UAV positioning algorithm based on ground reference

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Abstract
In this article, the algorithm of UAV positioning based on ground check points is considered. Differ from the most publications, in which UAV coordinates are determined by GPS or/and IMU, the proposed algorithm used results of image processing technology. Thank to image processing technique, the coordinates of ground check points in camera's frame are determined, then UAV coordinates will also be determined. With the measured parameters such as UAV's orientation and attitude, camera's elevation and azimuth angle, position of UAV relative to ground check points is determined, then other UAV coordinates are also determined. The proposed algorithm is mathematically proved and verified by symulation.

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Introduction

Currently, UAVs and its applications are becoming more and more popular in social life as well as national security. UAVs are used in reconnaissance, surveillance, detection, tracking and protection of ground objects as well as environmental observation, forest fire prevention or other commercial purposes such as transporting agricultural goods [1, 2, 3]. Whatever the field, in order to effectively perform its tasks, the UAV must first determine its state parameters, including its coordinates. In fact, there are areas where the UAV operates without GPS signal or the GPS signal is interfered with, faked, etc. At that time, there is no accurate information about the coordinates of the UAV to include in the inertial navigation system. This increases the error of the measurement system, affecting the ability of the UAV to perform the mission. To overcome this difficulty, it is possible to use CCTV cameras with predefined coordinates to calculate and determine the coordinates of the UAV to provide the navigation system as an alternative to GPS.

Cameras or image sensors have become popular in robotics applications thanks to the rapid development of image sensor, image processing and machine vision technology. Digital image sensors are being widely used on UAVs because they are easy to install. Especially in locations and conditions where GPS signals are absent or interfered with, inaccurate image sensors can still ensure operation of UAVs. Image sensors can serve as a navigation aid [4], position detection and target tracking [5].

Image-based positioning makes a lot of sense when using images to determine the location of a UAV on the basis of known objects such as markers on the ground. The application of image-guided navigation to aeronautical engineering has been found in inertial navigation systems combined with image matching as well as in automatic landing systems [6]. The inertial-visual navigation system [7] uses image data along with inertial sensors to assess the relative position of the UAV relative to obstacles and demonstrate the ability of the UAV to fly over those obstacles. There are navigation systems that rely solely on image sensors on the UAV. It is a system that only uses image sensors to support the flight of the UAV and it is a solution for a standalone system.

To do this requires a digital map and markers. On the basis of markers, using image processing technology, it is possible to determine the state vector of the UAV. To have a basis for building this algorithm, it is necessary to first study the problem of determining the target coordinates from the UAV on the basis of known UAV coordinates and target parameters on the camera, then determine the coordinates of the drone from the markers.

Methodology

A. Method of determining target coordinates according to UAV coordinates and parameters of the pan-tilt camera

Camera model and camera coordinate system

The image sensor performs a 3D spatial projection onto pixels on the 2D image plane. The model of converting coordinates of a point p^c from 3D space to the image plane is shown as follows:

$$\begin{bmatrix} \bar{v} \\ \bar{\omega} \end{bmatrix} = \frac{f}{x^c} \begin{bmatrix} y^c \\ z^c \end{bmatrix} \quad (1)$$

Where $p^c = \left[x^c y^c z^c\right]^T$ is the marked points position (or target position) in camera coordinate system, $(\bar{v}, \bar{\omega})$ is the marked points coordinates on image plane, and f is camera focal length.

The camera on board is not mounted directly on the UAV's frame, but rather on a pan-tilt gimbal block with two degrees of freedom: rotation around the gimbal's direction and range axis. Let's say the gimbal control algorithm ensures that it quickly switches from one direction to another or clings to a certain marker. Because the image obtained by the camera is rectangular, the camera's field of view with each pan-tilt angle is different by a number of observation loops shown in the following figure:

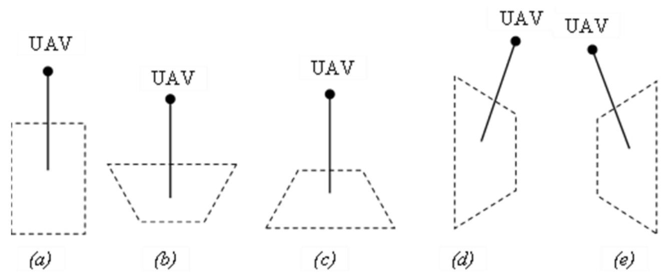

Fig 1: UAV's field of view with different angles and directions

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$$a)\beta = 0; \varepsilon = -90^0;\ \text{b)}\beta = 0; \varepsilon = -60^0\ \text{c)}\beta = 180^0; \varepsilon = -60^0$$
$$d)\beta = 90^0; \varepsilon = -60^0\ e)\beta = -90^0; \varepsilon = -60^0$$

As a result, the field of view is rectangular when the elevation angle is $\varepsilon = -90^0$. In different cases the image obtained on the image plane will be trapezoidal.

