International Journal of Multidisciplinary Research and Growth Evaluation.

Unsteady MHD flow of heat and mass transfer for second grade fluid in the presence of thermal radiation

Abbas Yusuf Balarabe [1*], Yale Ibrahim Danjumma [2], Gado Abubakar Abubakar [3], Yakubu Ibrahim Barga [4]
[1, 3] Department of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria
[2] Department of Physics, Kebbi State University of Science and Technology, Aliero, Nigeria
[2, 4] Department of Mathematics, Federal University Birnin Kebbi, Nigeria

* Corresponding Author: **Yale Ibrahim Danjuma**

Article Info

ISSN (online): 2582-7138
Volume: 04
Issue: 04
July-August 2023
Received: 17-05-2023;
Accepted: 07-06-2023
Page No: 122-128

Abstract
A study on Unsteady MHD flow of for second grade fluid with the effect of heat and mass transfer has been carried out in the presence of thermal radiation. The dimensionless governing equations are solved using Laplace transform technique. The solution for velocity, temperature and concentration field are obtained and analysed for the different values of flow parameters such as thermal Grashof number, mass Grashof number, Prandtl number, thermal radiation parameter, Schmidt number and time. The results obtained depicts that velocity profile increases with increasing parameter such as Pr, R and Gr and decrease with the increase of Gc, M and t. The temperature flow characteristics decreases with increasing number of Pr and t, and increases with increasing number of R. The concentration profile decreases with increasing value of Sc and t.

Keywords: Unsteady, Heat Transfer, Second Grade Fluid, Thermal Radiation

1. Introduction

The study of flow of heat and mass transfer for second grade fluid under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as magneto-hydro-dynamic (MHD) generator, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics [1]. Also, free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal [2]. Magneto-hydro-dynamic has attracted the attention of a large number of scholars due to its diverse applications [3-5]. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc. In engineering, it finds its application in MHD pumps, MHD bearings Kumar and Yadav [6]. Second grade fluids can model many fluids such as dilute polymer solutions, slurry flows, and industrial oils, and many flow problems with various geometries and different mechanical and thermal boundary conditions have been studied Ahmad and Vieru [7].

Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices VARMA and RAJU [8]. A study on MHD heat and mass transfer free convection flow along a vertical stretching sheet in the presence of magnetic field with heat generation was carried out by Samad and Mohebujjaman [9]. Saravana and Sreekanth [10] studied the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux. Hayat and Khan [11] examined the analytical solution for unsteady magneto-hydro-dynamic flow of second grade fluid past a porous plate with a porous medium due to an arbitrary well velocity. They obtained result by applying a lie symmetry and numerical method. Singh [12] analyzed the MHD free convection and mass transfer flow with heat source and thermal diffusion. The Paper deals with the study of free convection and mass transfer flow of an incompressible, viscous and electrically conducting fluid past a continuously moving infinite vertical plate in the presence of large suction and under the influence of uniform magnetic field considering heat source and thermal diffusion.

Kim [13] considered the unsteady magneto hydrodynamic convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. However, most of the previous works assume that the plate is at rest. In the present work, we consider the case of a semi-infinite moving porous plate with a constant velocity in the longitudinal direction when the magnetic field is imposed transversely to the plate. Many work has considered the free stream to consist of a mean velocity and temperature with a superimposed exponentially variation with time.

In view of the applications of free convective phenomenon, heat source and thermal diffusion, in the present work it is proposed to study the unsteady MHD free convective heat and mass transfer of polar fluids past a semi-infinite vertical moving porous plate via a porous medium taking into account the combined effect of heat source and thermal diffusion.

2. Formation of the Problem

An unsteady MHD flow of heat and mass transfer for second grade fluid in the presence of thermal radiation with variable velocity, temperature and concentration has been consider, it is assumed that the flow and plate are at the same temperature T; the velocity of the fluid raise with respect to time, it is also assumed that all the fluid properties are constant except first normal stress module and the density in buoyancy terms. Within the frame work of the above assumptions, under the usual Boussinesq's approximation the unsteady flow equation are momentum equation energy equation and mass equation respectively and are govern by the following equations;

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial t\partial y^2} + g\beta(T - T_0) + g\beta_\infty(C - C_0) - \frac{\sigma\beta_0^2 U\bar{U}}{\rho} \tag{1}$$

$$\frac{\partial T}{\partial t} = \alpha_1\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \tag{3}$$

