Variational homotopy perturbation study of the dengue disease model with incubation period

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Article Info

ISSN (online): 2582-7138
Volume: 03
Issue: 02
March-April 2022
Received: 04-03-2022;
Accepted: 20-03-2022
Page No: 296-305

Abstract
In this research article, the variational Homotopy perturbation method is proposed to solve the governing nonlinear differential equations describing the mathematical model of Dengue virus with incubation period. The equations were first written using the variational iteration method with the Lagrange multiplier based on the variational theory determined for each of the equations before applying the Homotopy perturbation method. Successive iterative solutions are then obtained upon equating the powers of both sides of the perturbation parameter. Using the limiting condition when the perturbation parameter tends to unity, the three-term approximation is then obtained for all model parameters. The result shows, the proposed method is reliable, efficient, convergent, and computationally convenient.

Keywords: Dengue Virus, Variational Iteration method (VIM), Homotopy Perturbation method (HPM), Variational Homotopy perturbation method (VHPM), Incubation period

1. Introduction

Dengue fever is a viral, vector-borne, and mosquito-transmitted disease that is spread by the Aedes Aegyptus mosquitoes. There are four variants of this disease denoted Den-1, Den-2, Den-3 and Den-4. Estimates show over 50millions or more have been infected in well over 100 countries and still counting. Copious studies of this disease reveal, infants, children, and the elderly are the most affected in sub-Saharan Africa, sub-tropical regions of the world, the eastern Mediterranean and the American. There are no known treatments for the Dengue disease. Nevertheless, supportive care in the form of bed rest and mild analgesic can ameliorate the effect of the disease, Gubler, Halstead, Naowarat and Rajabhat [1-3].

Numerous studies on this disease show that each of the serotypes of this disease contain different strains and there is no cross immunity to any of the strain except life-long immunity to any of the strain in which one is infected with. Diagnosis of the disease indicates the incubation period of the disease for an infected person is approximately (3-14) days, Pongsumpun, Rafiq and Ahmad, Rafiq & Raza, Garba & Gummel, Nuraini *et al*., Okuonghae & Omosigho, Driessche & Warmouth [4-10]. At the expiry of the incubation period, a sudden onset of fever will emerge which is an indication of viraemia which is usually detected using mosquito inoculation technique, Boutaveb *et al*., Lourdes and Cristobal [11-12]. The disease has a short lifespan and clears up quickly in the body of the infected person after seven days after the onset of fever for those with strong immune response. However, for those with weak immunity, the virus is expected to last longer, Pongsumpun, Tasman & Nuraini, Yaacob, WHO [13-16]. Equally, the virus is extremely deadly in that mosquito become susceptible whenever they have contact with an infected person. Statistics show that more than 2.5billion people across the globe have been affected with this disease and this staggering number keep rising due to increase in population and dearth of essential amenities for the teeming populace, Derouich & Boutaveb, Soewono & Supriatna, Andrawus & Eguda [17-19].

The variational Homotopy perturbation method (VHPM) is a hybrid semi-analytical method which coupled the variational iteration and Homotopy perturbation methods This method requires first taking the variational iteration method of the given system under consideration to give an iterative system. Secondly, applying the Homotopy perturbation to the resulting system and equating corresponding powers of the perturbation parameter, give different set of equations of the unknowns.

Solving this equation give the solution in approximate form. This method though nascent has been extensively applied to great effect in solving several problems in both science and engineering. Ebiwareme and Kormane [20] have employed VHPM to solve analytically the nonlinear equations governing MHD Jeffery-Hamel flow in the presence of magnetic field. The study showed that that the velocity profile is significantly affected for both the convergent/divergent channel for different values of the Reynold number, magnetic field, and angle of inclination. Malinfar and Mahdavi [21] have examined the application of the variational Homotopy perturbation method on the generalized Fisher's equation. Allahviranloo *et al.* [22] have investigated variational Homotopy perturbation method as a efficient iterative scheme in solving partial differential equations in fluid mechanics. The approximate solution of the foam drainage equation with time and space fractional derivative have been studied by Bouhassoun *et al.* [23] Fredholm integrodifferential equations of fractional order have been analysed using combined variational iteration and Homotopy perturbation methods, Kilicman *et al.* [24]. Linear and nonlinear heat transfer equations in engineering have been investigated using VHPM, Ghorbali *et al.* [25]. Yangqin [26] used VHPM to solve fractional initial boundary value problems. Mohyud-in and Mohammed [27] have examined higher order dimensional initial boundary value problems utilizing VHPM. Ji-Huan [28] used the coupled Variational iteration and Homotopy perturbation techniques for solving nonlinear problems.

In this present study, the motivation is to extend the novelty of the application of the variational Homotopy perturbation method (VHPM) to study the dynamics of the state parameters. This has not been studied before to the best of our knowledge. The study is organized as follows. Section 2 gives the detailed mathematical model governing the Dengue virus, section 3-4 presents the fundamentals of the semi-analytical variational iteration and Homotopy methods, the coupling of the variational Homotopy perturbation methods is given in section 5. Mathematical analysis of the model with a view to seeking explicit analytical solution to the state variables is presented in section 6 whereas, the section 7 gives the results of the model in tables and graphs for various values of the model parameters with respect to time.

