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Diagonal arithmetic an innovation to elementary mathematics

Inah Gowin Eni
Department of Mathematics, School of Secondary Education Science Programme, Cross River State College of Education, Akamkpa, Nigeria

* Corresponding Author: **Inah Gowin Eni**

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Abstract
Mathematics has always seemed like a mystery to many people, especially to children of basic school age. When children at this stage of learning encounter mathematical concepts, they tend to be so perplexed that many of them end up hating the subject irrespective of its relevance to life. The reasons for this perplexity are not far-fetched; teachers of the subject have serious difficulty demystifying the underlying of its most rudimentary concepts. Children progress from class to class with these unresolved challenges, which actually accumulate into a bigger challenges leading to hatred and phobia for the subject. Recall that mathematics developed several millennia ago from its most rudimentary form; number sense into arithmetic advancing gradually to its current climax; advanced mathematics. For countless ages now, mathematicians have sort to advance mathematics from its most rudimentary concepts to higher concepts, seemingly oblivious of the converse situation. In other words it seems quite likely that interest in the utilization of advanced concepts of mathematics to improve understanding of elementary concepts is either lacking or inadequate. A major question that can be impressed upon the mind is therefore; "How can we develop modified arithmetical approaches that will obliterate or reduce to the barest minimum the perplexity in arithmetic thereby making learning of mathematics easily accessible to all". This study therefore, seeks to develop new elementary semiotic approaches to number sense and place value with special recourse to arithmetical operations such as addition (+), subtraction (-), multiplication (x), division (÷), etc. The study attempts to provide alternative semiotic approaches to elementary arithmetic by synthesizing concepts from advanced mathematics such as linear algebra and number theory. Hence the concept of diagonal arithmetic.

Keywords: innovation, Diagonal arithmetic, mathematics

Introduction

Mathematics has advanced from its most elementary stage to what many would conceive of as the climax of it. All mathematical advancements have their foundations from arithmetic which involves operations of addition, subtraction, multiplication, division, squares and square roots, etc. Arithmetic itself came to the limelight with the invention of the Hindu-Arabic Numerals 0,1,2,3,4,5,6,7,8 and 9. These numerals exhibits qualities that make mathematics so very powerful and influential.

- They can be used by understanding a small number of ideas
- They can be generalized beyond the original setting for which they were devised; and
- They provide different perspectives from which they can be viewed.

Arithmetic is considered to be elementary number theory based principally on the Hindu-Arabic numeral system which is a decimal system. Arithmetical operations are thus performed on the bases of this decimal system of numerals. By this we mean that addition, subtraction etc., are done based on this decimal system of numbers [1].

The use of signs and symbols to communicate ideas is known as semiotics. Every language and discipline has a unique form of semiotics that is peculiar to it. Mathematics as the language of science has a unique form of semiotics. In this context, semiotics is the use of mathematical signs and symbols to communicate mathematical concepts. Arithmetic as elementary mathematics is no less in this regard. It employs the use of signs such as (+), (-), (x) and (÷), to pass information while working within the confines of symbols such as 0,1,2,3,4,… 9, as digits for numeration [2, 3, 4]. In my perspective, semiotics in mathematics transcends just the use of signs and symbols associated with meaning to communicate mathematical ideas. It includes amongst other things, the techniques and ways of presenting these ideas. In other words, what form of arrangement of numbers is being used to present these concepts? In this context, semiotics becomes more inclusive to allow for a different visualization of numbers and of arithmetical operation [4]. For example, the number 5142 can be visualized in the following ways:

5142 = 5000 + 100 + 40 + 2 (Horizontal view)

5142 = 5000
100
40
+ 2 (Vertical view)

Both of these arrangements have the same meaning because they are based on the place value system of numbers. These are good examples of different semiotic presentations. The second illustration is of great interest to this study because it forms the premise upon which the study is conceived.

The concept of matrices – a rectangular array of numbers – is in fact the main plank of linear algebra which is a branch of modern mathematics. Perhaps, this concept existed subconsciously since the beginning of arithmetic before it became established as a field of study. It is indeed fitting to think this way because we all write numbers having two or more digits as row vectors even though we often are not conscious of the fact that even the digits can be treated as such for example

10 = (1 0), 11= (1 1), 12 = (1 2) etc.
100 = (1 0 0), 101 = (1 0 1), 102 = (1 0 2) etc.
1000 = (1 0 0 0), 1001 = (1 0 0 1), 1002 (1 0 0 2) etc.

The fact that we fail to treat digits as vectors or perhaps as entries in a matrix might be limiting our understanding of how they behave during the course of addition, subtraction, multiplication and division. This actually raises the questions;

1. Can the digits of all numbers be represented as entries in a matrix?
2. How does treating digits of numbers as entries in a matrix facilitate our understandings their behaviors in addition, subtraction, multiplication and division etc.

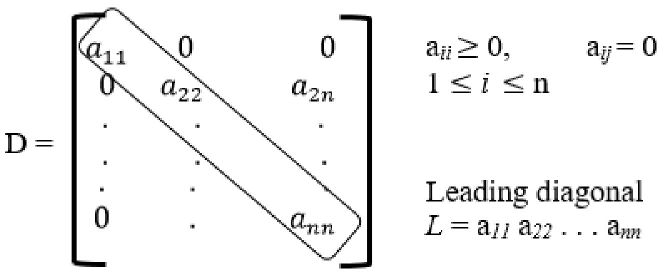
This paper, therefore seeks to demonstrate the possibilities in treating digits as such. The study attempts to borrow from our understanding of linear algebra to seek improvements in elementary number theory (i.e. arithmetic).

Representing digits as entries in a matrix

An nxn square matrix is one which has n rows and n columns. In other words, it is a matrix with the same rows and columns. Such a matrix can be generally written as shown below

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots a_{1n} \\ a_{21} & a_{22} & \cdots a_{2n} \\ \cdot & \cdots & \cdots \\ \cdot & \cdots & \cdots \\ \cdot & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots a_{nn} \end{bmatrix}$$

The entries a_{11} a_{22}, …. a_{nn} actually constitute the leading diagonal. Example of nxn matrices includes



An n x n square matrix with at least one non-=zero entry in the leading diagonal and zero entries outside the leading diagonal is called a diagonal matrix. To be more general, we could represent a diagonal matrix as shown below

Below are some examples of diagonal matrix

$$E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \text{ 3 x 3 diagonal matrix L } = 214$$

$$F = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \text{4 x 4 diagonal matrix} \\ L = 4000 \end{matrix}$$

The powers of diagonal matrices in linear algebra cannot be overemphasized. In the same vein, they are just as powerful in elementary number theory (i.e. Arithmetic) as they are in linear algebra. To be able to visualize this clearly, we need to try visualizing two different semiotic representations of numbers having two or more digit.