The initial and boundary conditions are;

$$\begin{aligned}
&u = 0 \;\; T = T_0 \;\; C = C_0 \; for\; all\; y, t \le 0\\
&u \to 0 \; T = T_1 \; C = C_\infty + (C_0 + C_\infty) \;\; at\; y = 0\\
&u(h,t) = UH(t) \;\; t \le 0; \;\; T \to T_\infty \; C \to C_\infty \; at\; y \to \infty
\end{aligned} \tag{4}$$

And by introducing non-dimensional quantities

$$u = U\bar{U}, \theta = \frac{T - T_0}{T_1 - T_0}, \varphi = \frac{C - C_0}{C_1 - C_0} \;\; \eta = \frac{y}{h}, \tau = \frac{Vt}{h^2}, l = h^{-1}\sqrt{\alpha}, \tag{5}$$

Equations (1), (2) and (3) becomes

$$\frac{\partial U}{\partial \tau} = \frac{\partial^2 U}{\partial \eta^2} + l^2\frac{\partial^3 U}{\partial \tau\partial \eta^2} + \theta Gr + \varphi Gc - M^2 U \tag{6}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} + R\theta \tag{7}$$

$$\frac{\partial \varphi}{\partial \tau} = \frac{1}{Sc}\frac{\partial^2 \varphi}{\partial \eta^2} \tag{8}$$

Where $Gr = \frac{g\beta(\mathrm{T}_1 - \mathrm{T}_0)h^2}{\bar{U}V}, Gc = \frac{g\beta(\mathrm{C}_1 - \mathrm{C}_0)h^2}{\bar{U}V}$, $\Pr = \frac{\mu}{\alpha}$, $R = \frac{16b^* T_0^3 h^2}{V}$ $\alpha_1 = \frac{\alpha}{\rho}, V = \frac{\mu}{\rho}$,

$$M^2 = \frac{\sigma\beta_0^2 h^2}{\rho V},$$

On the use of Laplace transformation, equations (6) – (8) becomes;

$$\overline{U} = \frac{e^{-\eta\sqrt{(M^2+S)}}}{s} + \frac{G_r e^{-\eta\sqrt{(M^2+S)}}}{S^2[(SP_r-R)-(M^2+S)]} + \frac{G_c e^{-\eta\sqrt{(M^2+S)}}}{S[S_cS-(M^2+S)]} - \frac{G_r e^{-\eta\sqrt{(SP_r-R)}}}{S^2[(SP_r-R)-(M^2+S)]} - \frac{G_c e^{-\eta\sqrt{S_cS}}}{S[S_cS-(M^2+S)]} \quad (9)$$

$$\overline{\theta}(\eta,\tau) = \frac{e^{-\eta\sqrt{SP_r}}e^{\sqrt{R}}}{s^2} = \frac{e^{-\eta\sqrt{SP_r}}e^{\sqrt{R}}}{s^2} \quad (10)$$

$$\overline{\varphi}(\eta,\tau) = \frac{1}{s}e^{(-\eta\sqrt{SS_c})} \quad (11)$$

Taking the inverse Laplace transformation of equations (9)-(11), the velocity, temperature and concentration equations is obtained respectively as follows:

$$U = \frac{1}{2}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right] + \frac{G_r t}{2a}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right] - \frac{\eta\sqrt{t}}{\sqrt{M}}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right]$$
$$+\frac{G_r}{2b}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right] + \frac{G_c t}{2c}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right]$$
$$-\frac{\eta\sqrt{t}}{\sqrt{M}}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right]$$
$$+\frac{G_c}{2M^2}\left[e^{2\eta M\sqrt{t}}erfc(\eta+M\sqrt{t}) + e^{-2\eta M\sqrt{t}}erfc(\eta-M\sqrt{t})\right] - \frac{G_r t}{2a}\left[e^{2\eta\sqrt{PrRt}}erfc(\eta\sqrt{Pr}+\sqrt{Rt}) + e^{-2\eta\sqrt{PrRt}}erfc(\eta\sqrt{Pr}-\sqrt{Rt})\right] - \frac{\eta\sqrt{Prt}}{\sqrt{R}}\left[e^{2\eta\sqrt{PrRt}}erfc(\eta\sqrt{Pr}+\sqrt{Rt}) + e^{-2\eta\sqrt{PrRt}}erfc(\eta\sqrt{Pr}-\sqrt{Rt})\right] + \frac{Gr}{2b}\left\{e^{2\eta\sqrt{Rt}}erfc(\eta\sqrt{Pr}+\sqrt{Rt}) + e^{-2\eta\sqrt{PrRt}}erfc(\eta\sqrt{Pr}-\sqrt{Rt})\right\} - \frac{G_c t}{c}\left\{(1+2\eta^2 S_c)erfc(\eta\sqrt{S_c}) - \frac{2\eta\sqrt{S_c}}{\sqrt{\pi}}e^{-\eta^2 S_c}\right\} - \frac{G_c}{M^2}\left[erfc(\eta)\sqrt{S_c}\right]$$