2. Mathematical Model of Dengue Fever

We examined the dynamics of the mathematical model of the Dengue virus [10]. It consists of seven-components of the state variable which includes. The model parameters, N_1 denotes human population size, N_2 represents size of vector population, λ is the birth rate of human population, μ_1 is the death rate of human population, α_1 represents rate of change of infected into infectious human populations, β_1 denotes rate of infectious Dengue virus from vector to human population, *R* (human population recovery rate), μ_2 (death rate of vector population), β_2 (rate of infectious Dengue disease virus from human population to vector population), α_2 (rate of change of infected vector to infectious vector population), and *C* (constant recruitment rate of vector population). The equations governing the transmission dynamics of Dengue disease are given as follows.

$$\begin{cases} \frac{dS}{dt} = \lambda N_1 - \beta_1 SW - \mu_1 S \\ \frac{dX}{dt} = \beta_1 SW - (\alpha_1 + \mu_1)X \\ \frac{dI}{dt} = \alpha_1 X - (r + \mu_1)I \\ \frac{dR}{dt} = rI - \mu_1 R \\ \frac{dU}{dt} = C - \beta_2 IU - \mu_2 U \\ \frac{dV}{dt} = \beta_2 IU - (\alpha_2 + \mu_2)V \\ \frac{dW}{dt} = \alpha_2 V - \mu_2 W \end{cases} \tag{1}$$

Subject to the initial conditions,

$$S(0) = 0, X(0) = 0.0001, I(0) = 0.0001, R(0) = 1, U(0) = 1, V(0) = 0.001, W(0) = 1. \tag{2}$$

To proceed with the analysis, we set the following values for the model parameters throughout this paper. $N_1 = 150, \alpha_1 = 0.2, \beta_1 = 0.025, \mu_1 = 0.00392, \mu_2 = 0.1, \beta_2 = 0.0018, \mu_2 = 0.0039, \lambda = 2, \gamma = 0.25, C = 3, \alpha_2 = 0.2$.

3. Variational Iteration Method (VIM)

The basic idea of the VIM is as follows

Consider the ordinary differential equation of the form

$$Ly + N(y) = f(x), x \in I \tag{3}$$

Where L and N are linear and nonlinear operators respectively, and $f(x)$ is any given inhomogeneous terms defined for $x \in I$

We defined a correctional functional for Eq. (3) as follows.

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda(\tau)\left(Ly_n(\tau) + N(\tilde{y}_n(\tau)) - f(\tau)\right) d\tau \tag{4}$$

Where $\lambda(\tau)$a Lagrange multiplier is obtained through variational theory, $y_n(x)$ is the nth approximation of $y(x)$ and $\tilde{y}_n(x)$ is a restricted variation meaning $\delta\tilde{y}_n(x) = 0$

By imposing the variation of both sides of Eq. (4) and taking the restricted variation we obtained

$$\delta y_{n+1}(x) = \delta y_n(x) + \delta(\int_0^x \lambda(\tau) L y_n(\tau) d\tau \qquad (5)$$

$$\delta y_{n+1}(x) = \delta y_n(x) + \left[\lambda(\tau)(\int_0^\tau L y_n(\xi) d\xi\right]_{\tau=0}^{\tau=x} - \int_0^x \lambda'(\tau)\big(\int_0^\tau L\delta y_n(\xi)\big) d\xi \qquad (6)$$

Now by applying the stationary condition, the value of the Lagrange multiplier, $\lambda(\tau)$ can be found. Then the successive approximations, $y_n(x), n = 0,1,2,3$ .... Can be found in the form

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda(\tau)\left(L y_n(\tau) + N\big(y_n(\tau)\big) - f(\tau)\right) d\tau \qquad (7)$$

The exact solution is then obtained as the limit of the successive approximations from Eq. (7)

$$y(x) = \lim_{n\to\infty} y_n(x) \qquad (8)$$

4. Homotopy Perturbation Method (HPM)

In this section, the fundamentals of the Homotopy perturbation method as proposed by He. J. Huan is discussed
Consider a functional differential equation of the form

$$\mathcal{A}(u) - f(r) = 0, r \in \Omega \qquad (9)$$

Subject to the boundary condition

$$\mathcal{B}\left(u, \frac{\partial u}{\partial t}\right) = 0, r \in \mathcal{T} \qquad (10)$$

Where $\mathcal{A}$ is a differential operator, $\mathcal{B}$ is boundary operator, $\mathcal{T}$ is the boundary of the domain Ω, $f(x,t)$ is a known analytic function and $u(x,t)$ is the unknown function
Decomposing the operator, $\mathcal{A}$ into two parts comprising linear, $(\mathcal{L})$ and nonlinear $(\mathcal{N})$

$$\mathcal{A} = \mathcal{L} + \mathcal{N} \qquad (11)$$

In view of Eq. (12), we rewrite Eq. (9) in the form

$$\mathcal{L}(u) + \mathcal{N}(u) - f(r) = 0 \qquad (12)$$

Embedding an artificial parameter p on Eq. 16) as follows

$$\mathcal{L}(u) + p(\mathcal{N}(u) - f(r)) = 0 \qquad (13)$$

Where $p \in [0,1]$ is the embedding or artificial parameter.