Proposition 1

All digits in any given number having one or more digits can be represented as nxn diagonal matrix with the digits appearing in the leading diagonal.

Consider the following semiotic representation of 5142 based on the place value system of numbers (i.e. base 10 system).

a. 5142 = 5000 + 100 + 40 + 2 (horizontal view)

b. 5142 = 5000

$$\begin{array}{r} 5000 \\ 100 \\ 40 \\ +\quad 2 \\ \hline \end{array} \quad \text{(Vertical view)}$$

The second illustration, (b), appears to provide us with a semiotic representation like the canonical form of a matrix.
The addition sign (+) can actually be ignored. Adding more 0s to complete the matrix, we get

$$5142 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

The result is a 4x4 diagonal matrix.

Notice that 5142 appears in the leading diagonal. More examples of this representation can be seen as shown below.

$$63 = \begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix} \qquad 251 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$71200 = \begin{bmatrix} 7 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad 407203 = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Note: The enumeration of the rows must be in the reverse order so that every digit corresponds to its own place valve. Thus the last row in the usual matrix becomes the first row and the last columns becomes the first column. Thus if we let d = digit, then d_{ii} = ith digit of any number N $= d_{kk} \ldots d_{33} d_{22}\, d_{11}$

Where $1 \leq i \leq k$

d_{11} being the unit digit, d_{22} the tens digit, d_{33} the hundreds digits, etc. such a number can be represented diagonally.

. . . . .

$$N = \begin{bmatrix} d_{kk} & \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & d_{33} & 0 & 0 \\ 0 & \cdot & 0 & d_{22} & 0 \\ 0 & \cdot & 0 & 0 & d_{11} \end{bmatrix}$$

Looking at these representations, one might consider them to be a little cumbersome to write. Therefore, for simplicity, it is proper to ignore the 0s keeping only the leading diagonal and to replace the brackets with a suitable symbol. In this case, we find the use of a single left-sided brace, {, to be convenient. Thus by this innovation, we have the following

$$5142 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} = \left\{ \begin{matrix} 5 & & & \\ & 1 & & \\ & & 4 & \\ & & & 2 \end{matrix} \right.$$

$$63 = \begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix} = \left\{ \begin{matrix} 6 & \\ & 3 \end{matrix} \right.$$

$$251 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left\{ \begin{matrix} 2 & & \\ & 5 & \\ & & 1 \end{matrix} \right.$$

$$71200 = \begin{bmatrix} 7 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \left\{ \begin{matrix} 7 & & & & \\ & 1 & & & \\ & & 2 & & \\ & & & 0 & \\ & & & & 0 \end{matrix} \right.$$

$$407203 = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} = \left\{ \begin{matrix} 4 & & & & & \\ & 0 & & & & \\ & & 7 & & & \\ & & & 2 & & \\ & & & & 0 & \\ & & & & & 3 \end{matrix} \right.$$

Notice here that this arrangement helps to reduce the stress in this method of semiotic representation whereby digits are positioned diagonally.

2.1. Diagonalization of arithmetical operation

The fact that digits of numbers can be represented as diagonal matrices of n-order is indeed of great importance because it helps us understand, in a very unique way, just how these digits behave during arithmetical operation such as addition, subtraction, multiplication and division as well as squares and square roots. Until now, arithmetical operations involving numbers having more than one digit have never been known to be possible by any other means than the rectangular array of digits of these numbers. For example

2152 + 321 = 2 1 5 2
+ 3 2 1 rectangular array

67531 – 41286 = 6 7 5 3 1
- 4 1 2 8 6

2576 x 32 = 2 5 7 6
x 3 2

This method is quite convenient and speedy to deal with. In other words, it is convenient to write and very fast at producing results. However, it is often noticed that the method hides out a lot of things that make for understanding especially when renaming is involved. It has been noticed from personal experience that the perplexity children develop during arithmetic classes boils to the fact that they are unable to demystify the hidden facts of addition, subtraction, multiplication and division. Diagonal arithmetic now appears to be a plausible addition to this challenge. We shall at this point demonstrate that the assertion might be a reasonable excuse for it.

Proposition 2

All numbers having one or more digits can be added, subtracted, multiplied and divided by viewing them as entries in the leading diagonal of a diagonal matrix.

2.2 Diagonal addition of numbers

Suppose that D and E are two numbers having K digits each, then

D = $d_{kk} \ldots d_{33}\, d_{22}\, d_{11}$

E = $e_{kk} \ldots e_{kk}\; e_{33}\, e_{22}\, e_{11}$

By diagonal addition

$$D + E = \left\{\begin{matrix} k_{kk} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & d_{33} & & \\ & & & & d_{22} & \\ & & & & & d_{11} \end{matrix}\right. + \left\{\begin{matrix} e_{kk} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & e_{33} & & \\ & & & & e_{22} & \\ & & & & & e_{11} \end{matrix}\right.$$

$$\left\{\begin{matrix} d_{kk}+e_{kk} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & d_{33}+e_{33} & & \\ & & & & d_{22}+e_{22} & \\ & & & & & d_{11}+e_{11} \end{matrix}\right.$$

Suppose that D has k digits while E has k-1 digits, then

$$D + E = \left\{\begin{matrix} k_{kk} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & d_{33} & & \\ & & & & d_{22} & \\ & & & & & d_{11} \end{matrix}\right. + \left\{\begin{matrix} e_{k-1\,k-1} & & & & & \\ & e_{k-2\,k-2} & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & e_{22} & \\ & & & & & e_{11} \end{matrix}\right.$$

$$= \left\{\begin{matrix} k_{kk} & & & & & & & \\ & d_{k-1\,k-1}+e_{k-1\,k-1} & & & & & & \\ & & d_{k-2\,k-2}+e_{k-2\,k-2} & & & & & \\ & & & \cdot & & & & \\ & & & & \cdot & & & \\ & & & & & d_{33}+e_{33} & & \\ & & & & & & d_{22}+e_{22} & \\ & & & & & & & d_{11}+e_{11} \end{matrix}\right.$$

Note

- Addition process starts from left to right
- The sum of the digits is allowed to remain temporarily just as it is even if the result has more than a single digit.
- Renaming is done only after the addition process has been completed.
- During renaming every digit is sent to its corresponding place value and written at the top-left of the existing digit in that place
- Diagonal addition retains cumutativity.