(12)

$$\theta = \frac{1}{2}\left\{e^{2\eta\sqrt{Rt}}erfc(\eta\sqrt{P_r}+\sqrt{Rt}) + e^{-2\eta\sqrt{Rt}}erfc(\eta\sqrt{P_r}-\sqrt{Rt})\right\} \quad (13)$$

$$\varphi = \text{erfc}\left(\frac{y}{2\sqrt{t}}\sqrt{S_c}\right) \Rightarrow \varphi = \text{erfc}(\eta\sqrt{S_c}) \quad (14)$$

$$\text{where } \eta = \frac{y}{2\sqrt{t}}, a = P_r - 1, b = R + M^2, c = S_c - 1 \ \eta = \frac{y}{2\sqrt{t}}$$

Result and Discussion

The problem of unsteady flow of heat and mass transfer for second grade flow has been formulated, analyzed and solve analytically. In order to point out the effects of physical parameters namely thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, time t, Radiation parameter R, chemical reaction parameter K, the computation of the flow fields are carried out. The value of the Prandtl number Pr is chosen to represent air (Pr = 0.71). The value of Schmidt number is chosen to represent water vapour (Sc = 0.57). The value of velocity, temperature and concentration are obtained for the physical parameters as mention.

Figure 1 to 11 is studied for the physical parameters namely thermal Grashof number Gr, mass Grashof number Gc, Schmidt number Sc, Prandtl number Pr, and time t. The effect of velocity for different values of Prandtl number (Pr = 0.71, 0.85, 1.0, 1.71) is presented in Figure 1. It observed that velocity increases with the increasing Pr. The effect of velocity for different values of time (t = 0.2, 0.4, 0.6, 0.8, and 1.0) is presented in Figure 2. It shows that velocity decreases with increasing values of t. The effect of velocity for different values of R (R = 0.2, 2.0, 5.0, and 8.0) is presented in Figure 3. It shows that velocity decreases with increasing values of R. the velocity profiles for different values of M (M = 0.5, 1.0, 3.0) is presented in Figure 4. And it observe that velocity increases with increasing values of M.

