-bv}}{(av)^n}\int_0^\infty \sinh(ut)e^{-(av)t}dt=T_{a,b}(v) \tag{13}$$

Substituting $av=s$ and $v=\frac{s}{a}$ in equation (13), obtain

$$=a^n e^{-vb}\left[\frac{1}{s^n}\int_0^\infty e^{-st}\left[\frac{e^{ut}-e^{-ut}}{2}\right]dt\right]$$

$$=a^n e^{-vb}.\frac{1}{2s^n}\left[\int_0^\infty e^{-(s-u)t}dt-\int_0^\infty e^{-(s+u)}dt\right]$$

$$=a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{-1}{s-u}e^{-(s-u)t}\Big|_0^\infty+\frac{1}{(s+u)}e^{-(s+u)}\Big|_0^\infty\right]$$

$$=a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{1}{s-u}-\frac{1}{s+u}\right]$$

$$=a^n e^{-vb}.\frac{1}{2s^n}\left[\frac{2u}{s^2-u^2}\right]$$

For $s=av$ follow from equation (13)

$$=\frac{a^n e^{-vb}}{(av)^n}\left[\frac{1}{(av)^2-u^2}\right]$$

Thus, finally we obtain

$$S_{a,b}\{\sinh(ut)\}=\frac{a^n e^{-vb}}{(av)^n}\left[\frac{1}{(av)^2-u^2}\right]$$

Theorem: SEE transform of two parameters for first, second up-to nth order derivative $T_{a,b}(v)$ is SEE transform of two parameters such as $S_{a,b}\{f(t)\}=T_{a,b}(v)$ then,

$$[S_{a,b}[f'(t)]]=\frac{-a^n e^{-bv}}{(av)^n}f(0)+avT_{a,b}(v)$$

$$[S_{a,b}[f''(t)]]=\frac{-a^n e^{-bv}}{(av)^n}f'(0)-\frac{a^n e^{-bv}}{(av)^{n-1}}f(0)+(av)^2T_{a,b}(v)$$

$$[S_{a,b}[f'''(t)]] = \frac{-a^n e^{-bv}}{(av)^n} f''(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} f'(0) - \frac{a^n e^{-bv}}{(av)^{n-2}} f(0) + (av)^3 T_{a,b}(v)$$

$$[S_{a,b}[f''''(t)]] = \frac{-a^n e^{-bv}}{(av)^n} f'''(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} f''(0) - \frac{a^n e^{-bv}}{(av)^{n-2}} f'(0) - \frac{a^n e^{-bv}}{(av)^{n-3}} f(0) + (av)^4 T_{a,b}(v)$$

$$[S_{a,b}[f^m(t)]] = \frac{-a^n e^{-bv}}{(av)^n} f^{(m-1)}(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} f^{m-2}(0) - \frac{a^n e^{-bv}}{(av)^{n-2}} f^{(m-3)}(0) -$$

$$\ldots \frac{a^n e^{-bv}}{(av)^{n-m+1}} f(0) + (av)^m T_{a,b}(v)$$

Proof: SEE transform of two parameters when $f(t) = f'(t)$

Using $f(t) = f'(t)$ follows from the equation (1)

$$T_{(a,b)}\{f'(t)\} = \frac{a^n e^{-bv}}{(av)^n} \int_0^\infty f'(t).e^{-avt} dt \tag{14}$$

Integrating by parts

$$= \frac{a^n e^{-bv}}{(av)^n} \left[f(t) e^{-avt} \Big|_0^\infty + (av) \int_0^\infty e^{-avt} f(t) dt \right]$$

Apply limit

$$= -\frac{a^n e^{-bv}}{(av)^n} f(0) + avT_{(a,b)}(v) \tag{15}$$

SEE transform of two parameters when $f(t) = f''(t)$

Using $f(t) = f''(t)$ follows from the equation (1)

$$T_{(a,b)}\{f''(t)\} = \frac{a^n e^{-bv}}{(av)^n} \int_0^\infty f''(t).e^{-avt} dt$$

Integrating by parts

$$= \frac{a^n e^{-bv}}{(av)^n} \left[f'(t) e^{-avt} \Big|_0^\infty + (av) \int_0^\infty e^{-avt} f'(t) dt \right]$$

Applying limit

$$= -\frac{a^n e^{-bv}}{(av)^n} f'(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} f(0) + (av)^2 T_{(a,b)}(v) \tag{16}$$

SEE transform of two parameters when $f(t) = f'''(t)$

Using $f(t) = f'''(t)$ follows from the equation (1)

$$T_{(a,b)}\{f'''(t)\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty f'''(t).e^{-avt}dt$$

