

Generalized autoregressive conditional heteroscedasticity (GARCH) for predicting volatility in Stock Market

Noorya Kargar

Lecturer, Department of Mathematics, Education Faculty, Jawzjan University, Afghanistan

Corresponding Author: Noorya Kargar DOI: https://doi.org/10.54660/.IJMRGE.2021.2.3.73-75

Abstract

Volatility plays an important role in financial markets and has held the attention of academics and practitioners. Using *Generalized autoregressive conditional heteroscedasticity* (GARCH) is one of the effective perspective for forecasting volatility. This study focused on comparing GARCH (P, Q) model with GJR-GARCH (P, Q) model and EGARCH (P, Q) model to make prediction more reliable and accurate. The

Keywords: GARCH, Volatility Heteroscedastic time series

1. Introduction

Volatility refers to the standard deviation of the continuously compounded returns of a financial instrument with a specific time horizon. It is used to quantify the risk of the instrument over that time period. Historical volatility is the volatility of a financial instrument based on historical returns. This phrase is used particularly when it is wished to distinguish between the actual volatility of an instrument in the past, and the current volatility implied by the market. For a financial instrument whose price follows a Gaussian random walk, or Wiener process, the volatility increases as time increases? Conceptually, this is because there is an increasing probability that the instrument's price will be farther away from the initial price as time increases. However, rather than increase linearly, the volatility increases with the square-root of time as time increases, because some fluctuations are expected to cancel each other out, so the most likely deviation after twice the time will not be twice the distance from zero. The annualized volatility σ is the standard deviation σ of the instrument's logarithmic returns in a year. The generalized volatility σ_T for time horizon *T* in years is expressed as: = $TT\sigma\sigma$ (Nasar, 1992)^[8]. Note that the formula used to annualize returns is not deterministic, but is an extrapolation valid for a random walk process whose steps have finite variance.

More broadly, volatility refers to the degree of (typically short-term) unpredictable change over time of a certain variable. It may be measured via the standard deviation of a sample, as mentioned above. However, price changes actually do not follow Gaussian distributions. Better distributions used to describe them actually have "fat tails" although their variance remains finite. Therefore, other metrics may be used to describe the degree of spread of the variable. As such, volatility reflects the degree of risk faced by someone with exposure to that variable. Volatility does not imply direction. This is due to the fact that all changes are squared. An instrument that is more volatile is likely to increase or decrease in value more than one that is less volatile.

Volatility has become a key input to many investment decisions and portfolio creations. Investors and portfolio managers have certain levels of risk which they can bear. A good forecast of the volatility of asset prices over the investment holding period is a good starting point for assessing investment risk. Volatility is the most important variable in the pricing of derivative securities, whose trading volume has quadrupled in recent years. Nowadays, one can buy derivatives that are written on volatility itself, in which case the definition and measurement of volatility will be clearly specified in the derivative contracts. In these new contracts, volatility now becomes the underlying "asset." So a volatility forecast is needed to price such derivative contracts. Policy makers often rely on market estimates of volatility as a barometer for the vulnerability of financial markets and the economy.

GARCH is a mechanism that includes past variance in the explanation of future variances. More specifically, GARCH is a timeseries technique that you use to model the serial dependence of volatility. Compared with other time-series models, GARCH models can provide a better description for heteroscedastic time series (Christoffersen and Jacobs, 2005)^[6].

results suggested that both GARCH (P, Q) model and GJR-GARCH (P, Q) model are good choices for forecasting volatility in financial market, especially for describing heteroscedastic time series. GARCH models are consistent with various forms of efficient market theory. These theories state that asset returns observed in the past cannot improve the forecasts of asset returns in the future.

This paper applies GARCH methods to stock market and builds a GARCH model for the volatility of Dow Jones Index daily closing value log-return series.

The form of GARCH Models

The form of GARCH (P, Q) Models Conditional Mean Equation

$$y_t = C + \varepsilon_t + \Sigma^R_{i=1}\theta_i y_{t-i} + \Sigma^M_{j=11}\theta_j \varepsilon_{t-j} + \Sigma^{N_x}_{k=1}\beta_k x(t,k)$$

This is a general ARMAX(R, M,xN) model. It applies to all variance models with autoregressive coefficients $\{\phi_i\}$, moving average coefficients $\{\theta_j\}$, innovations $\{\epsilon_t\}$, and stationary return series $\{y_t\}$.

X is an explanatory regression matrix in which each column is a time series. X(t, k) denotes the tth row and κ th column of this matrix.

The eigenvalues $\{\lambda_i\}$ associated with the characteristic AR polynomial

 $\lambda^{R} - \varphi 1 \lambda^{R-1} - \varphi 1 \lambda^{R-2} - \dots + \varphi_{R}$ must lie inside the unit circle to ensure stationarity. Similarly, the eigen values associated with the characteristic MA polynomial1

 $\lambda^{M} - \varphi 1 \lambda^{M-1} + \varphi 1 \lambda^{M-2} + \dots + \varphi_{M}$ must lie inside the unit circle to ensure invertibility.

