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# Note on encryption decryption algorithm using solutions of Pell's equation

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**Abstract**
In this paper we propose a decryption algorithm based on a simple analysis of the coding algorithm. Our algorithm will challenge encrypted messages using solutions of Pell's equation $x^2 - 3y^2 = 1$, recently published.

**Keywords:** Encryption, Decryption, Pell's equation

## 1. Introduction

The coding and decoding algorithms occupy a major importance to make better the security of communication. On this point, a family of mathematical algorithms has been developed to increasingly strengthen communication security in recent times ([8-11]). There are other research works on the same objective ([3-4], [6-7]). Recently, a new inspiration has been elaborated in [5], using the solutions of Pell's equation $x^2 - 3y^2 = 1$. The theory on solving Pell's equation can be found in ([1-2]).
All the authors of these works claim that their algorithms have significant security. But our analysis shows that it is possible to decipher them, this is what we will detail in this article. We will take the encryption algorithm proposed in [5] and apply our new algorithm to decrypt it easily. First, in section 2, we present preliminaries. In section 3, we will start our work by presenting the coding algorithm using the solutions of the Pell equation $x^2 - 3y^2 = 1$, proposed in [5]. In section 4, we analyze this algorithm in order to make the new decryption algorithm. Finally, in section 5, we present some examples of applications.

## 2. Preliminaries.

In this part, we explain the fundamental principle of the coding algorithm proposed in [5], as well as the used notations.
The coding algorithm proposed in the article [5] consists in inserting (from left to right) the initial text message in a square matrix of even order $2l$ denoted by B. The space between two words will be coded by θ. When the initial text cannot entirely fill the coefficients of the matrix B, we add θ to the missing coefficients. Then, we divide the matrix B into blocks $B_i$ of order 2, from left to right and the integer denoted by m represents the number of obtained blocks. After performing all these steps, the coding process begins.

We choose the natural number n such that $n = \begin{cases} 3, if\ m \leq 3 \\ m, if\ m > 3 \end{cases}$.

Correspondence between characters and numbers

| A | B | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $n+1$ | $n+2$ | $n+3$ | $n+4$ | $n+5$ | $n+6$ | $n+7$ | $n+8$ | $n+9$ | $n+10$ | $n+11$ |
| M | N | O | P | Q | R | S | T | U | V | W | X |
| $n+12$ | $n+13$ | $n+14$ | $n+15$ | $n+16$ | $n+17$ | $n+18$ | $n+19$ | $n+20$ | $n+21$ | $n+22$ | $n+23$ |
| Y | Z | θ | | | | | | | | | |
| $n+24$ | $n+25$ | $n-1$ | | | | | | | | | |

Let us then set

- $B_i = \begin{pmatrix} b_{i1} & b_{i2} \\ b_{i3} & b_{i4} \end{pmatrix}$, the block matrix of order 2;
- $d_i = \det(B_i) = b_{i1} \times b_{i4} - b_{i3} \times b_{i2}$ ;
- $\mathrm{E} = [d_i, b_{ik}]_{k\in\{1,2,4\}}$, the coded matrix.

**3. Coding algorithm according to the proposed model [5].**

This algorithm is divided into six steps to encode the initial message

**Algorithm 1: Coding**

**Step 1:** Construct the matrix B of even order for the given text.
**Step 2:** Divide the matrix B into blocks $B_i( 1 \leq i \leq \ell^2)$.
**Step 3:** Choose n using b.
**Step 4:** Choose the integer n.
**Step 5:** Determine $b_{ij}$ $( 1 \leq j \leq 4)$.
**Step 6:** Count $\det(B_i) \rightarrow d_i$.
**Step 7:** Construct $E = [\, d_i \,, b_{ik}]_{k\in\{1,2,4\}}$.

Now we will analyze this algorithm in order to decrypt it. According to the authors of the article [5], this algorithm is associated with the solutions of the Pell's equation $x^2 - 3y^2 = 1$. But nowhere in this process, the solutions of the equation of $x^2 - 3y^2 = 1$ appear.