Integrating by parts

$$= \frac{a^n e^{-bv}}{(av)^n}\left[f''(t)e^{-avt}\Big|_0^\infty + (av)\int_0^\infty e^{-avt} f''(t)dt \right]$$

Again, integrating by parts

$$= \frac{a^n e^{-bv}}{(av)^n}\left[f''(t)e^{-avt}\Big|_0^\infty + (av)\left(f'(t)e^{-avt}\Big|_0^\infty + (av)\int_0^\infty e^{-avt} f'(t)dt \right)\right]$$

Applying limit

$$= -\frac{a^n e^{-bv}}{(av)^n} f''(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} f'(0) - \frac{a^n e^{-bv}}{(av)^{n-2}} f(0) + (av)^3 T_{(a,b)}(v) \qquad (17)$$

SEE transform of two parameters when $f(t) = f^{(m)}(t)$
From the equation (15), (16) and (17) we generalize

$$[S_{a,b}\{f^m(t)\}] = \frac{-a^n e^{-bv}}{(av)^n} f^{(m-1)}(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} f^{m-2}(0) - \frac{a^n e^{-bv}}{(av)^{n-2}} f^{(m-3)}(0) -$$

$$....\frac{a^n e^{-bv}}{(av)^{n-m+1}} f(0) + (av)^m T_{a,b}(v) \qquad (18)$$

Linearity property
Linearity property of two parametric extensions of SEE transformation define as

$$S_{a,b}\{af(t) + bg(t)\} = aS_{a,b}\{f(t)\} + bS_{a,b}\{g(t)\}$$

a and *b* are constant.

Change of scale property
Change of scale property of Two parametric extensions of SEE transformation if $S_{a,b}\{f(t)\} = T_{a,b}(v)$ then

$$S_{a,b}\{f(t)\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty f(t)e^{-(av)t}dt = T_{a,b}(v)$$

Then

$$S_{a,b}\{af(t)\} = \frac{1}{a^{n-1}} T_{a,b}\left(\frac{v}{a}\right)$$

Shifting Property
Two parametric extensions of SEE transformation of $f(t)$ is

$$S_{a,b}\{e^{ct} f(t)\} = \frac{a^n e^{-bv}}{(av)^n}\int_0^\infty e^{ct} f(t)e^{-vt}dt$$

$$= \frac{(v-c)^n}{(av)^n} T_{a,b}(v-c)$$

Convolution Theorem

If $S_{a,b}\{f(t)\} = T_{1(a,b)}(v)$ and $S_{a,b}\{H(t)\} = T_{2(a,b)}(v)$ then

$$S_{a,b}\{f(t) * H(t)\} = (av)^n S_{a,b}\{f(t)\} * S_{a,b}\{H(t)\}$$

$$= (av)^n T_{1(a,b)}(v).T_{2(a,b)}(v)$$

Inverse of Two parametric extensions of SEE transformation
The inverse of SEE Transform over a set of functions, defined is

$$S_{a,b}^{-1}\{T_{a,b}(v)\} = f(t) \tag{18}$$

Inverse of Two parametric extensions of SEE transformation some functions.
Inverse transform of function following from the equation (18) are

$$S_{a,b}^{-1}\left[\frac{a^n e^{-bv}}{(av)^{n+1}}\right] = 1$$

$$S_{a,b}^{-1}\left[\frac{a^n e^{-bv}}{(av)^{n+2}}\right] = t$$

$$S_{a,b}^{-1}\left[\frac{a^n e^{-bv}}{(av)^{n+m-1}}\right] = t^m$$

$$S_{a,b}^{-1}\left[\frac{a^n e^{-bv}}{(av)^n (av+u)}\right] = e^{-ut}$$

$$S_{a,b}^{-1}\left[\frac{ua^n e^{-bv}}{(av)^n ((av)^2 + u^2)}\right] = \sin(ut)$$

$$S_{a,b}^{-1}\left[\frac{a^n e^{-bv}}{(av)^{n-1} ((av)^2 + u^2)}\right] = \cos(ut)$$

$$S_{a,b}^{-1}\left[\frac{ua^n e^{-bv}}{(av)^n ((av)^2 - u^2)}\right] = \sinh(ut)$$

$$S_{a,b}^{-1}\left[\frac{a^n e^{-bv}}{(av)^{n-1} ((av)^2 - u^2)}\right] = \cosh(ut)$$

Applications
SEE integral Transform of Two Parameter of Ordinary Differential Equations.
1st order linear conventional differential equation

$$\frac{dx}{dty} + px = f(t), \text{ Where } t > 0$$

With condition $x(0) = a$

Where p and a are constants and (t) is an outside input work with the goal that its SEE integral transform of two parameter exists.