We shall demonstrate diagonal addition using actual numbers

a. $25 + 32 = \left\{\begin{matrix} 2 & \\ & 5 \end{matrix}\right. + \left\{\begin{matrix} 3 & \\ & 2 \end{matrix}\right. = \left\{\begin{matrix} 2+3 & \\ & 5+2 \end{matrix}\right.$

$= \left\{\begin{matrix} 5 & \\ & 7 \end{matrix}\right.$

$\overline{\;5 \quad 7\;}$

b. $416 + 348 = \left\{\begin{matrix} 4 & & \\ & 1 & \\ & & 6 \end{matrix}\right. + \left\{\begin{matrix} 3 & & \\ & 4 & \\ & & 8 \end{matrix}\right.$

$= \left\{\begin{matrix} 4+3 & & \\ & 1+4 & \\ & & 6+8 \end{matrix}\right. = \left\{\begin{matrix} 7 & & \\ & 5 & \\ & & 14 \end{matrix}\right.$

$\left\{\begin{matrix} 7 & & \\ & {}^{1}5 & \\ & & 4 \end{matrix}\right. = \left\{\begin{matrix} 7 & & \\ & 6 & \\ & & 4 \end{matrix}\right.$

$\overline{\;7 \quad 6 \quad 4\;}$

6 + 8 = 14, 14 is written just as it is, then after renaming, 1 is sent to its rightful place and written at the top-left of 5.

$$\text{c. } 6345+7754=\left\{\begin{array}{cccc}6&&&\\&3&&\\&&4&\\&&&5\end{array}\right. + \left\{\begin{array}{cccc}7&&&\\&7&&\\&&5&\\&&&4\end{array}\right.$$

$$\left\{\begin{array}{cccc}6+7&&&\\&3+7&&\\&&4+5&\\&&&5+4\end{array}\right. = \left\{\begin{array}{cccc}13&&&\\&10&&\\&&9&\\&&&9\end{array}\right.$$

$$=\left\{\begin{array}{ccccc}1&&&&\\&{}^{1}3&&&\\&&0&&\\&&&9&\\&&&&9\end{array}\right.$$
$$\overline{\begin{array}{ccccc}1&4&0&9&9\end{array}}$$

After renaming the 1 in 10 is sent to 3 while the 1 in 13 is sent to a higher place.

$$\text{d. } 315+3=\left\{\begin{array}{ccc}3&&\\&1&\\&&5\end{array}\right. + \left\{\begin{array}{ccc}&&\\&&\\&&3\end{array}\right. = \left\{\begin{array}{ccc}3&&\\&1&\\&&5+3\end{array}\right.$$

$$=\left\{\begin{array}{ccc}3&&\\&1&\\&&8\end{array}\right.$$
$$\overline{\begin{array}{ccc}3&1&8\end{array}}$$

$$\text{e. } 56142+248=\left\{\begin{array}{ccccc}5&&&&\\&6&&&\\&&1&&\\&&&4&\\&&&&2\end{array}\right. + \left\{\begin{array}{ccccc}&&&&\\&&&&\\&&2&&\\&&&4&\\&&&&8\end{array}\right.$$

$$\text{f. } =\left\{\begin{array}{ccccc}5&&&&\\&6&&&\\&&1+2&&\\&&&4+4&\\&&&&2+8\end{array}\right.$$

$$\text{g. } \left\{\begin{array}{ccccc}5&&&&\\&6&&&\\&&3&&\\&&&8&\\&&&&10\end{array}\right. = \left\{\begin{array}{ccccc}5&&&&\\&6&&&\\&&3&&\\&&&{}^{1}8&\\&&&&0\end{array}\right.$$
$$\overline{\begin{array}{ccccc}5&6&3&9&0\end{array}}$$

The 1 in 10 is sent to 8 and written on the top-left of 8.
Clearly, it can be seen that the renaming process is very much more visible in diagonal addition than in the usual rectangular method.

2.3 Diagonal subtraction of numbers

Suppose again that
D = $d_{kk} \ldots d_{33}\, d_{22}\, d_{11}$ and
E = $e_{kk} \ldots e_{33}\, e_{22}\, e_{11}$

Then by diagonal subtraction we have

$$D+E=\left\{\begin{array}{cccccc}k_{kk}&&&&&\\&.&&&&\\&&.&&&\\&&&d_{33}&&\\&&&&d_{22}&\\&&&&&d_{11}\end{array}\right. - \left\{\begin{array}{cccccc}e_{kk}&&&&&\\&.&&&&\\&&.&&&\\&&&e_{33}&&\\&&&&e_{22}&\\&&&&&e_{11}\end{array}\right.$$

$$=\left\{\begin{array}{llllll} d_{kk}+e_{kk} & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & d_{33}-e_{33} & & \\ & & & & d_{22}-e_{22} & \\ & & & & & d_{11}-e_{11} \end{array}\right.$$

Note

- Diagonal subtraction is NOT commutative i.e. D – E ≠ E – D
- Subtraction proceeds from left to right
- A digit in a higher place value can be renamed and sent down to a digit in a lower place value
- Renaming occurs before subtraction.
- The digit sent down is attached in front of the digit in that place.

The following example will illustrate the rules clearly.

a. $45-21=\left\{\begin{array}{ll}4 & \\ & 5\end{array}\right. - \left\{\begin{array}{ll}2 & \\ & 1\end{array}\right. = \left\{\begin{array}{ll}4-2 & \\ & 5+1\end{array}\right.$

$=\left\{\begin{array}{ll}2 & \\ & 4\end{array}\right.$

$\overline{\begin{array}{ll}2 & 4\end{array}}$

b. $485-25=\left\{\begin{array}{lll}4 & & \\ & 8 & \\ & & 5\end{array}\right. - \left\{\begin{array}{lll} & & \\ & 2 & \\ & & 5\end{array}\right.$

$=\left\{\begin{array}{lll}4 & & \\ & 8-2 & \\ & & 5-5\end{array}\right. = \left\{\begin{array}{lll}4 & & \\ & 6 & \\ & & 0\end{array}\right.$

$\overline{\begin{array}{lll}4 & 6 & 0\end{array}}$

c. $248-54=\left\{\begin{array}{lll}2 & & \\ & 4 & \\ & & 8\end{array}\right. - \left\{\begin{array}{lll} & & \\ & 5 & \\ & & 4\end{array}\right.$

$=\left\{\begin{array}{lll}{}^{\square}1 & & \\ & 14 & \\ & & 8\end{array}\right. - \left\{\begin{array}{lll} & & \\ & 5 & \\ & & 4\end{array}\right.$

$=\left\{\begin{array}{lll}1 & & \\ & 14-5 & \\ & & 8-4\end{array}\right. - \left\{\begin{array}{lll}1 & & \\ & 9 & \\ & & 4\end{array}\right.$