**4. Analysis of Algorithm 1 and proposed decryption method.**

We will begin our analysis with the 7th step. The seventh step is the construction of the coded matrix. Let $E = [\, d_i \,, b_{ik}]_{k\in\{1,2,4\}}$ . For a little more information, let's expand the expression of E.

we have $E = \begin{pmatrix} d_1 & b_{11} & b_{12} & b_{14} \\ d_2 & b_{21} & b_{22} & b_{24} \\ d_3 & b_{31} & b_{32} & b_{34} \\ \vdots & \vdots & \vdots & \vdots \\ . & . & . & . \\ d_m & b_{m1} & b_{m2} & b_{m4} \end{pmatrix}$, this is an m×4 matrix.

First of all, we can directly estimate the number of blocks $B_i$. This number is equal to the number of the determinant $d_i$ appearing in the first column of E. We then notice that on each line, the second index of b remains constant. We can therefore write the matrix E in another way, as a function of the row matrices $E_i$, defined for i ranging from 1 to m by: $E_i = (\, d_i \quad b_{i1} \quad b_{i2} \quad b_{i4})$. Or again: $E_i = (b_{i1} \times b_{i4} - b_{i3} \times b_{i2} \quad b_{i1} \quad b_{i2} \quad b_{i4})$.

The new expression of E is therefore given by: $E = \begin{pmatrix} E_1 \\ E_2 \\ \cdot \\ \vdots \\ E_m \end{pmatrix}$.

Let us then set: $E_i = (\, d_i \quad e_{i1} \quad e_{i2} \quad e_{i4})$ and we know that

$E_i = (b_{i1} \times b_{i4} - b_{i3} \times b_{i2} \quad b_{i1} \quad b_{i2} \quad b_{i4})$.

The equality of two matrices gives, for all $i \in \{1, \ldots, m\}$ : $\begin{cases} b_{i1} = e_{i1} \\ b_{i2} = e_{i2} \\ b_{i4} = e_{i4} \\ b_{i1} \times b_{i4} - b_{i3} \times b_{i2} = d_i. \end{cases}$

From this system, one can determine all the coefficients of the initial block matrices $B_i$. Our decryption algorithm is therefore based on this system.
Let us now propose the decryption algorithm.

**Algorithm 2: Decryption**
Step 1: Determination of the number of blocks of B.
Step 2: Calculation of the coefficients of $B_i$:

For all $i \in \{1, \dots, m\}$, we have : $\begin{cases} b_{i1} = e_{i1} \\ b_{i2} = e_{i2} \\ b_{i4} = e_{i4} \\ b_{i1} \times b_{i4} - b_{i3} \times b_{i2} = d_i \end{cases}$

**Step 3:** Construction of blocks of matrices $B_i$.
**Step 4:** Construction of the matrix B from the $B_i$.
**Step 5:** End of algorithm.

## 5. Examples of applications

In this part, we will take all the examples discussed in article [5], and we apply algorithm 2 to decipher the coded message. We are not going to detail the steps the author took to encode the message (Readers can read the details in [5]). We will directly present the initial message, the matrix B as well as the coded matrix E. Then, we decrypt the matrix E using the algorithm that we have made.

**Example 1**
The initial message to be coded is: "TAKE".
The initial matrix: $B = \begin{pmatrix} T & A \\ K & E \end{pmatrix}$, after having converted all the letters into numbers, we then obtain: $B = \begin{pmatrix} 22 & 3 \\ 13 & 7 \end{pmatrix}$.
The coded matrix: $E = (115 \quad 22 \quad 3 \quad 7)$.
Let us now find the initial message, that is to say the matrix B from E using algorithm 2.
Step 1: The number of blocks in Matrix B.
The number of rows in E gives the number of blocks in B. Since E has only one row, then m=1.
Step 2: The coefficients of the matrices:

We have : $\begin{cases} b_{i1} = e_{i1} \\ b_{i2} = e_{i2} \\ b_{i4} = e_{i4} \\ b_{i1} \times b_{i4} - b_{i3} \times b_{i2} = d_i. \end{cases}$

For i=1, then, this system becomes: $\begin{cases} b_{11} = e_{11} = 22 \\ b_{12} = e_{12} = 3 \\ b_{14} = e_{14} = 7 \\ b_{11} \times b_{14} - b_{13} \times b_{12} = d_1 = 115. \end{cases}$