The form of EGARCH (P, Q) Models

The general EGARCH (P, Q) model ([7]) for the conditional variance of the innovations, with leverage terms and an explicit probability distribution assumption, is

$$\log \sigma_t^2 = \mathbf{k} + \Sigma_{i=1}^P \mathbf{G}_i \log \sigma_{t-1}^2 + \Sigma_{i=1}^Q \mathbf{A}_i \log \sigma_{t-1}^2$$

for the Student's t distribution, with degrees of freedom > 2.

$$\begin{split} & \Sigma^{P} \,_{i=1}G_{i} \, + \Sigma^{Q} \,_{j=1}A_{j} + \frac{1}{2} \, \Sigma^{Q} \,_{j=1}L_{j} < 1 \\ & k > 0 \\ & G_{i} \geq 0 \, i = 1, \, 2, \, \ldots , \, P \\ & A_{j} \geq 0 \, j = 1, \, 2, \, \ldots , \, Q \\ & A_{i} + L_{j} \geq 0 \, j = 1, \, 2, \, \ldots , \, Q \end{split}$$

The form of GJR-GARCH (P, Q) Models

The general GJR-GARCH (P, Q) model for the conditional variance of the innovations with leverage terms is

$$\begin{split} &\sigma^2{}_t = k + \Sigma^P{}_{i=1}G_i \, \sigma^2{}_{t-1} + \Sigma^Q{}_{j=1}A_j \epsilon^2{}_{t-j} + \Sigma^Q{}_{j=1}L_j S_{t-j} \epsilon^2{}_{t-j} \\ &\Sigma^P{}_{i=1}G_i + \Sigma^Q{}_{j=1}A_j + \frac{1}{2} \Sigma^Q{}_{j=1}L_j < 1 \\ &k > 0 \\ &G_i \ge 0 \; i = 1, \, 2, \, \dots, \, P \\ &A_j \ge 0 \; j = 1, \, 2, \, \dots, \, Q \\ &A_i + L_i \ge 0 \; j = 1, \, 2, \, \dots, \, Q \end{split}$$

Use of Maximum likelihood Estimation

To estimate the parameters of GARCH models, this study used MATLAB7.0 garchfit function. The garchfit function calls the appropriate log-likelihood objective function to estimate the model parameters using *maximum likelihood* estimation (MLE).

Use of model selection criteria AIC and BIC

AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) are proposed by Akaike (1974) and Schwarz (1978) respectively.

Results

Model Identification

This study used AIC and BIC criterion and list the result as follows:

	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)
AIC	$1.0e^{+0.04*}(-1.313796)$	$1.0e^{+004*}(-1.312641)$	$1.0e^{+004*}(-1.312441)$
BIC	-1.3109925	-1.310398	-1.309638

Model estimation

Table 2: GARCH (1, 1) model Diagnostic Checking (significance level α =0.05)

lag	Н	p-value	Q-Statitic	Critical value
10	0	0.1482	14.5789	18.3070
15	0	0.1842	19.6896	24.9958
20	0	0.4073	20.8290	31.4104

Table 3: GJR-GARCH (1, 1) model Diagnostic Checking (=0.05)

lag	Н	p-value	Q-Statitic	Critical value
10	0	0.2621	12.3532	18.3070
15	0	0.2917	17.4671	24.9958
20	0	0.5438	18.6629	31.4104

GARCH models are consistent with various forms of efficient market theory. These theories state that asset returns observed in the past cannot improve the forecasts of asset returns in the future. Since GARCH innovations $\{\epsilon_t\}$ are

serially uncorrelated, GARCH modeling does not violate efficient market theory.

Conclusion

Both GARCH (P, O) model and GJR-GARCH (P, O) model are good choices for forecasting volatility in financial market, especially for describing heteroscedastic time series. It should not be neglected that GARCH (P, Q) model responds equally to positive and negative shocks. To overcome its weakness, we use GJR-GARCH(1, 1) model and catch some leverage effects successfully which makes our prediction more reliable and accurate. However, recent empirical studies of high-frequency financial time series indicate that the tail behavior of GARCH models remains too short even with standardized Student-t innovations. So, GARCH models are only part of a solution. To make financial decisions, it is always necessary to connect GARCH models with other methods such as fundamental analysis. For instance, fundamental analysis can examine all relevant factors affecting the stock price in order to determine an intrinsic value for that stock.

References

- 1. Abdalla SZS. Modelling Exchange Rate Volatility using GARCH Models: Empirical Evidence from Arab Countries. International Journal of Economics and Finance. 2012; 4(3):216-229. doi:10.5539/ijef.v4n3p216
- Akaike H, A new look at the statistical model identification, IEEE Trans. Automatic Control, AC-19, 1974, 716-723.
- Andersen TG, Bollerslev T, Diebold FX, Labys P. Exchange Rate Returns Standardized by Realized Volatility Are (Nearly) Gaussian. Multinational Finance Journal. 2000; 4:159-179.
- 4. Antonakakis N, Darby J. Forecasting Volatility in Developing Countries' Nominal Exchange Returns. MPRA Paper No.40875, 2012.
- Bollerslev T. Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics. 1986; 31(3):307-327. doi:10.1016/0304-4076(86)90063-1
- Christoffersen P, Jacobs K. Which GARCH Model for Option Valuation? Management Science. 2004; 50:204-21.
- Griebeler CM. Models for Forecasting Exchange Rate Volatility: A Comparison between Developed and Emerging Countries. IMPA, 2010.
- 8. Nasar S. For Fed, a New Set of Tea Leaves, New York Times, 1992.
- 9. Schwarz G., Estimating the dimension of a model, Ann. Statist. 1978; 6:461-464.