Next, we construct a Homotopy, $\mathcal{H}(r,p): \Omega \times [0,1] \longrightarrow \Re$ to Eq. (13) that satisfies

$$\mathcal{H}(r,p) = (1-p)[\mathcal{L}(v) - \mathcal{L}(u_0)] + p[\mathcal{L}(v) + \mathcal{N}(v) - f(r)] = 0 \qquad (14)$$

And

$$\mathcal{H}(r,p) = \mathcal{L}(v) - \mathcal{L}(u_0) + p\mathcal{L}(u_0) + p[\mathcal{N}(v) - f(r)] = 0 \qquad (15)$$

Where $u_0(x)$ is the initial approximation which satisfies the boundary condition.
Putting $p = 0$ and $p = 1$ into Eq. (15), we obtain the following equations

$$\left.\begin{array}{l} \mathcal{H}(r,0) = \mathcal{L}(v) - \mathcal{L}(u_0) \\ \mathcal{H}(r,1) = \mathcal{A}(u) - f(r) \end{array}\right\} \qquad (16)$$

Clearly as p changes monotonically from zero to unity, $\mathcal{H}(r,p)$ changes from $u_0(x)$ to $u(x)$. This is called deformation, whereas the terms $\mathcal{L}(v) - \mathcal{L}(u_0)$ and $\mathcal{A}(u) - f(r)$ are homotopic to each other.
Now we consider a power series solution in p as follows

$$v = \sum_{n=0}^{\infty} p^{(n)} v_n \qquad (17)$$

The approximate solution of Eq. (17) can be obtained by setting $p = 1$

$$u(x) = \lim_{p\to 1} v_n = v_0 + v_1 + v_2 + \cdots \tag{18}$$

Similarly, the nonlinear term, $\mathcal{N}(u)$ can be expressed in He's polynomial

$$\mathcal{N}(u) = \sum_{n=0}^{\infty} p^{(m)} H_m(v_0 + v_1 + \cdots + v_m) \tag{19}$$

Where $H_m(v_0 + v_1 + \cdots + v_m) = \frac{1}{m!}\frac{\partial^m}{\partial p^m}[\mathcal{N}(\sum_{k=0}^{m} p^k v_k)]_{p=0}, m = 0,1,2,\ldots$ (20)

Where,

$$\begin{aligned}
H_0 &= \mathcal{N}(u_0)\\
H_1 &= u_1\mathcal{N}'(u_0)\\
H_2 &= u_2\mathcal{N}'(u_0) + \frac{1}{2}\mathcal{N}_1^2\mathcal{N}''(u_0)\\
H_3 &= u_3\mathcal{N}'(u_0) + u_1u_2\mathcal{N}''(u_0) + \frac{1}{6}\mathcal{N}_1^3\mathcal{N}'''(u_0)\\
H_4 &= u_4\mathcal{N}'(u_0) + (\frac{1}{2}u_2^2 + u_1u_3)\mathcal{N}''(u_0) + \frac{1}{2}u_1^2u_2\mathcal{N}_1^3\mathcal{N}'''(u_0) + \frac{1}{24}u_4^3\mathcal{N}^{(iv)}(u_0)
\end{aligned} \tag{21}$$

5. Variational Homotopy Perturbation Method (VHPM)

Consider a functional equation of the form

$$\mathcal{L}(u) + \mathcal{N}(u) - f(r) = 0$$

According to VIM, we construct a correction functional of Eq. (7) as follows

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(\xi)[Lu_n(\xi) + Nu_n(\xi) - f(\xi)]d\xi \tag{22}$$

Where λ is the Lagrange multiplier, L is the integral or differential operator, N is the nonlinear operator and $f(r)$ is the inhomogeneous term.
Now applying the VHPM to the above expression, we have

$$\sum_{n=0}^{\infty} p^{(n)}u_n = u_0(x) + p\int_0^x \lambda(\xi)[\sum_{n=0}^{\infty} p^{(n)} L(u_n(\xi)) + \sum_{n=0}^{\infty} p^{(n)}N(u_n(\xi)) - f(\xi)]d\xi \tag{23}$$

Equating the coefficients of the powers of p, the different iterates of the problem are obtained. Using the relation,

$$u(x) = \lim_{p\to 1} u_n(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$$

6. Mathematical Analysis via VHPM

The correctional functional for the nonlinear system in terms of the model parameters is given by

$$S_{n+1}(t) = S_n(t) + \int_0^t \lambda_1(\xi)\left[\frac{dS_n(\xi)}{dt} - \lambda N_1 + \beta_1 S_n(\xi)W_n(\xi) - \mu_1 S_n(\xi)\right]d\xi \tag{24}$$

$$X_{n+1}(t) = X_n(t) + \int_0^t \lambda_2(\xi)\left[\frac{dX_n(\xi)}{dt} - \beta_1 S_n(\xi)W_n(\xi) + (\alpha_1 + \mu_1)X_n(\xi)\right]d\xi \tag{25}$$

$$I_{n+1}(t) = I_n(t) + \int_0^t \lambda_3(\xi)\left[\frac{dI_n(\xi)}{dt} - \alpha_1 X_n(\xi) + (\gamma + \mu_1)I_n(\xi)\right]d\xi \tag{26}$$

$$R_{n+1}(t) = R_n(t) + \int_0^t \lambda_4(\xi)\left[\frac{dR_n(\xi)}{dt} - \gamma I_n(\xi) + \mu_1 R_n(\xi)\right]d\xi \tag{27}$$

$$U_{n+1}(t) = U_n(t) + \int_0^t \lambda_5(\xi)\left[\frac{dU_n(\xi)}{dt} - C + \beta_2 I_n(\xi)U_n(\xi) + \mu_2 U_n(\xi)\right]d\xi \tag{28}$$

$$V_{n+1}(t) = V_n(t) + \int_0^t \lambda_6(\xi)\left[\frac{dV_n(\xi)}{dt} - \beta_2 I_n(\xi)U_n(\xi) + (\alpha_2 + \mu_2)V_n(\xi)\right]d\xi \tag{29}$$

$$W_{n+1}(t) = W_n(t) + \int_0^t \lambda_7(\xi)\left[\frac{dW_n(\xi)}{dt} - \alpha_2 V_n(\xi) + \mu_2 W_n(\xi)\right]d\xi \tag{30}$$