Let: $22 \times 7 - b_{13} \times 3 = 115$. So $b_{13} = 13$.
Then: $b_{11} = 22$ ; $b_{12} = 3$ ; $b_{13} = 13$ et $b_{14} = 7$.
Step 3: Construction of $B_1$.
We have: $B_1 = \begin{pmatrix} b_{11} & b_{12} \\ b_{13} & b_{14} \end{pmatrix} = \begin{pmatrix} 22 & 3 \\ 13 & 7 \end{pmatrix}$.
Step 4: Construction of matrix B.
Since there is only one block, then $B = B_1 = \begin{pmatrix} 22 & 3 \\ 13 & 7 \end{pmatrix}$.
The initial matrix is therefore: $B = \begin{pmatrix} T & A \\ K & E \end{pmatrix}$.
The initial message is therefore: "TAKE".

**Example 2**
The initial message is: "HAVE A NICE DAY".

The initial matrix: $B = \begin{pmatrix} \mathrm{H} & \mathrm{A} & \mathrm{V} & \mathrm{E} \\ \theta & \mathrm{A} & \theta & \mathrm{N} \\ \mathrm{I} & \mathrm{C} & \mathrm{E} & \theta \\ \mathrm{D} & \mathrm{A} & \mathrm{Y} & \theta \end{pmatrix}$.

After converting all the letters into numbers, we have: $B = \begin{pmatrix} 11 & 4 & 25 & 8 \\ 3 & 4 & 3 & 17 \\ 12 & 6 & 8 & 3 \\ 7 & 4 & 28 & 3 \end{pmatrix}$.

The coded matrix: $E = \begin{pmatrix} 32 & 11 & 4 & 4 \\ 401 & 25 & 8 & 17 \\ 6 & 12 & 6 & 4 \\ -60 & 8 & 3 & 3 \end{pmatrix}$.

Find the matrix B from E.
**Step 1:** Number of blocks of B.
The matrix E has four rows, so B has 4 blocks and we have m=4.
**Step 2:** The coefficients of $B_i$:

For all i∈{1,2,3,4}, we have: $\begin{cases} b_{i1} = e_{i1} \\ b_{i2} = e_{i2} \\ b_{i4} = e_{i4} \\ b_{i1} \times b_{i4} - b_{i3} \times b_{i2} = d_i. \end{cases}$

$B_1$ coefficients:

For i=1, then, we have: $\begin{cases} b_{11} = e_{11} = 11 \\ b_{12} = e_{12} = 4 \\ b_{14} = e_{14} = 4 \\ b_{11} \times b_{14} - b_{13} \times b_{12} = d_1 = 32. \end{cases}$

Let: $11 \times 4 - b_{13} \times 4 = 32$. So $b_{13} = 3$.
So: $b_{11} = 11$ ; $b_{12} = 4$; $b_{13} = 3$ et $b_{14} = 4$.
$B_2$ coefficients:

For i=2, then, we have: $\begin{cases} b_{21} = e_{21} = 25 \\ b_{22} = e_{22} = 8 \\ b_{24} = e_{24} = 17 \\ b_{21} \times b_{24} - b_{23} \times b_{22} = d_2 = 401. \end{cases}$

Let: $25 \times 17 - b_{23} \times 8 = 401$. So $b_{23} = 3$.
So: $b_{21} = 25$ ; $b_{22} = 8$; $b_{23} = 3$ et $b_{24} = 17$.
$B_3$ coefficients:

For i=3, then, we have: $\begin{cases} b_{31} = e_{31} = 12 \\ b_{32} = e_{32} = 6 \\ b_{34} = e_{34} = 4 \\ b_{31} \times b_{34} - b_{33} \times b_{32} = d_3 = 6. \end{cases}$

Let: $12 \times 4 - b_{33} \times 6 = 401$. So $b_{33} = 7$.
So: $b_{31} = 12$ ; $b_{32} = 6$; $b_{33} = 7$ et $b_{34} = 4$.
$B_4$ coefficients:

For i=4, then, we have: $\begin{cases} b_{41} = e_{41} = 8 \\ b_{42} = e_{42} = 3 \\ b_{44} = e_{44} = 3 \\ b_{41} \times b_{44} - b_{43} \times b_{42} = d_4 = -60. \end{cases}$