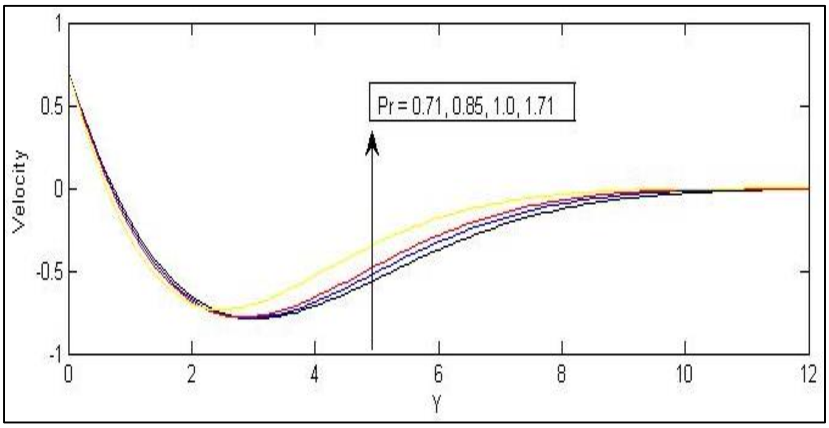

Fig 1: Velocity profiles for different values of Pr

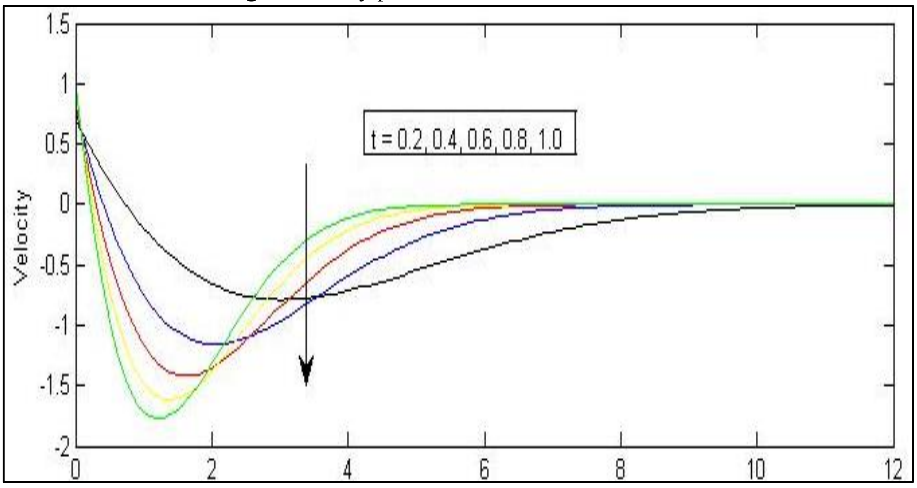

Fig 2: Velocity profiles for different values of t.

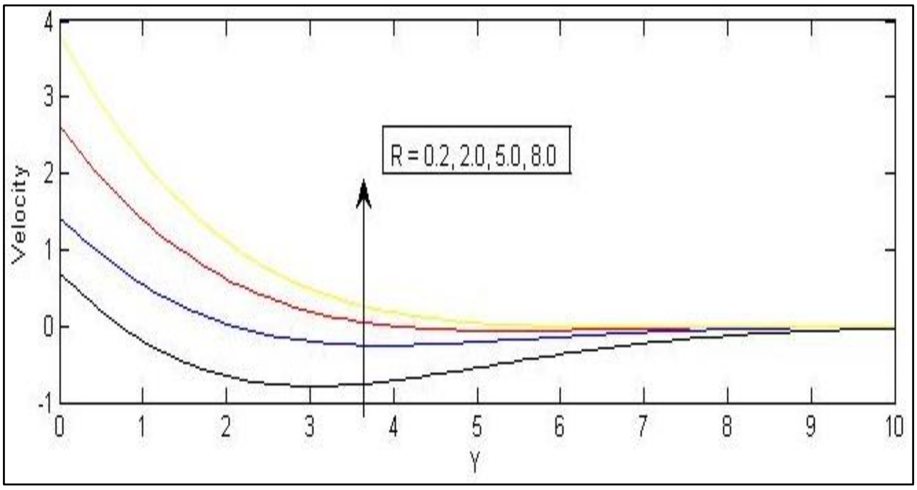

Fig 3: Velocity profiles for different values of R.

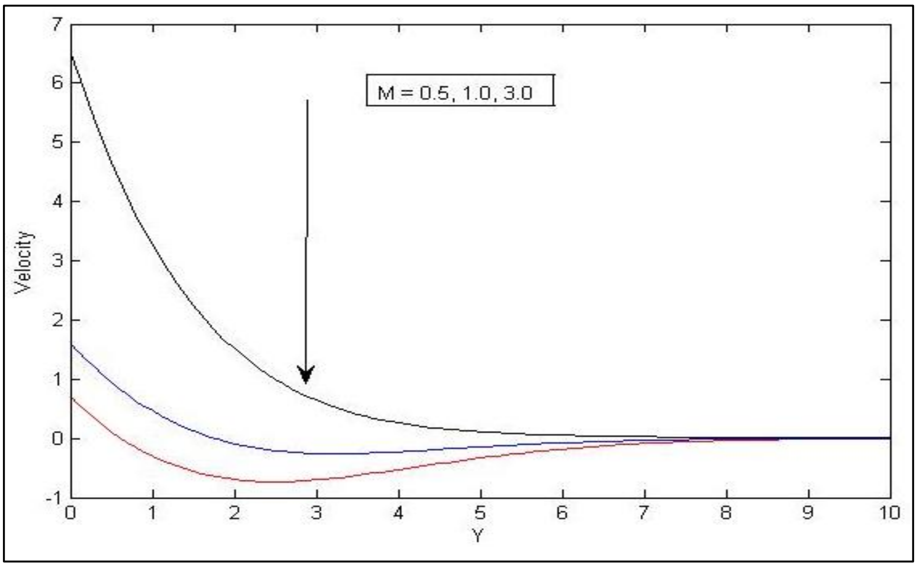

Fig 4: Velocity profiles for different values of M.