Using the variational theory and making the integral stationary, we have the Lagrange multiplier as

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = -1.$$

Substitution into the above system, we have the equivalent expression as follows

$$S_{n+1}(t) = S_n(t) - \int_0^t \left[\frac{dS_n(\xi)}{dt} - \lambda N_1 + \beta_1 S_n(\xi)W_n(\xi) - \mu_1 S_n(\xi)\right]d\xi \tag{31}$$

$$X_{n+1}(t) = X_n(t) - \int_0^t \left[\frac{dX_n(\xi)}{dt} - \beta_1 S_n(\xi)W_n(\xi) + (\alpha_1 + \mu_1)X_n(\xi)\right]d\xi \tag{32}$$

$$I_{n+1}(t) = I_n(t) - \int_0^t \left[\frac{dI_n(\xi)}{dt} - \alpha_1 X_n(\xi) + (\gamma + \mu_1) I_n(\xi)\right] d\xi \tag{33}$$

$$R_{n+1}(t) = R_n(t) - \int_0^t \left[\frac{dR_n(\xi)}{dt} - \gamma I_n(\xi) + \mu_1 R_n(\xi)\right] d\xi \tag{34}$$

$$U_{n+1}(t) = U_n(t) - \int_0^t \left[\frac{dU_n(\xi)}{dt} - C + \beta_2 I_n(\xi) U_n(\xi) + \mu_2 U_n(\xi)\right] d\xi \tag{35}$$

$$V_{n+1}(t) = V_n(t) - \int_0^t \left[\frac{dV_n(\xi)}{dt} - \beta_2 I_n(\xi) U_n(\xi) + (\alpha_2 + \mu_2) V_n(\xi)\right] d\xi \tag{36}$$

$$W_{n+1}(t) = W_n(t) - \int_0^t \left[\frac{dW_n(\xi)}{dt} - \alpha_2 V_n(\xi) + \mu_2 W_n(\xi)\right] d\xi \tag{37}$$

Next, we form the Homotopy perturbation method to the above system in Eqs. (31)−(37) where the perturbation parameter, $p \epsilon [0,1]$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = S_0(t) - \int_0^t \left[\frac{dS_n(\xi)}{dt} - \lambda N_1 + \beta_1 \sum_{n=0}^{\infty} p^{(n)} A_n - \mu_1 S_n(\xi)\right] d\xi \tag{38}$$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = X_0(t) - \int_0^t \left[\frac{dX_n(\xi)}{dt} - \beta_1 \sum_{n=0}^{\infty} p^{(n)} A_n + (\alpha_1 + \mu_1) X_n(\xi)\right] d\xi \tag{39}$$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = I_0(t) - \int_0^t \left[\frac{dI_n(\xi)}{dt} - \alpha_1 X_n(\xi) + (\gamma + \mu_1) I_n(\xi)\right] d\xi \tag{40}$$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = R_0(t) - \int_0^t \left[\frac{dR_n(\xi)}{dt} - \gamma I_n(\xi) + \mu_1 R_n(\xi)\right] d\xi \tag{41}$$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = U_0(t) - \int_0^t \left[\frac{dU_n(\xi)}{dt} - C + \beta_2 \sum_{n=0}^{\infty} p^{(n)} B_n + \mu_2 U_n(\xi)\right] d\xi \tag{42}$$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = V_0(t) - \int_0^t \left[\frac{dV_n(\xi)}{dt} - \beta_2 \sum_{n=0}^{\infty} p^{(n)} B_n + (\alpha_2 + \mu_2) V_n(\xi)\right] d\xi \tag{43}$$

$$\sum_{n=0}^{\infty} p^{(n)} u_n = W_0(t) - \int_0^t \left[\frac{dW_n(\xi)}{dt} - \alpha_2 V_n(\xi) + \mu_2 W_n(\xi)\right] d\xi \tag{44}$$