Let: $8 \times 3 - b_{43} \times 3 = 401$. So $b_{43} = 28$.
So: $b_{41} = 8$ ; $b_{42} = 3$; $b_{43} = 28$ et $b_{44} = 3$.
**Step 3:** Construction of blocks $B_i$:
Based on the results of the previous step (step 2), we obtain the following four matrices:

$$B_1 = \begin{pmatrix} 11 & 4 \\ 3 & 4 \end{pmatrix};\ B_2 = \begin{pmatrix} 25 & 8 \\ 3 & 17 \end{pmatrix};\ B_3 = \begin{pmatrix} 12 & 6 \\ 7 & 4 \end{pmatrix}\ and\ B_4 = \begin{pmatrix} 8 & 3 \\ 28 & 3 \end{pmatrix}.$$

**Step 4:** Construction of matrix B from $B_i$.

We have: $B = \begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix}$.

Let: $B = \begin{pmatrix} 11 & 4 & 25 & 8 \\ 3 & 4 & 3 & 17 \\ 12 & 6 & 8 & 3 \\ 7 & 4 & 28 & 3 \end{pmatrix}$.

The initial matrix is therefore: $B = \begin{pmatrix} \text{H} & \text{A} & \text{V} & \text{E} \\ \theta & \text{A} & \theta & \text{N} \\ \text{I} & \text{C} & \text{E} & \theta \\ \text{D} & \text{A} & \text{Y} & \theta \end{pmatrix}$.

The initial message: "HAVE A NICE DAY".

**Example 3**

Initial message: "THE SUN RISES IN THE EAST".

The initial matrix: $B = \begin{pmatrix} T & H & E & \theta & S & U \\ N & \theta & R & I & S & E \\ S & \theta & I & N & \theta & T \\ H & E & \theta & E & A & S \\ T & \theta & \theta & \theta & \theta & \theta \\ \theta & \theta & \theta & \theta & \theta & \theta \end{pmatrix}$.

After converting all the letters to numbers, we have: $B = \begin{pmatrix} 28 & 16 & 13 & 8 & 27 & 29 \\ 22 & 8 & 26 & 17 & 27 & 13 \\ 27 & 8 & 17 & 22 & 8 & 28 \\ 16 & 13 & 8 & 13 & 9 & 27 \\ 28 & 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$.

The coded matrix: $E = \begin{pmatrix} -128 & 28 & 16 & 8 \\ 13 & 13 & 8 & 17 \\ -432 & 27 & 29 & 13 \\ 223 & 27 & 8 & 13 \\ 45 & 17 & 22 & 13 \\ -36 & 8 & 28 & 27 \\ 160 & 28 & 8 & 8 \\ 0 & 8 & 8 & 8 \\ 0 & 8 & 8 & 8 \end{pmatrix}$.

Find the matrix B from E.

**Step 1:** Number of blocks of B.

The matrix E has nine rows, so B has 9 blocks and we have m=9.

**Step 2:** The coefficients of $B_i$:

For all i∈{1,…,9}, we have: $\begin{cases} b_{i1} = e_{i1} \\ b_{i2} = e_{i2} \\ b_{i4} = e_{i4} \\ b_{i1} \times b_{i4} - b_{i3} \times b_{i2} = d_i. \end{cases}$

$B_1$ coefficients:

For i=1, then, we have: $\begin{cases} b_{11} = e_{11} = 28 \\ b_{12} = e_{12} = 16 \\ b_{14} = e_{14} = 8 \\ b_{11} \times b_{14} - b_{13} \times b_{12} = d_1 = -128. \end{cases}$

Let: $28 \times 8 - b_{13} \times 16 = -128$. So $b_{13} = 22$.

So: $b_{11} = 28$ ; $b_{12} = 16$; $b_{13} = 22$ et $b_{14} = 8$.

$B_2$ coefficients:

For i=2, then, we have: $\begin{cases} b_{21} = e_{21} = 13 \\ b_{22} = e_{22} = 8 \\ b_{24} = e_{24} = 17 \\ b_{21} \times b_{24} - b_{23} \times b_{22} = d_2 = 13. \end{cases}$

Let: $13 \times 17 - b_{23} \times 8 = 13$. So $b_{23} = 26$.
So: $b_{21} = 13$ ; $b_{22} = 8$; $b_{23} = 26$ et $b_{24} = 17$.