$\overline{\begin{array}{lll}1 & 9 & 4\end{array}}$

d. $78-9=\left\{\begin{array}{ll}7 & \\ & 8\end{array}\right. - \left\{\begin{array}{ll} & \\ & 9\end{array}\right. = \left\{\begin{array}{ll}\overset{1}{\square}6 & \\ & 8\end{array}\right. - \left\{\begin{array}{ll} & \\ & 9\end{array}\right.$

$=\left\{\begin{array}{ll}7 & \\ & 18\end{array}\right. - \left\{\begin{array}{ll} & \\ & 9\end{array}\right. = \left\{\begin{array}{ll}6 & \\ & 18-9\end{array}\right. - \left\{\begin{array}{ll}6 & \\ & 9\end{array}\right.$

$\overline{\begin{array}{ll}6 & 9\end{array}}$

e. $72514-4738=\left\{\begin{array}{lllll}7 & & & & \\ & 2 & & & \\ & & 5 & & \\ & & & 1 & \\ & & & & 4\end{array}\right. - \left\{\begin{array}{lllll} & & & & \\ & 4 & & & \\ & & 7 & & \\ & & & 3 & \\ & & & & 8\end{array}\right.$

$=\left\{\begin{array}{lllll}\overset{1}{\square}6 & & & & \\ & \overset{1}{\square}1 & & & \\ & & \overset{1}{\square}4 & & \\ & & & \overset{1}{\square}0 & \\ & & & & 4\end{array}\right. - \left\{\begin{array}{lllll} & & & & \\ & 4 & & & \\ & & 7 & & \\ & & & 3 & \\ & & & & 8\end{array}\right.$

$\left\{\begin{array}{lllll}6 & & & & \\ & 11 & & & \\ & & 14 & & \\ & & & 10 & \\ & & & & 14\end{array}\right. - \left\{\begin{array}{lllll} & & & & \\ & 4 & & & \\ & & 7 & & \\ & & & 3 & \\ & & & & 8\end{array}\right.$

f. $\left\{\begin{array}{lllll}6 & & & & \\ & 11-4 & & & \\ & & 14-7 & & \\ & & & 10-3 & \\ & & & & 14-8\end{array}\right.$

$=\left\{\begin{array}{lllll}6 & & & & \\ & 7 & & & \\ & & 7 & & \\ & & & 7 & \\ & & & & 6\end{array}\right.$

$\overline{\begin{array}{lllll}6 & 7 & 7 & 7 & 6\end{array}}$

Notice here too that the renaming process becomes more visible than that which occurs in the rectangular method of subtraction.

2.4 Diagonal multiplication of numbers

Suppose that D is a two digit number and one E is another number having just one digit, then,

$$D = d_{22}\, d_{11} = \left\{\begin{array}{ll} d_{22} & \\ & d_{11} \end{array}\right.$$

And

$$E = e_{11} = \left\{\begin{array}{ll} & \\ & e_{11} \end{array}\right.$$

And so

$$D \text{ x } E \left\{\begin{array}{ll} d_{22} & \\ & e_{11} \end{array}\right. \text{x} \left\{\begin{array}{ll} & \\ & e_{11} \end{array}\right.$$

$$= \left\{\begin{array}{ll} d_{22}\ x\ e_{11} & \\ & d_{11}\ x\ e_{11} \end{array}\right.$$

$$E \text{ x } D = \left\{\begin{array}{ll} & \\ & e_{11} \end{array}\right. \text{x} \left\{\begin{array}{ll} d_{22} & \\ & e_{11} \end{array}\right.$$

$$= \left\{\begin{array}{ll} e_{11} \text{ x } d_{22} & \\ & e_{11} \text{ x } d_{11} \end{array}\right.$$

Since d_{22} x e_{11} = e_{11} x d_{22}
And d_{11} x e_{11} = e_{11} x d_{11}
It follows that

D x E = E x D

Thus diagonal multiplication is commutative.

Note

- Multiplication proceeds from left to right
- Multiplication is completed before renaming
- During renaming digits are sent to their original places and written at the top-left of the digit there.

Consider the following example

a. $21 \text{ x } 3 = \left\{\begin{array}{ll} 2 & \\ & 1 \end{array}\right. \text{x} \left\{\begin{array}{ll} & \\ & 3 \end{array}\right. = \left\{\begin{array}{ll} 2\text{x}3 & \\ & 1\text{x}3 \end{array}\right.$

$\left\{\begin{array}{ll} 6 & \\ & 3 \end{array}\right.$

6 3

b. $25 \text{ x } 4 = \left\{\begin{array}{ll} 2 & \\ & 5 \end{array}\right. \text{x} \left\{\begin{array}{ll} & \\ & 4 \end{array}\right. = \left\{\begin{array}{ll} 2\text{x}4 & \\ & 5\text{x}4 \end{array}\right.$

$\left\{\begin{array}{ll} 8 & \\ & 20 \end{array}\right. = \left\{\begin{array}{ll} {}^{2}8 & \\ & 0 \end{array}\right. = \left\{\begin{array}{ll} 10 & \\ & 0 \end{array}\right.$

$\left\{\begin{array}{lll} 1 & & \\ & 0 & \\ & & 0 \end{array}\right.$

1 0 0

Suppose now that
D = $d_{22}\, d_{11}$ and E = $e_{22}\, e_{11}$
Then

$$D \text{ x } E = \left\{\begin{array}{ll} d_{22} & \\ & d_{11} \end{array}\right. \text{x} \left\{\begin{array}{ll} e_{22} & \\ & e_{11} \end{array}\right.$$

$$= \left\{\begin{array}{lll} d_{22} \text{ x } e_{22} & & \\ & d_{22} \text{ x } e_{11} + d_{11} \text{ x } e_{22} & \\ & & d_{11} \text{ x } e_{11} \end{array}\right.$$

Now, since
$e_{22} \text{ x } d_{22} = d_{22} \text{ x } e_{22}$
$e_{22} \text{ x } d_{11} + e_{11} \text{ x } d_{22} = d_{22} \text{ x } e_{22} + d_{11} \text{ x } e_{22}$
And
$e_{11} \text{ x } d_{11} = d_{11} \text{ x } e_{11}$
It follows that diagonal multiplication is indeed commutative. The following examples will illustrate the process clearly

c. $23 \text{ x } 21 = \left\{\begin{array}{ll}2 & \\ & 3\end{array}\right. \text{ x } \left\{\begin{array}{ll}2 & \\ & 1\end{array}\right.$

$$= \left\{\begin{array}{lll}2\text{x}2 & & \\ & 12\text{x}1 + 3\text{x } 2 & \\ & & 3\text{x}1\end{array}\right.$$

$$= \left\{\begin{array}{lll}4 & & \\ & 2+6 & \\ & & 3\end{array}\right. = \left\{\begin{array}{lll}4 & & \\ & 8 & \\ & & 3 \\ \hline 4 & 8 & 3\end{array}\right.$$