Where the nonlinear terms are given by

$$A = SW, B = IU \tag{45}$$

The first seven terms of these nonlinear terms are given by the following algorithms

$$\begin{aligned}
A_0 &= S_0 W_0 \\
A_1 &= S_0 W_1 + S_1 W_0 \\
A_2 &= S_0 W_2 + S_1 W_1 + S_2 W_0 \\
A_3 &= S_0 W_3 + S_1 W_2 + S_2 W_1 + S_3 W_0 \\
A_4 &= S_0 W_4 + S_1 W_3 + S_2 W_2 + S_3 W_1 + S_4 W_0 \\
A_5 &= S_0 W_5 + S_1 W_4 + S_2 W_3 + S_3 W_2 + S_4 W_1 + S_5 W_0 \\
A_6 &= S_0 W_6 + S_1 W_5 + S_2 W_4 + S_3 W_3 + S_4 W_2 + S_5 W_1 + S_6 W_0 \\
A_7 &= S_0 W_7 + S_1 W_6 + S_2 W_5 + S_3 W_4 + S_4 W_3 + S_5 W_2 + S_6 W_1 + S_7 W_0
\end{aligned} \tag{46}$$

$$\begin{aligned}
B_0 &= 1_0 U_0 \\
B_1 &= I_0 U_1 + I_1 U_0 \\
B_2 &= I_0 U_2 + I_1 U_1 + I_2 U_0 \\
B_3 &= I_0 U_3 + I_1 U_2 + I_2 U_1 + I_3 U_0 \\
B_4 &= I_0 U_4 + I_1 U_3 + I_2 U_2 + I_3 U_1 + I_4 U_0 \\
B_5 &= I_0 U_5 + I_1 U_4 + I_2 U_3 + I_3 U_2 + I_4 U_1 + I_5 U_0 \\
B_6 &= I_0 U_6 + I_1 U_5 + I_2 U_4 + I_3 U_3 + I_4 U_2 + I_5 U_1 + I_6 U_0 \\
B_7 &= I_0 U_7 + I_1 U_6 + I_2 U_5 + I_3 U_4 + I_4 U_3 + I_5 U_2 + I_6 U_1 + I_7 U_0
\end{aligned} \tag{47}$$

Equating the corresponding terms of the powers of p, we have the following equations

$$\begin{aligned}
&p^{(0)}: \quad u_{1,0} = S_0(t) = 0. \\
&u_{2,0} = X_0(t) = 0.0001 \\
&u_{3,0} = I_0(t) = 0.0001 \\
&u_{4,0} = R_0(t) = 1 \\
&u_{5,0} = U_0(t) = 1 \\
&u_{6,0} = V_0(t) = 0.001 \\
&u_{7,0} = W_0(t) = 1
\end{aligned} \tag{48}$$

$$p^{(1)}: u_{1,1}(t) = \int_0^t \left[-\frac{dS_0(\xi)}{dt} + \lambda N_1 - \beta_1 A_0 - \mu_1 S_0(\xi)\right] d\xi$$

$$u_{2,1}(t) = \int_0^t \left[-\frac{dX_0(\xi)}{dt} + \beta_1 A_0 - (\alpha_1 + \mu_1) X_0(\xi)\right] d\xi$$

$$u_{3,1}(t) = \int_0^t \left[-\frac{dI_0(\xi)}{dt} + \alpha_1 X_0(\xi) - (\gamma + \mu_1) I_0(\xi)\right] d\xi$$

$$
\begin{aligned}
u_{4,1}(t) &= \int_0^t \left[-\frac{dR_0(\xi)}{dt} + \gamma I_0(\xi) - \mu_1 R_0(\xi)\right] d\xi \\
u_{5,1}(t) &= \int_0^t \left[-\frac{dU_0(\xi)}{dt} + C - \beta_2 B_0 - \mu_2 U_0(\xi)\right] d\xi \\
u_{6,1}(t) &= \int_0^t \left[-\frac{dV_0(\xi)}{dt} + \beta_2 B_0 - (\alpha_2 + \mu_2) V_0(\xi)\right] d\xi \\
u_{7,1}(t) &= \int_0^t \left[-\frac{dW_0(\xi)}{dt} + \alpha_2 V_0(\xi) - \mu_2 W_0(\xi)\right] d\xi
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
p^{(2)}: u_{1,2}(t) &= \int_0^t \left[-\frac{dS_1(\xi)}{dt} + \lambda N_1 - \beta_1 A_1 - \mu_1 S_1(\xi)\right] d\xi \\
u_{2,2}(t) &= \int_0^t \left[-\frac{dX_1(\xi)}{dt} + \beta_1 A_1 - (\alpha_1 + \mu_1) X_1(\xi)\right] d\xi \\
u_{3,2}(t) &= \int_0^t \left[-\frac{dI_1(\xi)}{dt} + \alpha_1 X_1(\xi) - (\gamma + \mu_1) I_1(\xi)\right] d\xi \\
u_{4,2}(t) &= \int_0^t \left[-\frac{dR_1(\xi)}{dt} + \gamma I_1(\xi) - \mu_1 R_1(\xi)\right] d\xi \\
u_{5,2}(t) &= \int_0^t \left[-\frac{dU_1(\xi)}{dt} + C - \beta_2 B_1 - \mu_2 U_1(\xi)\right] d\xi \\
u_{6,2}(t) &= \int_0^t \left[-\frac{dV_1(\xi)}{dt} + \beta_2 B_1 - (\alpha_2 + \mu_2) V_1(\xi)\right] d\xi \\
u_{7,2}(t) &= \int_0^t \left[-\frac{dW_1(\xi)}{dt} + \alpha_2 V_1(\xi) - \mu_2 W_1(\xi)\right] d\xi
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
p^{(3)}: u_{1,3}(t) &= \int_0^t \left[-\frac{dS_2(\xi)}{dt} + \lambda N_1 - \beta_1 A_2 - \mu_1 S_2(\xi)\right] d\xi \\
u_{2,3}(t) &= \int_0^t \left[-\frac{dX_2(\xi)}{dt} + \beta_1 A_1 - (\alpha_1 + \mu_1) X_2(\xi)\right] d\xi \\
u_{3,3}(t) &= \int_0^t \left[-\frac{dI_2(\xi)}{dt} + \alpha_1 X_2(\xi) - (\gamma + \mu_1) I_2(\xi)\right] d\xi \\
u_{4,3}(t) &= \int_0^t \left[-\frac{dR_2(\xi)}{dt} + \gamma I_2(\xi) - \mu_1 R_2(\xi)\right] d\xi \\
u_{5,3}(t) &= \int_0^t \left[-\frac{dU_2(\xi)}{dt} + C - \beta_2 B_1 - \mu_2 U_2(\xi)\right] d\xi \\
u_{6,3}(t) &= \int_0^t \left[-\frac{dV_2(\xi)}{dt} + \beta_2 B_1 - (\alpha_2 + \mu_2) V_2(\xi)\right] d\xi \\
u_{7,3}(t) &= \int_0^t \left[-\frac{dW_2(\xi)}{dt} + \alpha_2 V_2(\xi) - \mu_2 W_2(\xi)\right] d\xi
\end{aligned} \tag{51}
$$