Assume that the UAV coordinate system has Euler angles relative to the inertial coordinate system of $(\phi_i^b, \theta_i^b, \psi_i^b)$; The Euler angles of the camera coordinate system relative to the body coordinate system will be $(\phi_b^c, \theta_b^c, \psi_b^c)$; in cases where the camera is mounted on a pan-tilt base:

Assuming $\mathbf{T}_{NED}^C$ is a cosine matrix oriented from the inertial coordinate system or NED coordinate system to the camera coordinate system, then the coordinates of the target in the camera coordinate system are determined as follows:

$$\mathbf{p}_T^C = \mathbf{T}_{NED}^C \left(\mathbf{p}_T - \mathbf{p}_u\right) \quad (2)$$

Where: $\mathbf{p}_u \in \mathbf{R}^3$ is UAV position in inertial coordinate system

$\mathbf{p}_T \in \mathbf{R}^3$ is the coordinates of the target in the inertial coordinate system,

$\mathbf{p}_T \in \mathbf{R}^3$ is the coordinates of the target in the camera coordinate system.

Also:

$$\mathbf{T}_{NED}^C = \mathbf{T}_B^C . \mathbf{T}_{NED}^B \quad (3)$$

$\mathbf{T}_B^C$ is the cosine matrix oriented from the body coordinate system (the coordinate system associated with the UAV) to the camera coordinate system; $\mathbf{T}_{NED}^B$ is the cosine matrix oriented from the inertial coordinate system to the body coordinate system.

With the formulas (1),(2),(3), the determination of the target coordinates on the image plane (forward problem) has been performed. The specific algorithm will be presented in detail below.

Determining target coordinates from camera coordinate system:

Assume the Euler angles of the body coordinate system to the inertial coordinate system, the elevation and direction angles of the pan-tilt base, the target coordinates on the screen, and the coordinates of the UAV are known. Need to find target coordinates. The steps are carried out in turn as follows:

Step 1: Determine the directional cosine matrix from the NED coordinate system to the camera coordinate system. Use formula (3)

Step 2: Assuming the target is on the axis Ox of the camera coordinate system, or $y^c = z^c = 0$, then the target coordinates in the camera coordinate system are determined as follows:

$$\mathbf{p}_T^C = \begin{bmatrix} x^c & 0 & 0 \end{bmatrix}^T = \mathbf{T}_{NED}^C \left(\mathbf{p}_T - \mathbf{p}_u \right) \tag{4}$$

In system of equations (4), there are 3 eqtuations and 3 unknowns x^c, x_T, y_T. Note that z_T is approximated according to the digital map of the observation area. To solve this system of equations, set:

$$\mathbf{T}_{NED}^C = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{5}$$

$$\mathbf{p}_T - \mathbf{p}_u = \begin{bmatrix} \Delta x & \Delta y & \Delta z \end{bmatrix}^T$$

Then, one can get:

$$x^c = \frac{(a_{21}a_{13} - a_{11}a_{23})(a_{31}a_{12} - a_{11}a_{32}) + (a_{31}a_{13} - a_{11}a_{33})(a_{21}a_{12} - a_{11}a_{22})}{a_{21}(a_{31}a_{12} - a_{11}a_{32}) + a_{31}(a_{21}a_{12} - a_{11}a_{22})} \Delta z \tag{6}$$

After determining the coordinates of the target are determined as follows:

$$y_T = \frac{a_{21}x^c - (a_{21}a_{13} - a_{11}a_{23})\Delta z}{(a_{21}a_{12} - a_{11}a_{22})} + y_u$$

$$x_T = \frac{a_{22}x^c - (a_{22}a_{13} - a_{12}a_{23})\Delta z}{(a_{22}a_{11} - a_{12}a_{21})} + x_u \tag{7}$$

Step 3: Determine the target coordinates when its coordinates on the image plane are $[\bar{v} \quad \bar{\omega}]^T$ according to formula (1) (When the target is not on OX_c axis)

$$y^c = \frac{x^c}{f}.\bar{v}; \; z^c = \frac{x^c}{f}.\bar{\omega};$$

$$\mathbf{p}_T^c = \begin{bmatrix} x^c & y^c & z^c \end{bmatrix}^T = \begin{bmatrix} x^c & x^c\bar{v}/f & x^c\bar{\omega}/f \end{bmatrix}^T \tag{8}$$