$B_3$ coefficients:

For i=3, then, we have: $\begin{cases} b_{31} = e_{31} = 27 \\ b_{32} = e_{32} = 29 \\ b_{34} = e_{34} = 13 \\ b_{31} \times b_{34} - b_{33} \times b_{32} = d_3 = -432. \end{cases}$

Let: $27 \times 13 - b_{33} \times 29 = -432$. So $b_{33} = 27$.
So: $b_{31} = 12$ ; $b_{32} = 6$; $b_{33} = 7$ et $b_{34} = 4$.
.
$B_4$ coefficients:

For i=4, then, we have: $\begin{cases} b_{41} = e_{41} = 27 \\ b_{42} = e_{42} = 8 \\ b_{44} = e_{44} = 13 \\ b_{41} \times b_{44} - b_{43} \times b_{42} = d_4 = 223. \end{cases}$

Let: $27 \times 13 - b_{43} \times 8 = 401$. So $b_{43} = 16$.
So: $b_{41} = 27$ ; $b_{42} = 8$; $b_{43} = 16$ et $b_{44} = 13$.

$B_5$ coefficients:

For i=5, then, we have: $\begin{cases} b_{51} = e_{51} = 17 \\ b_{52} = e_{52} = 22 \\ b_{54} = e_{54} = 13 \\ b_{51} \times b_{54} - b_{53} \times b_{52} = d_5 = 45. \end{cases}$

Let: $17 \times 13 - b_{53} \times 22 = 45$. So $b_{53} = 8$.
So: $b_{51} = 17$ ; $b_{52} = 22$; $b_{53} = 8$ et $b_{54} = 13$.

$B_6$ coefficients:

For i=6, then, we have: $\begin{cases} b_{61} = e_{61} = 8 \\ b_{62} = e_{62} = 28 \\ b_{64} = e_{64} = 27 \\ b_{61} \times b_{64} - b_{63} \times b_{62} = d_6 = -36. \end{cases}$

Let: $8 \times 27 - b_{63} \times 28 = -36$. So $b_{63} = 9$.
So: $b_{61} = 8$ ; $b_{62} = 28$; $b_{63} = 9$ et $b_{64} = 27$.

$B_7$ coefficients:

For i=7, then, we have: $\begin{cases} b_{71} = e_{71} = 28 \\ b_{72} = e_{72} = 8 \\ b_{74} = e_{74} = 8 \\ b_{71} \times b_{74} - b_{73} \times b_{72} = d_7 = 160. \end{cases}$

Let: $28 \times 8 - b_{73} \times 8 = 160$. So $b_{73} = 8$.
So: $b_{71} = 28$ ; $b_{72} = 8$; $b_{73} = 8$ et $b_{74} = 8$.

$B_8$ coefficients:

For i=8, then, we have: $\begin{cases} b_{81} = e_{81} = 8 \\ b_{82} = e_{82} = 8 \\ b_{84} = e_{84} = 8 \\ b_{81} \times b_{84} - b_{83} \times b_{82} = d_8 = 0. \end{cases}$

Let: $8 \times 8 - b_{83} \times 8 = 0$. So $b_{83} = 8$.
So: $b_{81} = 8$ ; $b_{82} = 8$; $b_{83} = 8$ et $b_{84} = 8$.

$B_9$ coefficients:

For i=9, then, we have: $\begin{cases} b_{91} = e_{91} = 8 \\ b_{92} = e_{92} = 8 \\ b_{94} = e_{94} = 8 \\ b_{91} \times b_{94} - b_{93} \times b_{92} = d_9 = 0. \end{cases}$

Let: $8 \times 8 - b_{93} \times 8 = 0$. So $b_{93} = 8$.
So: $b_{91} = 8$ ; $b_{92} = 8$; $b_{93} = 8$ et $b_{94} = 8$.