Continuing the same way, subsequent iterates of the model parameters can be obtained via integration. The three-term approximate solution for the parameters of interest are obtained using the limiting series

$$
\left.
\begin{aligned}
S(t) &= \lim_{p\to 1} p^{(n)} u_n = S_0(t) + S_1(t) + S_2(t) + \cdots \\
X(t) &= \lim_{p\to 1} p^{(n)} u_n = X_0(t) + X_1(t) + X_2(t) + \cdots \\
I(t) &= \lim_{p\to 1} p^{(n)} u_n = I_0(t) + I_1(t) + I_2(t) + \cdots \\
R(t) &= \lim_{p\to 1} p^{(n)} u_n = R_0(t) + R_1(t) + R_2(t) + \cdots \\
U(t) &= \lim_{p\to 1} p^{(n)} u_n = U_0(t) + U_1(t) + U_2(t) + \cdots \\
V(t) &= \lim_{p\to 1} p^{(n)} u_n = V_0(t) + V_1(t) + V_2(t) + \cdots \\
W(t) &= \lim_{p\to 1} p^{(n)} u_n = W_0(t) + W_1(t) + W_2(t) + \cdots
\end{aligned}
\right] \tag{52}
$$

$$
\left.
\begin{aligned}
S(t) &= u_{1,0}(t) + u_{1,1}(t) + u_{1,2}(t) + u_{1,3}(t) + \cdots \\
X(t) &= u_{2,0}(t) + u_{2,1}(t) + u_{2,2}(t) + u_{2,3}(t) + \cdots \\
I(t) &= u_{3,0}(t) + u_{3,1}(t) + u_{3,2}(t) + u_{3,3}(t) + \cdots \\
R(t) &= u_{4,0}(t) + u_{4,1}(t) + u_{4,2}(t) + u_{4,3}(t) + \cdots \\
U(t) &= u_{5,0}(t) + u_{5,1}(t) + u_{5,2}(t) + u_{5,3}(t) + \cdots \\
V(t) &= u_{6,0}(t) + u_{6,1}(t) + u_{6,2}(t) + u_{6,3}(t) + \cdots \\
W(t) &= u_{7,0}(t) + u_{7,1}(t) + u_{7,2}(t) + u_{7,3}(t) + \cdots
\end{aligned}
\right] \tag{53}
$$

Plugging the constant values of the model parameters, we have the expression of the state variables of the model.

$$
\begin{aligned}
&S(t) = 0.1 + 599.999583t - 14.9999672t^2 + 23.91098691t^3 \\
&X(t) = 0.01 - 0.1875392t + 0.056622646t^2 - 0.007275123t^3 \\
&I(t) = 0.01 - 0.2480392t + 0.137993997t^2 - 0.1154231003t^3 \\
&R(t) = 1 - 0.00142t - 0.030949236t^2 - 0.1012347891652t^3
\end{aligned} \tag{54}
$$

$$U(t) = 1 + 5.90037627t - 0.1449991t^2 + 0.2056291260988t^3$$
$$V(t) = 1 - 0.101392471t + 0.100196947t^2 - 0.01200672912t^3$$
$$W(t) = 0.01 + 0.000961t - 0.010101694t^2 - 0.0023671259621t^3$$

7. Results and Discussion

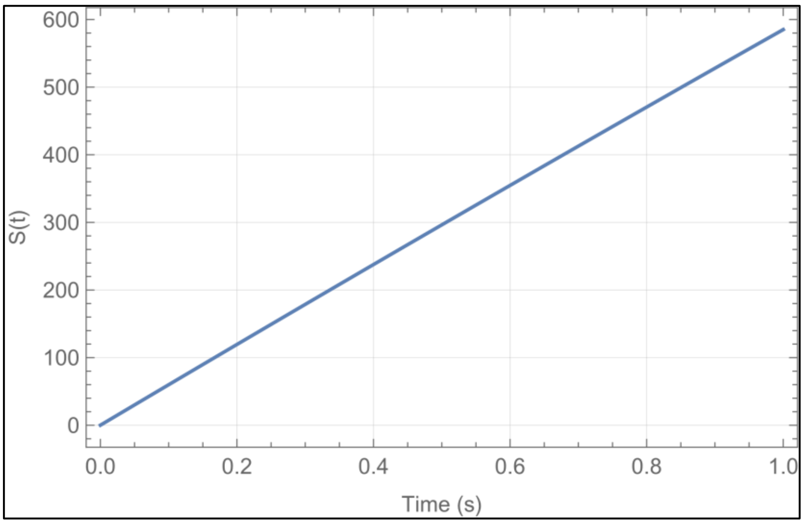

Fig 1: Variation of susceptible population over time for chosen values of the parameters