Also

$21 \text{ x } 23 = \left\{\begin{array}{ll}2 & \\ & 1\end{array}\right. \text{ x } \left\{\begin{array}{ll}2 & \\ & 3\end{array}\right.$

$$= \left\{\begin{array}{lll}2\text{x}2 & & \\ & 2\text{x}3 + 1\text{x } 2 & \\ & & 1\text{x}3\end{array}\right.$$

$$= \left\{\begin{array}{lll}4 & & \\ & 6+2 & \\ & & 3\end{array}\right. = \left\{\begin{array}{lll}4 & & \\ & 8 & \\ & & 3 \\ \hline 4 & 8 & 3\end{array}\right.$$

d. $24 \text{ x } 63 = \left\{\begin{array}{ll}2 & \\ & 4\end{array}\right. \text{ x } \left\{\begin{array}{ll}6 & \\ & 3\end{array}\right.$

$$= \left\{\begin{array}{lll}2\text{x}6 & & \\ & 2\text{x}3 + 4\text{x } 6 & \\ & & 4\text{x}3\end{array}\right.$$

$$= \left\{\begin{array}{lll}12 & & \\ & 6+24 & \\ & & 12\end{array}\right. = \left\{\begin{array}{lll}12 & & \\ & 30 & \\ & & 12\end{array}\right.$$

$$= \left\{\begin{array}{llll}1 & & & \\ & {}^{3}2 & & \\ & & {}^{1}0 & \\ & & & 2 \\ \hline 1 & 5 & 1 & 2\end{array}\right.$$

And

a. $63 \text{ x } 24 = \left\{\begin{array}{ll}6 & \\ & 3\end{array}\right. \text{ x } \left\{\begin{array}{ll}2 & \\ & 4\end{array}\right.$

$$= \left\{\begin{array}{lll}6\text{x}2 & & \\ & 4\text{x } 6 + 3\text{x}2 & \\ & & 3\text{x}4\end{array}\right.$$

$$= \left\{\begin{array}{lll}12 & & \\ & 24+6 & \\ & & 12\end{array}\right. = \left\{\begin{array}{lll}12 & & \\ & 30 & \\ & & 12\end{array}\right.$$

$$= \left\{\begin{array}{llll}1 & & & \\ & {}^{3}2 & & \\ & & {}^{1}0 & \\ & & & 2 \\ \hline 1 & 5 & 1 & 2\end{array}\right.$$

Let $D = d_{33}\, d_{22}\, d_{11}$ and $E = e_{33}\, e_{22}\, e_{11}$
Then

$$\text{D x E} = \left\{\begin{array}{lll}d_{33} & & \\ & d_{22} & \\ & & d_{11}\end{array}\right. \text{ x } \left\{\begin{array}{lll}e_{33} & & \\ & e_{22} & \\ & & e_{11}\end{array}\right.$$

$$\begin{cases} d_{33} \text{ x } e_{33} \\ \quad d_{33} \text{ x } e_{22} + d_{22} \text{ x } e_{33} \\ \qquad d_{33} \text{ x } e_{11} + d_{22} \text{ x } e_{22} + d_{11} \text{ x } e_{33} \\ \qquad\quad d_{22} \text{ x } e_{11} + d_{11} \text{ x } e_{22} \end{cases}$$

Note also that diagonal multiplication of 3- digits is cumulative
i.e. D x E = E x D
Consider the case below

e. $213 \text{ x } 452 = \begin{cases} 2 \\ \quad 1 \\ \qquad 3 \end{cases} \text{ x } \begin{cases} 4 \\ \quad 5 \\ \qquad 2 \end{cases}$

$$= \begin{cases} 2\text{x}4 \\ \quad 2\text{x}5 + 1\text{x}4 \\ \qquad 2\text{x}2 + 1\text{x}5 + 3\text{x}4 \\ \qquad\quad 1\text{x}2 + 3\text{x}5 \\ \qquad\qquad 3\text{x}2 \end{cases}$$

$$= \begin{cases} 8 \\ \quad 10 + 4 \\ \qquad 4 + 5 + 12 \\ \qquad\quad 2 + 15 \\ \qquad\qquad 6 \end{cases}$$

$$= \begin{cases} 8 \\ \quad 14 \\ \qquad 21 \\ \qquad\quad 17 \\ \qquad\qquad 6 \end{cases} \qquad = \begin{cases} {}^{1}8 \\ \quad {}^{2}4 \\ \qquad {}^{1}1 \\ \qquad\quad 7 \\ \qquad\qquad 6 \end{cases}$$

$$\overline{9 \quad 6 \quad 2 \quad 7 \quad 6}$$

f. $513 \text{ x } 24 = \begin{cases} 5 \\ \quad 3 \\ \qquad 1 \end{cases} \text{ x } \begin{cases} 2 \\ \quad 4 \end{cases}$

$$= \begin{cases} 5\text{x}2 \\ \quad 5\text{x}4 + 3\text{x}2 \\ \qquad 3\text{x}4 + 1\text{x}2 \\ \qquad\quad 1\text{x}4 \end{cases}$$

$$= \begin{cases} 10 \\ \quad 20 + 6 \\ \qquad 12 + 2 \\ \qquad\quad 4 \end{cases} \qquad = \begin{cases} 10 \\ \quad 26 \\ \qquad 14 \\ \qquad\quad 4 \end{cases}$$

$$= \begin{cases} 1 \\ \quad {}^{2}0 \\ \qquad {}^{1}6 \\ \qquad\quad 4 \\ \qquad\qquad 4 \end{cases}$$

$$\overline{1 \quad 2 \quad 7 \quad 4 \quad 4}$$

In general, if
D = $d_{kk} \ldots d_{33}\, d_{22}\, d_{11}$ and
E = $e_{kk} \ldots e_{33}\, e_{22}\, e_{11}$
Then by diagonal multiplication of D and E we have

$$\text{D + E} = \begin{cases} k_{kk} \\ \quad . \\ \qquad . \\ \qquad\quad d_{33} \\ \qquad\qquad d_{22} \\ \qquad\qquad\quad d_{11} \end{cases} \text{ x } \begin{cases} e_{kk} \\ \quad . \\ \qquad . \\ \qquad\quad e_{33} \\ \qquad\qquad e_{22} \\ \qquad\qquad\quad e_{11} \end{cases}$$

$$= \begin{cases} d_{kk} \text{x } e_{kk} \\ \quad . \\ \quad\quad . \\ \quad\quad\quad d_{kk} \text{x } e_{33} \ldots + \ d_{33} \text{x } e_{33} \\ \quad\quad\quad\quad d_{kk} \text{x } e_{22} \ldots + d_{33} \text{x } e_{33} + d_{22} \text{x } e_{22} \\ \quad\quad\quad\quad\quad d_{kk} \text{x } e_{11} \ldots + d_{33} \text{x } e_{11} \end{cases}$$