**Step 3:** Construction of blocks $B_i$:
Based on the results of the previous step (step 2), we obtain the following nine matrices:

$$B_1 = \begin{pmatrix} 28 & 16 \\ 22 & 8 \end{pmatrix};\ B_2 = \begin{pmatrix} 13 & 8 \\ 26 & 17 \end{pmatrix};\ B_3 = \begin{pmatrix} 27 & 29 \\ 27 & 13 \end{pmatrix};\ B_4 = \begin{pmatrix} 27 & 8 \\ 16 & 13 \end{pmatrix};\ B_5 = \begin{pmatrix} 17 & 22 \\ 8 & 13 \end{pmatrix}.$$
$$B_6 = \begin{pmatrix} 8 & 28 \\ 9 & 27 \end{pmatrix};\ B_7 = \begin{pmatrix} 28 & 8 \\ 8 & 8 \end{pmatrix};\ B_8 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}\ and\ B_9 = \begin{pmatrix} 8 & 8 \\ 8 & 8 \end{pmatrix}.$$

**Step 4:** Construction of matrix B from $B_i$.

We have: $B = \begin{pmatrix} B_1 & B_2 & B_3 \\ B_4 & B_5 & B_6 \\ B_7 & B_8 & B_9 \end{pmatrix}$.

Let $B = \begin{pmatrix} 28 & 16 & 13 & 8 & 27 & 29 \\ 22 & 8 & 26 & 17 & 27 & 13 \\ 27 & 8 & 17 & 22 & 8 & 28 \\ 16 & 13 & 8 & 13 & 9 & 27 \\ 28 & 8 & 8 & 8 & 8 & 8 \\ 8 & 8 & 8 & 8 & 8 & 8 \end{pmatrix}$.

After having converted all the numbers into letters we have: $B = \begin{pmatrix} T & H & E & \theta & S & U \\ N & \theta & R & I & S & E \\ S & \theta & I & N & \theta & T \\ H & E & \theta & E & A & S \\ T & \theta & \theta & \theta & \theta & \theta \\ \theta & \theta & \theta & \theta & \theta & \theta \end{pmatrix}$.

Initial message: “THE SUN RISES IN THE EAST”.

## 6. Conclusion
In this paper, we proposed a new algorithm to challenge coded messages associated with solutions of Pell's equation $x^2 - 3y^2 = 1$. The discovery of our decryption algorithm proves the existence of a flaw in the coding method associated with the solutions of the Pell equation. In future work, we will consider proposing an improvement to this on to provide a little more security.

## 7. References

1. Ahmet Tekcan. The Pell Equation $x^2 - Dy^2 = \pm 4$, Applied Mathematical Sciences. 2007; 1(8):363-369.
2. Ahmet Tekcan, Continued Fractions Expansion of √D and Pell Equation $x^2 - Dy^2 = 11$, Mathematica Moravica. 2011; 15(2):19-27.
3. Bandhu Prasad, Coding theory on Lucas p numbers, 2016.
4. Emarah Kiliç. The generalized Pell (p,i)-numbers and their Binet formulas, combinatorial representation, sums, 2009.
5. J Kannan, Manju Somanath, M Mahalakshmi, K Raja. Encryption Decryption Algorithm Using Solutions of Pell Equation. Int. J.Math. And Appl. 2022; 10(1):1-8.
6. M Basu, B Prasad. Coding theory on h(x) Fubonacci p-numbers polynomials, 2012.
7. Mostafa Esmaeili, Morteza Esmaeili, A Fibonacci-polynomial based coding method with error detection and correction, 2010.
8. Nahal Tas, Sümeyra Ucar, Nihal Yilmaz Özgür, and Öznur Öztunç Kaymak, A new coding/ decoding algorithm using Fibonacci numbers. Discrete Mathematics, Algorithms and Applications. 2017; 10(2):1850028.
9. Nihal Tas, Sümeyra Ucar, Nihal Yilmaz Özgür, Pell coading and decoding methods with some application. 2019; 15(1):52-56, ISSN 1715-0868.
10. Sümeyra Ucar, Nihal Tas, Ninah Yimaz Özgür. A new cryptography model via Fubonacci and Lucas numbers, 2017.
11. Sümeyra Uçar, Nihal Tas, Nihal Yılmaz Özgür. A New Application to Coding Theory via Fibonacci and Lucas Numbers. Mathematical sciences and applications. E-notes. 2019; 7 (1)62-70. MSAEN (2018).