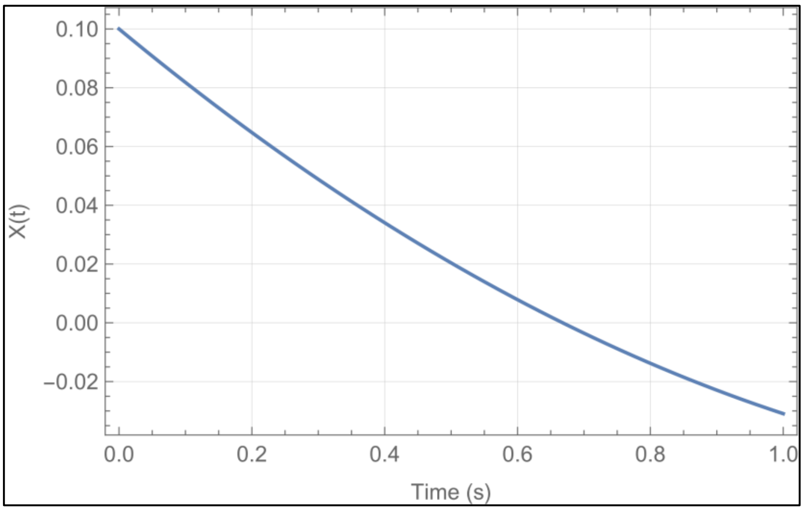

Fig 2: Variation of Infected population over time for different values of the chosen parameters

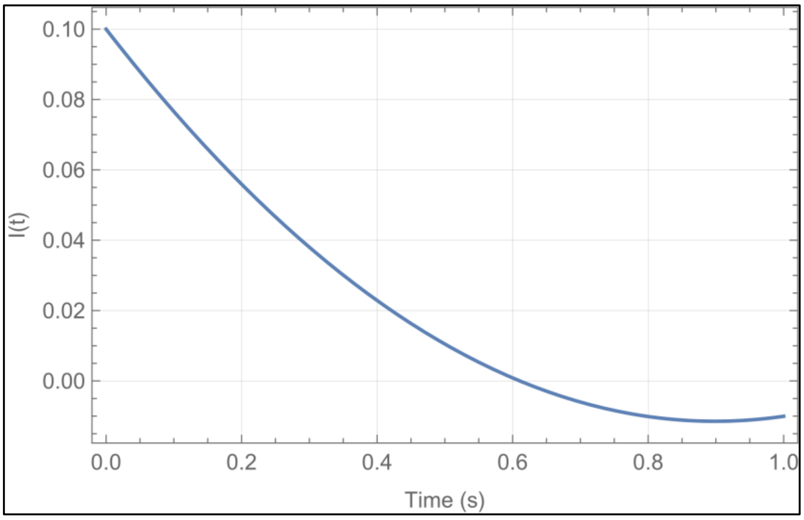

Fig 3: Variation of Infectious population over time for constant values of the chosen parameters

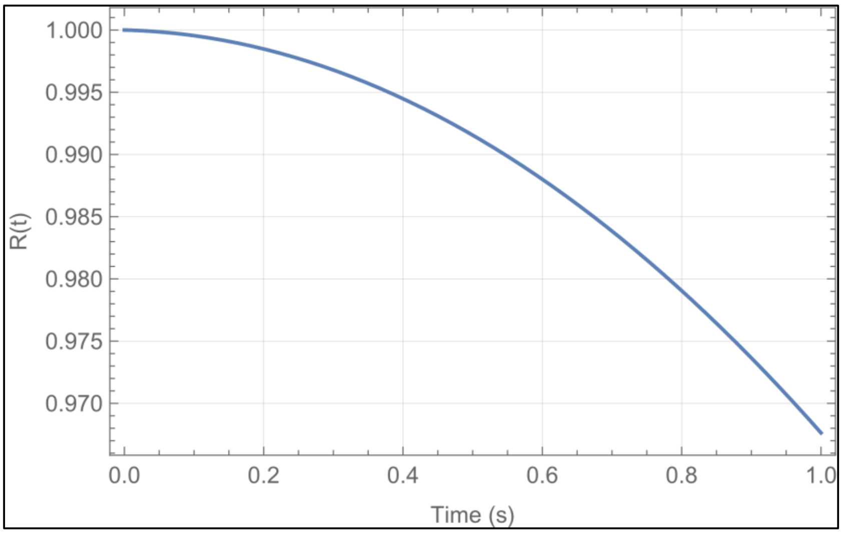

Fig 4: Variation of Recovered population over time for various values of the chosen parameters