The velocity profiles for different value of thermal Grashof number Gr (Gr = 0.2, 0.5, 1.0, 3.0) is seen in Figure 5. It observed that velocity increases with increasing Gr. The velocity profiles for different values of mass Grashof number Gc (0.2, 0.4, 0.6, 0.8) is presented in Figure 6, it observed that velocity decreases with increasing Gc.

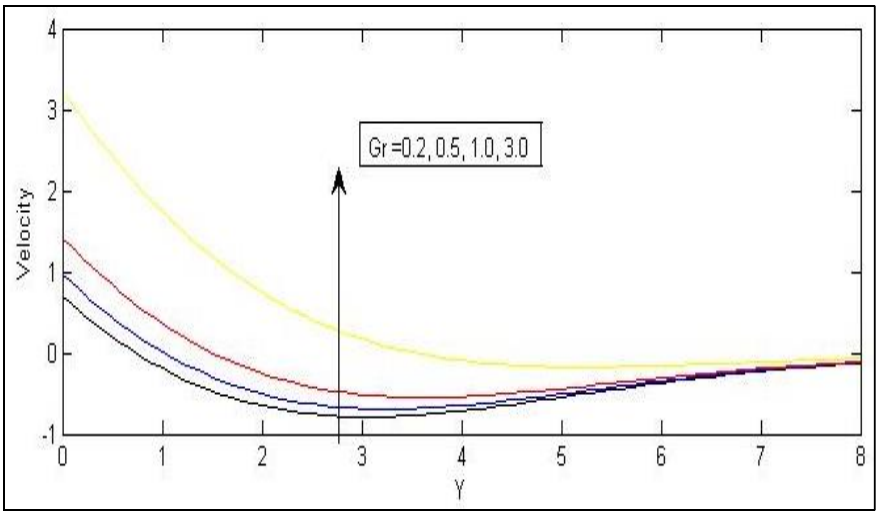

Fig 5: Velocity profiles for different value of Gr

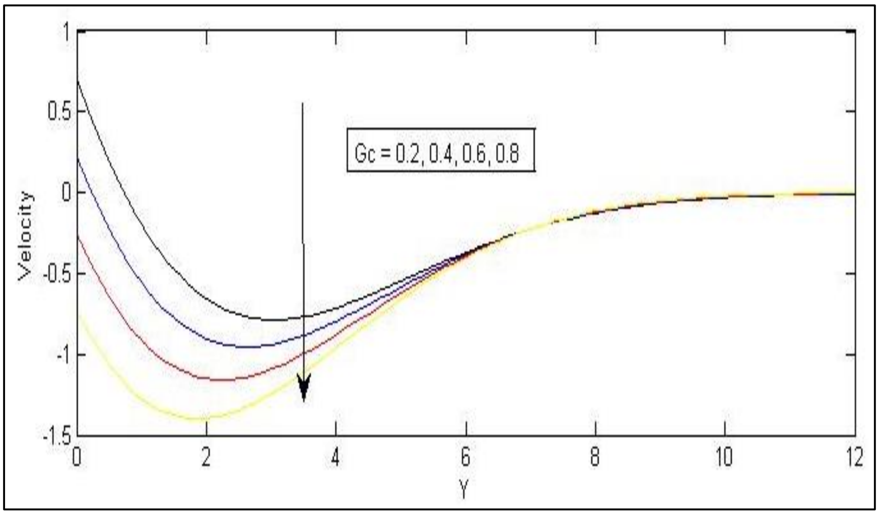

Fig 6: velocity profiles for different value of Gc

The effect of temperature for different values of time (t = 0.2, 0.4, 0.6, 0.8, 1.0) is presented in Figure 7. It shows that temperature decreases with increasing t. The temperature profiles for different values of Prandtl number (Pr = 1.71, 0.85, 1.0, 1.71) is presented in figure 8. It is observed that increases in Prandtl number Pr Temperature decreases. The temperature profiles for different values of Thermal radiation R (R = 0.2, 2.0, 5.0, 8.0) is presented in figure 9. It is observed that increase in Thermal radiation Temperature also increases.