To illustrate this further, let us take another example

$$3125 \text{ x } 1468 = \begin{cases} 3 \\ \quad 1 \\ \quad\quad 2 \\ \quad\quad\quad 5 \end{cases} \text{ x } \begin{cases} 1 \\ \quad 4 \\ \quad\quad 6 \\ \quad\quad\quad 8 \end{cases}$$

$$= \begin{cases} 3\text{x}1 \\ \quad 3\text{x}4 + 1\text{x}1 \\ \quad\quad 3\text{x}6 + 1\text{x}4 + 2\text{x}1 \\ \quad\quad\quad 3\text{x}8 + 1\text{x}6 + 2\text{x}4 + 5\text{x}1 \\ \quad\quad\quad\quad 1\text{x}8 + 2\text{x}6 + 5\text{x}4 \\ \quad\quad\quad\quad\quad 2\text{x}8 + 5x6 \\ \quad\quad\quad\quad\quad\quad 5\text{x}8 \end{cases}$$

$$= \begin{cases} 3 \\ \quad 12 + 1 \\ \quad\quad 18 + 4 + 2 \\ \quad\quad\quad 24 + 6 + 8 + 5 \\ \quad\quad\quad\quad 8 + 12 + 20 \\ \quad\quad\quad\quad\quad 16 + 30 \\ \quad\quad\quad\quad\quad\quad 40 \end{cases}$$

$$= \begin{cases} 3 \\ \quad 13 \\ \quad\quad 24 \\ \quad\quad\quad 43 \\ \quad\quad\quad\quad 40 \\ \quad\quad\quad\quad\quad 46 \\ \quad\quad\quad\quad\quad\quad 40 \end{cases}$$

$$= \begin{cases} 4 \\ \quad 5 \\ \quad\quad 8 \\ \quad\quad\quad 7 \\ \quad\quad\quad\quad {}^{1}4 \\ \quad\quad\quad\quad\quad 0 \\ \quad\quad\quad\quad\quad\quad 0 \end{cases}$$

$$\begin{array}{ccccccc} \hline 4 & 5 & 8 & 7 & 5 & 0 & 0 \end{array}$$

As is the case with addition and subtraction, we can see clearly too that diagonal multiplication is not only possible but also brings out the renaming process in a manner that is easy to understand.

Note

- Diagonal multiplication is commutative irrespective of the number digits
- Multiplication proceeds from left to right
- Renaming occurs only after multiplication has been completed.
- Every digit is sent from a lower place value to its rightful place in a higher place value.
- The multiplication could be cumbersome. Therefore to reduce the laboy, we could decide to perform the multiplication outside and arrange the results inside. See the illustration below

First multiply the numbers as shown below

3 x 1 = 3	1 x 1 = 1	2 x 1 = 2	5 x 1 = 5
3 x 4 = 12	1 x 4 = 4	2 x 4 = 8	5 x 4 = 20
3 x 6 = 18	1 x 6 = 6	2 x 6 = 12	5 x 6 = 30
3 x 8 = 24	1 x 8 = 8	2 x 8 = 16	5 x 8 = 40

Then arrange the results as shown

3	= 3
12 + 1	= 13
18 + 4 + 2	= 24
24 + 6 + 8 +5	= 43
8 + 12 +20	= 40
16 + 30	= 46
40	= 40

Then arrange the results inside as shown

$$.:\ 3125 \times 1468 = \left\{\begin{array}{cccc} 3 & & & \\ & 1 & & \\ & & 2 & \\ & & & 5 \end{array}\right. \times \left\{\begin{array}{cccc} 1 & & & \\ & 4 & & \\ & & 6 & \\ & & & 8 \end{array}\right.$$

$$= \left\{\begin{array}{ccccccc} 3 & & & & & & \\ & 13 & & & & & \\ & & 24 & & & & \\ & & & 43 & & & \\ & & & & 40 & & \\ & & & & & 46 & \\ & & & & & & 40 \end{array}\right.$$

$$= \left\{\begin{array}{ccccccc} {}^{1}3 & & & & & & \\ & {}^{2}3 & & & & & \\ & & {}^{4}4 & & & & \\ & & & {}^{4}3 & & & \\ & & & & {}^{4}0 & & \\ & & & & & {}^{4}0 & \\ & & & & & & 0 \end{array}\right.$$

$$= \left\{\begin{array}{ccccccc} 4 & & & & & & \\ & 5 & & & & & \\ & & 8 & & & & \\ & & & 7 & & & \\ & & & & 4 & & \\ & & & & & 10 & \\ & & & & & & 0 \end{array}\right. = \left\{\begin{array}{ccccccc} 4 & & & & & & \\ & 5 & & & & & \\ & & 8 & & & & \\ & & & 7 & & & \\ & & & & {}^{1}4 & & \\ & & & & & 0 & \\ & & & & & & 0 \\ \hline 4 & 5 & 8 & 7 & 5 & 0 & 0 \end{array}\right.$$

2.5 Diagonal division of numbers

Suppose that

$D = d_{33}\ d_{22}\ d_{11}$ and $E = e_{11}$

Then

$D \div E = d_{33}\ d_{22}\ d_{11} \div e_{11}$

$$= \left\{\begin{array}{ccc} d_{33} & & \\ & d_{22} & \\ & & d_{11} \end{array}\right. \div \left\{\begin{array}{ccc} & & \\ & & \\ & & e_{11} \end{array}\right.$$

For simplicity, let

$$\left\{\begin{array}{ccc} d_{33} & & \\ & d_{22} & \\ & & d_{11} \end{array}\right. \div \left\{\begin{array}{ccc} & & \\ & & \\ & & e_{11} \end{array}\right. = {}_{e_{11}}\left\{\begin{array}{ccc} d_{33} & & \\ & d_{22} & \\ & & d_{11} \end{array}\right.$$

Then

$$D \div E = {}_{e_{11}}\begin{cases} d_{33} & d_{22} \quad d_{11} \end{cases}$$

Note

If d_{33} d_{22} and d_{11} are divisible by e_{11} then

$$D \div E = {}_{e_{11}}\begin{cases} d_{33} & d_{22} \quad d_{11} \end{cases} = {}_{e_{11}}\begin{cases} \frac{d_{33}}{e_{11}} & \frac{d_{22}}{e_{11}} \quad \frac{d_{11}}{e_{11}} \end{cases}$$

- If d_{33} d_{22} and d_{11} are not all divisible by e_{11} then renaming must be done before division
- Division can actually proceed from right to left or from left to right. In other words, division goes both ways
- During renaming, a digit in a higher places value does 10, 20 etc. to a digit in the adjacent place value below it.