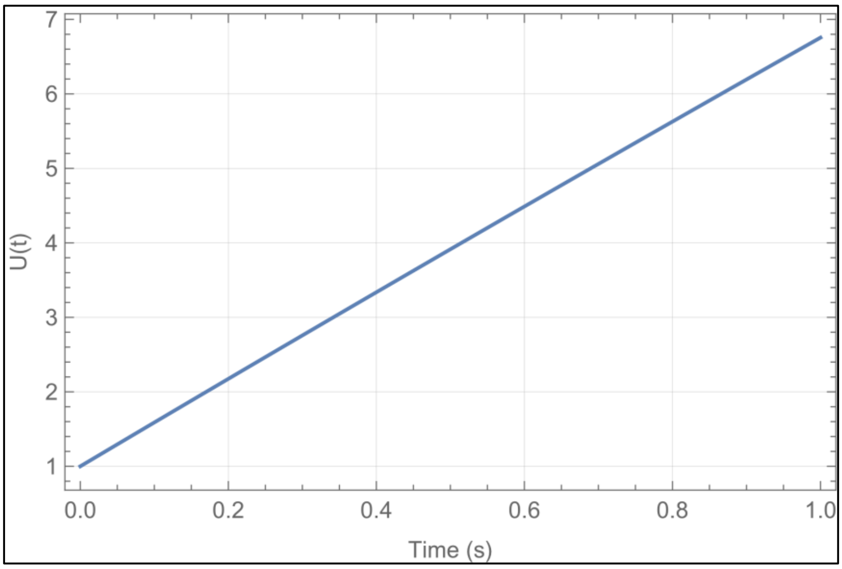

Fig 5: Variation of Susceptible Mosquitoes over time for different values of the chosen parameters

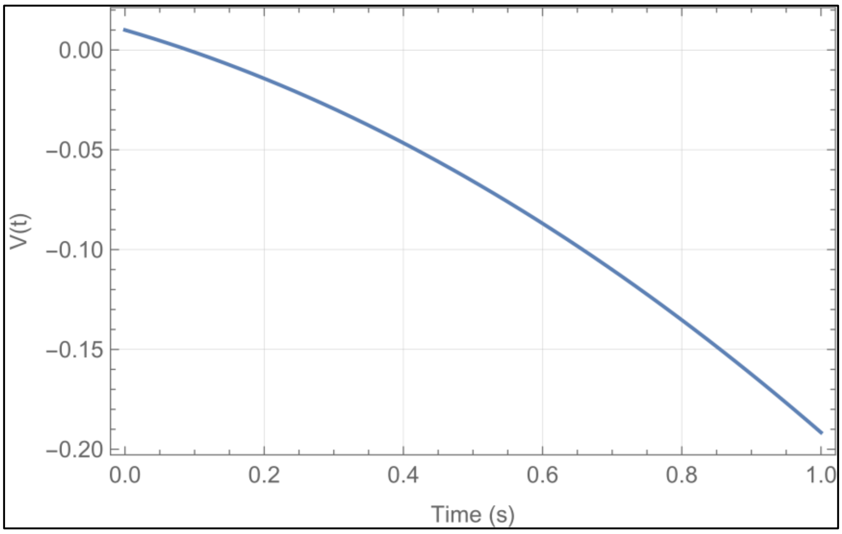

Fig 6: Variation of Infected Mosquitoes over time for various values of the chosen parameters

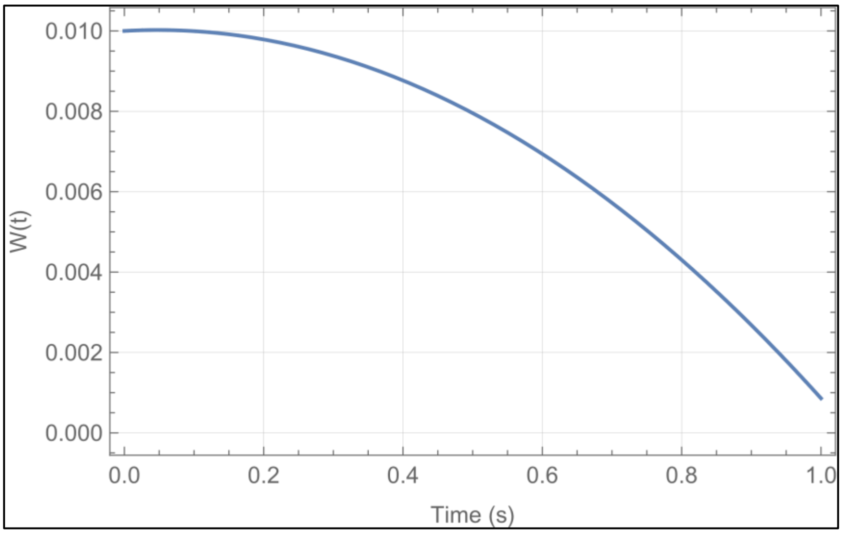

Fig 7: Variation of Infectious Mosquitoes over time for constant values of the parameters

Conclusion

It appears you've detailed a comprehensive approach using the Variational Homotopy Perturbation Method (VHPM) to solve a nonlinear system of equations. The method involves iteratively refining solutions via a series expansion in terms of a perturbation parameter ppp, leading to approximate solutions un(x)u_n(x)un(x). By integrating these solutions and equating coefficients, you derive expressions for the state variables S(t),X(t),I(t),R(t),U(t),V(t),S(t), X(t), I(t), R(t), U(t), V(t),S(t),X(t),I(t),R(t),U(t),V(t), and W(t)W(t)W(t). This iterative approach converges to solutions that describe the dynamics of your system over time, as shown in your figures illustrating population variations.

8. References

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