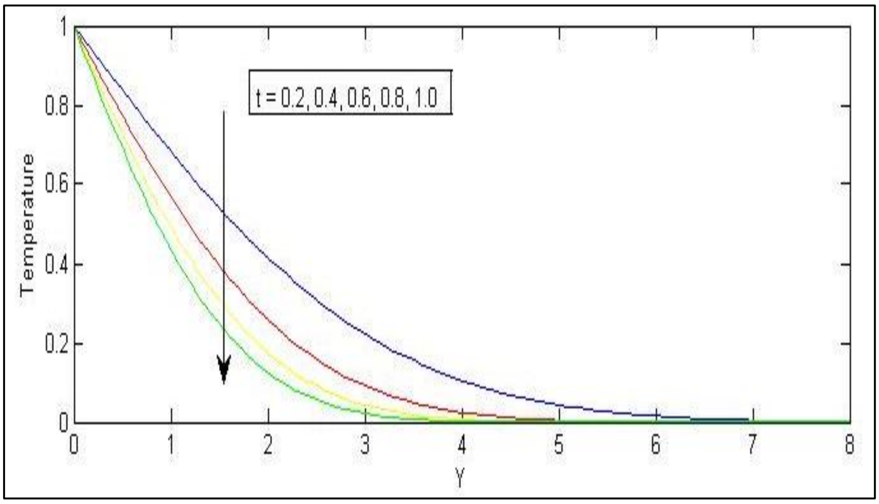

Fig 7: temperature profiles for different value of t.

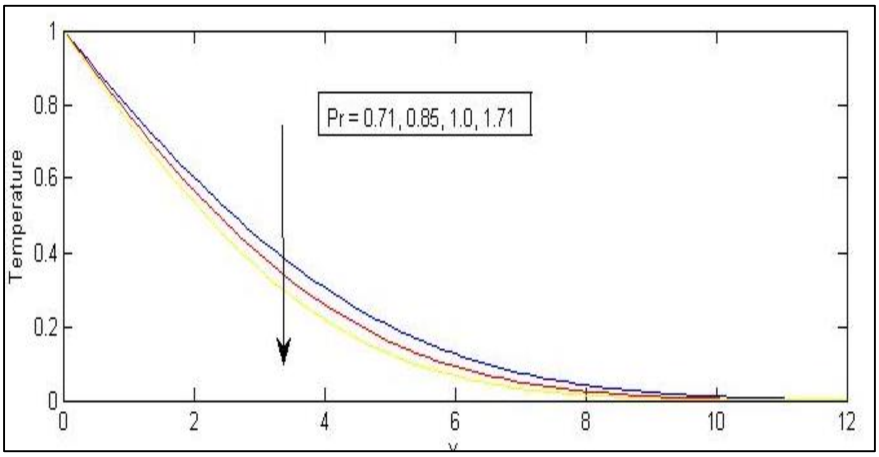

Fig 8: Temperature profiles for different value of Pr.

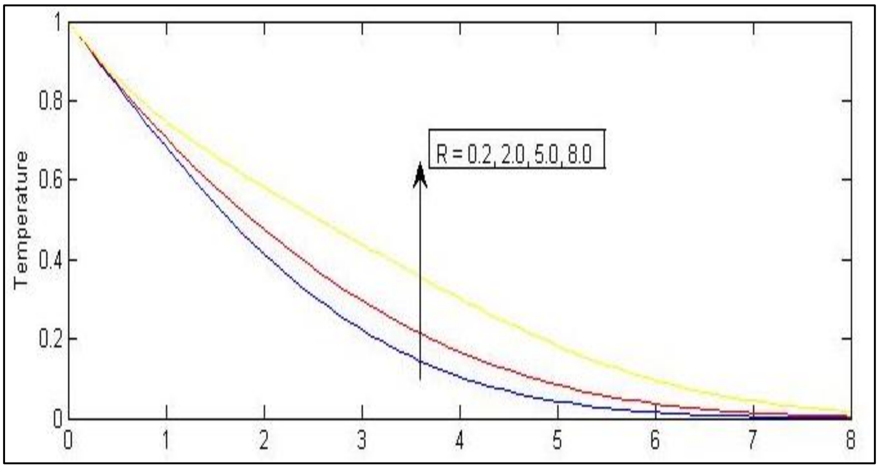

Fig 9: Temperature profiles for different value of R.