The following examples will illustrate these points.

a. $24 \div 2 = {}_{2}\{2 \quad 4 = \{\frac{2}{2} \quad \frac{4}{2} = \{1 \quad 2 = 1\ 2$

b. $72 \div 3 = {}_{3}\{7 \quad 2 = {}_{3}\{{}^{1}6 \quad 2 = {}_{3}\{6 \quad 12$

$= \{\frac{6}{3} \quad \frac{12}{3} = \{2 \quad 4 = 2\ 4$

c. $84 \div 3 = {}_{3}\{8 \quad 4 = {}_{3}\{{}^{2}6 \quad 4 = {}_{3}\{6 \quad 24$

$= \{\frac{6}{3} \quad \frac{24}{3} = \{2 \quad 8 = 2\ 8$

d. $124 \div 2 = {}_{2}\{1 \quad 2 \quad 4 = {}_{2}\{{}^{1}0 \quad 2 \quad 4 = {}_{2}\{0 \quad 12 \quad 4$

$= \{\frac{0}{2} \quad \frac{12}{2} \quad \frac{4}{2} = \{0 \quad 6 \quad 2 = 0\ 6\ 2$

e. $112 \div 2 = {}_{2}\{1 \quad 1 \quad 2 = = {}_{2}\{11 \quad 4 = {}_{2}\{{}^{1}10 \quad 2$

${}_{2}\{10 \quad 12 = \{\frac{10}{2} \quad \frac{12}{2} = \{5 \quad 6 = 5\ 6$

f. $102 \div 4 = {}_{4}\{10 \quad 2 = \{\frac{10}{2} \quad \frac{2}{3} \quad \frac{4}{2} = {}_{2}\{5 \quad 1 = {}_{2}\{4 \quad 11$

${}_{2}\{4 \quad 10 \quad 1 = \{\frac{4}{2} \quad \frac{10}{2} \quad \frac{1}{2} = \{2 \quad 5 \quad \frac{1}{2} = 2\ 5\ \frac{1}{2}$

If there is a common factor between the divisor and the dividend, then it is used to divide both of them.

g. 128 ÷ 16 = $_{16}\{1 \quad 2 \quad 8$ = $_{16}\{12 \quad 8$ = $_{\frac{16}{4}}\{\frac{12}{4} \quad \frac{8}{4}$

= $_{2}\{1 \quad 6$ = = $_{2}\{ \quad 16$ = $_{2}\{ \quad \frac{16}{2}$ = 8

h. 180 ÷ 15 = $_{15}\{18 \quad 0$ = $_{15}\{{}^{3}15 \quad 0$ = $_{15}\{15 \quad 30$

= $\{1 \quad 2$

1 2

i. 1348 ÷ 25 = $_{25}\{13 \quad 4 \quad 8$ = $_{25}\{134 \quad 8$

= $_{25}\{125 \quad 98$ = $_{25}\{125 \quad 75 \quad 23$ = $\{5 \quad 3 \quad \frac{23}{25}$

= $\{5 \quad 3 \quad . \quad \frac{230}{25}$ = $\{5 \quad 3 \quad . \quad \frac{255}{25} \quad \frac{5}{25}$

= $\{5 \quad 3 \quad . \quad 9 \quad \frac{50}{25}$ = 5 3 . 9 2

5 3 . 9 2

j. 49 ÷ 5 = $_{5}\{4 \quad 9$ = $_{5}\{4 \quad 5 \quad 4$ = $_{5}\{45 \quad 4$

$\{9 \quad \frac{4}{5}$ = $_{5}\{9 \quad \frac{40}{5}$ = $\{9 \quad . \quad 8$

9 . 8

k. 4.92 ÷ 3 = $_{3}\{4 \quad . \quad 9 \quad 2$ = $_{3}\{4 \quad . \quad 8 \quad 12$

= $_{3}\{3 \quad . \quad 18 \quad 12$ = $\{1 \quad . \quad 6 \quad 4$

1 . 6 4

The foregoing examples have indeed demonstrated that division of numbers is equally possible by diagonal method. It must be borne in mind; however, that division of numbers is NOT commutative by the diagonal method.

3. Results and discussion

It is important to examine the distinction between diagonal arithmetic's and the conventional rectangular form of arithmetic. Note here that we use the term rectangular arithmetic to refer to that form of arithmetic whereby the semiotic arrangement of the digits of numbers is a rectangular array. On the other hand, diagonal arithmetic is used to refer to a form of arithmetic whereby the semiotic arrangement id the digits of numbers is mainly the leading diagonal of a matrix. It must be borne in mind, however, that both methods of arithmetic operate in a sense like matrices.

It is again important to begin by considering the case of addition in the following illustration.

Addition without renaming
Rectangular method:

$$26 + 73 = \begin{array}{cc} 2 & 6 \\ +\,7 & 3 \\ \hline 9 & 9 \end{array}$$

Diagonal method

$$26 + 73 = \left\{\begin{array}{ll} 2 & \\ & 6 \end{array}\right. + \left\{\begin{array}{ll} 7 & \\ & 3 \end{array}\right. = \left\{\begin{array}{ll} 9 & \\ & 9 \end{array}\right. \\ \overline{\;9 \quad 9\;}$$

Addition with renaming
Rectangular method

$$27 + 94 = \begin{array}{cc} {}^{1}2 & 7 \\ +\,9 & 4 \\ \hline 1\;\;2 & 1 \end{array}$$

Diagonal method

$$27 + 94 = \left\{\begin{array}{ll} 2 & \\ & 7 \end{array}\right. + \left\{\begin{array}{ll} 9 & \\ & 4 \end{array}\right. = \left\{\begin{array}{ll} 11 & \\ & 11 \end{array}\right.$$

$$\left\{\begin{array}{lll} 1 & & \\ & {}^{1}1 & \\ & & 1 \end{array}\right. \\ \overline{\;1 \quad 2 \quad 1\;}$$

Notice that for addition without renaming both methods appear to be quite simple and easy and so it is not easy to see the distinction between them. On the other hand a closer look at the process with renaming, one can see clearly that the two methods are distinct. The rectangular method is short and concise but a bit unclear because the renaming process is not very visible, whereas the diagonal method is a little cumbersome to write but very clear because the renaming process is quite visible and detailed.