$$S_{a,b}\left\{\frac{dx}{dty}\right\} + S\{px\} = S\{f(t)\}, t > 0$$

By using equation (16)

$$-\frac{a^n e^{-bv} x(0)}{(av)^n} + a^n e^{-bv}(av)T(v) + pa^n e^{-bv}T(v)$$

$$T(v)[av + p] = \frac{a^n e^{-bv} x(0)}{(av)^n} + \overline{f}(v)$$

$$T(v) = \frac{\overline{f}(v)}{[av + p]} + \frac{a}{(av)^n(av + p)}$$

Taking inverse of SEE integral transform of two parameters change prompts the arrangement.

4.5.8 Theorem

The 2nd order request straight normal differential condition has the structure

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x), x > 0$$

Initial conditions are

$$y(0) = c, \frac{dy}{dx} = d$$

Constants are p, c and d

Proof

$$\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + qy = f(x), x > 0$$

Applying SEE integral transform of two parameters

$$S_{a,b}\left\{\frac{d^2y}{dx^2}\right\} + S_{a,b}\left\{2p\frac{dy}{dx}\right\} + S_{a,b}\{qy\} = S_{a,b}\{f(x)\}, x > 0$$

From equation (16)

$$-\frac{a^n e^{-bv}}{(av)^n} y'(0) - \frac{a^n e^{-bv}}{(av)^{n-1}} y(0) + (av)^2 T_{(a,b)}(v)$$

$$+2p\left[-\frac{a^n e^{-bv}}{(av)^n}y(0)+avT_{(a,b)}(v)\right]+qT_{(a,b)}(v)=\overline{f}(v)$$

Using initial conditions

$$-\frac{a^n e^{-bv}}{(av)^n}c-\frac{a^n e^{-bv}}{(av)^{n-1}}d+(av)^2T_{(a,b)}(v)$$

$$-2p\frac{a^n e^{-bv}}{(av)^n}d+2p(av)T_{(a,b)}(v)+qT_{(a,b)}(v)=\overline{f}(v)$$

$$T_{(a,b)}(v)[2p(av)+(av)^2+q]=\overline{f}(v)+\frac{a^n e^{-bv}}{(av)^n}(d+2p(av))+\frac{a^n e^{-bv}}{(av)^{n-1}}c$$

$$T_{(a,b)}(v)=\frac{\overline{f}(v)}{[2p(av)+(av)^2+q]}+\frac{c.a^n e^{-bv}}{(av)^{n-1}[2p(av)+(av)^2+q]}$$

$$+\frac{a^n e^{-bv}}{(av)^n[2p(av)+(av)^2+q]}(d+2p(av))$$

Taking inverse of SEE transform of two parameters gives the arrangements.

Conclusion

In this research, two parametric SEE transformation presented which is quite important and efficient in solving differential equation and general form which can be turned into ordinary form by fixing the parameters.

References

1. Aggarwal S, Sharma N. Laplace transform for the solution of first kind linear Volterra integral equation. Journal of Advanced Research in Applied Mathematics and Statistics. 2019;4(3&4):16-23.
2. Dar AA, Ahmed A, Reshi JA. Transmuted weighted exponential distribution and its application. Journal of Statistics Applications & Probability. 2017;6(1):219-32.
3. Davis GB. A Laplace transform technique for the analytical solution of a diffusion-convection equation over a finite domain. Applied mathematical modelling. 1985;9(1):69-71.
4. Abdelilah K, Sedeeg LH. The New Integral Transform 'Kamal Transform'. Advances in Theoretical and Applied Mathematics. 2016;11:451-458.
5. Ghanwat AJ, Gaikwad SB. Two Dimensional Kamal Transform of Space of Distribution of Bohemians. International Journal of Engineering Research & Technology. 2020;9(4):451-458.
6. Aggarwal S, Singh GP. Kamal transform of error function. Journal of Applied Science and Computations. 2019;6(5):2223-35.
7. Aggarwal S, Sharma SD. Sumudu transform of error function. Journal of Applied Science and Computations. 2019;6(6):1222-1231.
8. Aibinu MO, Thakur SC, Moyo S. Analyzing population dynamics models via Sumudu Transform. Mathematics Subject Classification. 2021;6:8877-8888.
9. Sharma K, Kaith BS, Kumar V, Kalia S, Kumar V, Swart HC. Water retention and dye adsorption behavior of Gg-cl-poly (acrylic acid-aniline) based conductive hydrogels. Geoderma. 2014;232:45-55.