The effect of concentration for different values of time (t = 0.2, 0.4, 0.6, 0.8, 1.0) is presented in Figure 10. And it observed that concentration decreases with increasing t. the concentration profiles for different values of Schmidt number (Sc = 0.57, 0.85, 0.99, 1.14) is presented in Figure 11. And it observed that as Schmidt number increase the concentration decreases.

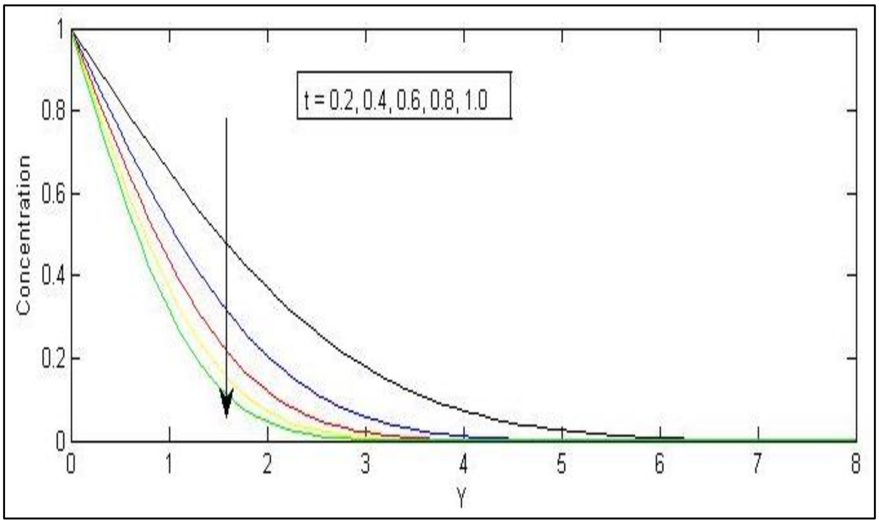

Fig 10: Concentration profiles for different value of t.

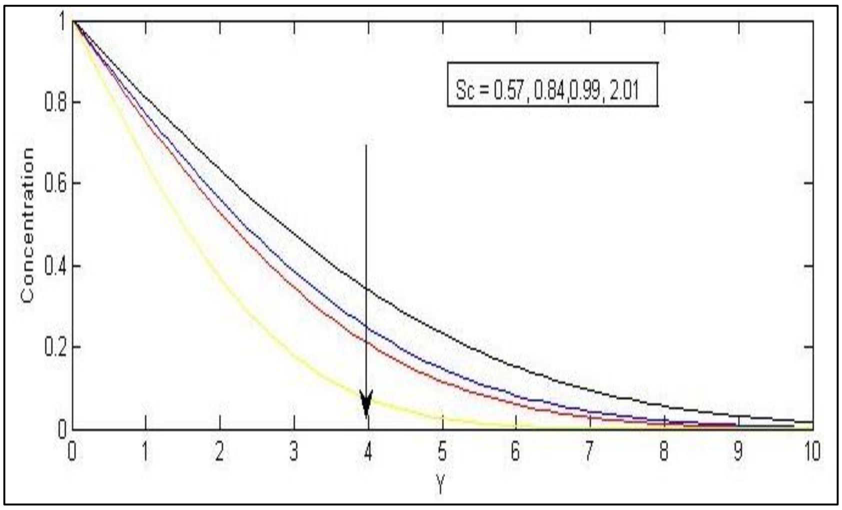

Fig 11: Concentration profiles for different value of Sc.

3. Conclusion

The Unsteady MHD flow of heat and mass transfer of second grade fluid was studied with the effect of thermal radiation. The analytical method sing Laplace transform technique was used to solve the dimensionless governing equations. The physical parameters such as thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, time t, Radiation parameter R, chemical reaction parameter K, was analyzed on the flow fields and computed for different values. The results obtained for the velocity profile shows an increases with increasing parameter such as Pr, R and Gr and decrease with the increase on Gc, M and t. The temperature profile displayed shows decreases with increasing values of Pr and t, and increases with incresing value of R. The concentration profile decreases with increasing value of Sc and t.

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