In addition, consider the case of subtraction as shown below

Subtraction without renaming
Rectangular method

$$86 - 45 = \begin{array}{cc} 8 & 6 \\ -\,4 & 5 \\ \hline 4 & 1 \end{array}$$

Subtraction with renaming
Rectangular method

$$72 - 45 = \begin{array}{cc} 7 & 2 \\ -\,4 & 5 \\ \hline 2 & 7 \end{array}$$

Diagonal method

$$72 - 45 = \left\{\begin{array}{ll} 7 & \\ & 2 \end{array}\right. - \left\{\begin{array}{ll} 4 & \\ & 5 \end{array}\right. = \left\{\begin{array}{ll} 6 & \\ & 12 \end{array}\right. - \left\{\begin{array}{ll} 4 & \\ & 5 \end{array}\right.$$

$$= \left\{\begin{array}{ll} 2 & \\ & 7 \end{array}\right. \\ \overline{\;2 \quad 7\;}$$

Diagonal method

It can be seen here too that subtraction without renaming does not pose any challenge at all for both methods owing to its simplicity. However, the case of subtraction with renaming by the rectangular method does post a little challenge because the renaming process is hidden and thus a bit more imaginary. The diagonal method on the contrary does reveal the renaming process clearly even though it is not as concise as the rectangular method.

Furthermore, consider the case of multiplication of 2-digits as shown in the following illustrations.

Multiplication without renaming
Rectangular method

$$
\begin{array}{lrrr}
21 \times 32 = & & 2 & 1 \\
& \times & \underline{3} & \underline{2} \\
& & 4 & 2 \\
& \underline{6} & \underline{3} & \\
& \underline{6} & \underline{7} & \underline{2}
\end{array}
$$

Diagonal method

$$21 \times 32 = \begin{cases} 2 & \\ & 1 \end{cases} \times \begin{cases} 3 & \\ & 2 \end{cases} = \begin{cases} 2x3 & & \\ & 2x2 + 1x3 & \\ & & 1x2 \end{cases}$$

$$= \begin{cases} 6 & & \\ & 4+3 & \\ & & 2 \end{cases} = \begin{cases} 6 & & \\ & 7 & \\ & & 2 \end{cases}$$

$$\begin{array}{ccc} \hline 6 & 7 & 2 \end{array}$$

Multiplication with renaming
Rectangular method

$$
\begin{array}{lrrrr}
26 \times 73 = & & & 2 & 6 \\
& & \times & \underline{7} & \underline{3} \\
& & & 7 & 8 \\
& \underline{1} & \underline{8} & \underline{2} & \\
& \underline{1} & \underline{8} & \underline{9} & \underline{8}
\end{array}
$$

Diagonal method

$$26 \times 73 = \begin{cases} 2 & \\ & 6 \end{cases} \times \begin{cases} 7 & \\ & 3 \end{cases} = \begin{cases} 2x7 & & \\ & 2x3 + 6x7 & \\ & & 6x3 \end{cases}$$

$$= \begin{cases} 14 & & \\ & 6+42 & \\ & & 18 \end{cases} = \begin{cases} 14 & & \\ & 48 & \\ & & 18 \end{cases}$$

$$= \begin{cases} 1 & & & \\ & {}^{4}4 & & \\ & & {}^{1}8 & \\ & & & 8 \\ \hline 1 & 8 & 9 & 8 \end{cases}$$

Notice that the multiplication process with the conventional rectangular method is quite concise with or without renaming. However, the renaming aspect poses a challenge in that it leaves much to be imagined and so it is not clear enough to enhance understanding. On the contrary, the diagonal method of multiplication is a little cumbersome but it reveals all of what takes place during renaming and so it is clear enough to enhance understanding.

Similar observations can be made in the case of division. Consider, for instance, the following division of a 2-digit number by a 1-digit number.

Division without renaming
Division algorithm

$$
\begin{array}{r|l}
 & \;\; 2\;1 \\ \hline
3 & \;\; 6\;3 \\
 & -\,\underline{6\;1} \\
 & \quad\;\; 3 \\
 & \;\; \underline{-\;3} \\
 & \quad\;\; 0
\end{array}
$$

$\therefore 63 \div 3 = 21$

Diagonal method

$$63 \div 3 = {}_{3}\begin{cases} 6 & \\ & 3 \end{cases} \div \begin{cases} \frac{6}{3} & \\ & \frac{3}{3} \end{cases} = \begin{cases} 2 & \\ & 1 \end{cases}$$
$$\overline{\;2\,1\;}$$

Division with renaming
Division algorithm

$$\begin{array}{r|l} & \;\;2\;\;4 \\ \hline 3 & \;\;7\;\;2 \\ & -\underline{6\;\;\;\;\;} \\ & \;\;1\;\;2 \\ & -\underline{1\;\;2} \\ & \;\;\;\;\;\;0 \end{array}$$

$$\therefore\; 72 \div 3 = 24$$

Diagonal method

$$72 \div 3 = {}_{3}\begin{cases} 7 & \\ & 2 \end{cases} = \begin{cases} 6 & \\ & 12 \end{cases} = \begin{cases} 2 & \\ & 4 \end{cases}$$
$$\overline{\;2\,4\;}$$

From the illustration above, it does in fact seem like diagonal division has a little advantage over the division algorithm with or without renaming. Again, the renaming process here is more visible and thus clearer to understand. Besides, the level of maneuvering involved in the division algorithm seems to be a bit confusing. On the other hand, this maneuvering is less in diagonal division.

The point to emphasize here is if clarity is the vital element of doing elementary mathematics, then clarity must as well be the goal of arithmetic, and if this is the case then diagonal arithmetic provides a good and innovative alternative for enhancing clarity and understanding. This is evidently so because of the manner in which diagonal arithmetic simplifies the renaming processes in arithmetical operations such as addition, subtraction, multiplication and division.

4. Conclusion

In the light of the foregoing expositions, the following conclusions are hereby drawn.

1. Every number can be written as a diagonal matrix;
2. Diagonal matrix is very useful for performing arithmetical operations such as addition, subtraction, multiplication and division etc;
3. Diagonal arithmetic is very efficient in arithmetic because it reveals more details in the renaming process taking place during arithmetical operation;
4. Diagonal arithmetic could be considered a useful alternative to the conventional form of arithmetic which we refer to in this context as rectangular arithmetic;
5. Diagonal arithmetic possesses the capacity to reveal what is yet to be known in number theory and algebra as well.

5. References

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