Step 4: Similar to step 3, solve the system of equations to determine the target coordinates in the inertial coordinate system according to (2):

$$\mathbf{p}_T^c = \begin{bmatrix} x^c & y^c & z^c \end{bmatrix}^T = \begin{bmatrix} x^c & x^c\bar{v}/f & x^c\bar{\omega}/f \end{bmatrix}^T = \mathbf{T}_{NED}^C \left(\mathbf{p}_T - \mathbf{p}_u \right) \tag{9}$$

System (9) has 3 equations and 3 unknowns x^c, x_T, y_T. z^c, y^c can be found with x^c from system of equations (6). This is accurate only when the ground plane is parallel to the image plane, this only happens when the Ox_c axis of the camera coordinate system is perpendicular to the viewing plane. In fact, this is just a rare case. To overcome this, use a new coordinate system called extended camera coordinate system.

Extended camera coordinate system Ox_{CE}, y_{CE}, z_{CE} is the coordinate system obtained from the camera coordinate system rotated at an elevation angle of $\Delta\varepsilon$ and rotated at an direction angle of $\Delta\beta$ such that the target lies on the Ox_{CE} axis.

The problem is to determine $\Delta\varepsilon$ and $\Delta\beta$ when the target coordinates on the image plane $\bar{v}, \bar{\omega}$ are known.

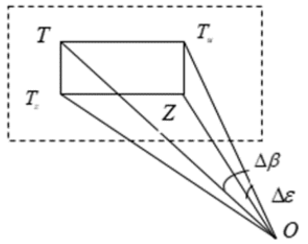

Fig 2: Target coordinates on the image plane

From Fig.2, one can get:

$$\begin{cases} tg\Delta\varepsilon = \dfrac{ZT_u}{OZ} = \dfrac{\bar{v}}{f} \\ tg\Delta\beta = \dfrac{ZT_v}{OT_u} = \dfrac{ZT_v}{OZ/\cos\Delta\varepsilon} = \dfrac{\bar{\omega}.\cos\Delta\varepsilon}{f} \end{cases} \tag{10}$$

Therefore:

$$\begin{cases} \Delta\varepsilon = \operatorname{arc}tg\dfrac{\bar{v}}{f} \\ \Delta\beta = \operatorname{arc}tg\dfrac{\bar{\omega}.\cos\Delta\varepsilon}{f} \end{cases} \tag{11}$$

After having the Euler angles of the extended camera coordinate system, determine the rotation orientation cosine matrix from the inertial coordinate system to the extended camera coordinate system, ie redo step 1:

$$T_{NED}^{CE} = T_C^{CE}.T_{NED}^{C} \tag{12}$$

From step 2 , subtitute T_{NED}^{C} in (3) by T_{NED}^{CE} and perform the next steps. Note that step 3 is not required because: $\mathbf{p}_T^c = \begin{bmatrix} x^c & 0 & 0 \end{bmatrix}$

A further problem is that the coordinates of the target on the image plane cannot be measured directly. In fact, the image on the image plane is given by the field of view, the coordinates of the target point determined by the image processing engine are given by the pixel offset, the total number of pixels of the sensor is a known parameter. Then it is necessary to determine the coordinates of the target pixel on the image plane. However, when using the extended camera coordinate system, we do not use the coordinates of the length of the pixels, but use the angular coordinates. To determine these Euler angles, it is first necessary to determine the focal distance in pixels:

$$f_{pix} = \frac{n_z}{tg\alpha_z} = \frac{n_y}{tg\alpha_y} \tag{13}$$

Where f_{pix} is the focal distance in pixels,

$n_z, n_y, \alpha_z, \alpha_y$, respectively, are the number of pixels in the *z, y* axis and the opening angle in the *z* and *y* axes.

Then, from (13):

$$\begin{cases} \Delta\varepsilon = \operatorname{arc}tg\dfrac{n_{zt}}{f_{pix}} = \operatorname{arc}tg\left(\dfrac{n_{zt}}{n_z}.tg\alpha_z\right) \\ \Delta\beta = \operatorname{arc}tg\dfrac{n_{yt}}{f_{pix}}.\cos\Delta\varepsilon = \operatorname{arc}tg\left(\dfrac{n_{yt}}{n_y}.tg\alpha_y.\cos\Delta\varepsilon\right) \end{cases} \tag{14}$$

Thus, with the use of the extended camera coordinate system, the steps to determine the target coordinates according to the coordinates of the UAV and the directional cosine matrices are as follows:

Step 1: Determine the eletric angles of the camera coordinate system extended from the camera coordinate system according to formula (14).
Step 2: Determine the directional cosine matrices from the inertial coordinate system to the UAV-body coordinate system, from the UAV-body coordinate system to the camera coordinate system, and from the camera coordinate system to the extended camera coordinate system , determine the oriented cosine matrix from the inertial coordinate system to the extended camera coordinate system.

$$\mathbf{T}_{NED}^{CE} = \mathbf{T}_{C}^{CE}.\mathbf{T}_{B}^{C}.\mathbf{T}_{NED}^{B} \tag{15}$$

Then it is possible to determine the components of the matrix $\mathbf{T}_{NED}^{CE}$

$$\mathbf{T}_{NED}^{CE} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Step 3: Determine the target coordinate deviation according to the following equation.

$$\begin{cases} a_{11}.\Delta x + a_{12}.\Delta y + a_{13}.\Delta z = x^{c} & (16a) \\ a_{21}.\Delta x + a_{22}.\Delta y + a_{23}.\Delta z = 0 & (16b) \\ a_{31}.\Delta x + a_{32}.\Delta y + a_{33}.\Delta z = 0 & (16c) \end{cases} \tag{16}$$

After some transformations, get:

$$\Delta x = -\frac{\left(a_{22}.a_{33} - a_{32}.a_{23}\right).\Delta z}{a_{22}.a_{31} - a_{32}.a_{21}} \tag{17}$$

$$\Delta y = \frac{\left(a_{21}.a_{33} - a_{31}.a_{23}\right)}{a_{21}.a_{32} - a_{31}.a_{22}}.\Delta z \tag{18}$$

Target coordinate offset from UAV coordinates:

$$\Delta \mathbf{P} = \mathbf{P}_{T} - \mathbf{P}_{u} = \left[\Delta x \quad \Delta y \quad \Delta z\right]^{T} \tag{19}$$

Step 4: Determine the target coordinates.

$$\mathbf{P}_{T} = \Delta \mathbf{P} + \mathbf{P}_{u} \tag{20}$$

Thus, when knowing the target coordinates in the camera frame along with the Eule angles of the camera coordinate system from the UAV coordinate system and the UAV pose parameters and the UAV coordinates, the ground target coordinates will be determined. To determine the coordinates of the UAV using ground markers, it is necessary to develop the inverse problem. This method will be presented below.

Determine the coordinates of the UAV from the coordinates of the marker points

From the system of equations (16), if the directional cosine matrix is known from the inertial coordinates to the displaced (expanded) camera coordinates, it is possible to determine the coordinate deviation between the UAV and the target. However, in order to solve this system of equations, it is necessary to know the elevation deviation Δz

The height of the marker is predefined while the altitude of the UAV can be determined by the barometer.
Thus, with the help of a barometer, with the same method of using cameras and markers, the UAV coordinates are completely determined according to the algorithm diagram in Figure 4. There the parameters are calculated as follows:
Euler angles of the displacement camera coordinate system $\Delta\varepsilon, \Delta\beta$ are determined according to (14), the Euler angles of the body coordinate system ψ, θ, ϕ are determined by the UAV-mounted inertial measurement system, α, β are determined by encoders measuring the angle of the pan-tilt base.
The coordinate system transformation matrices are defined as follows:

$$T_{NED}^{B}=\begin{bmatrix} c\theta.c\psi & c\theta.s\psi & -s\theta \\ -c\theta.s\psi+s\phi.s\theta.c\psi & c\phi.c\psi+s\phi.s\theta.s\psi & s\phi.c\theta \\ s\phi.s\psi+c\phi.s\theta.c\psi & -s\phi.c\psi+c\phi.s\theta.s\psi & c\phi.c\theta \end{bmatrix}; \quad (21)$$

$$T_{B}^{C}=\begin{bmatrix} s\varepsilon.c\beta & s\varepsilon.s\beta & c\varepsilon \\ -s\beta & c\beta & 0 \\ -c\varepsilon.c\beta & -c\varepsilon.s\beta & s\varepsilon \end{bmatrix}; \quad T_{C}^{CE}=\begin{bmatrix} s\Delta\varepsilon.c\Delta\beta & c\Delta\varepsilon.s\Delta\beta & -s\Delta\varepsilon \\ -s\Delta\beta & c\Delta\beta & 0 \\ s\Delta\varepsilon.c\beta & s\Delta\varepsilon.s\Delta\beta & c\Delta\varepsilon \end{bmatrix} \quad (22)$$

$$T_{NED}^{CE}=T_{C}^{CE}.T_{B}^{C}.T_{NED}^{B} \quad (23)$$

- Deviation of target coordinates from UAV coordinates: $\Delta\mathbf{P}=\mathbf{P}_T-\mathbf{P}_u=[\Delta x \quad \Delta y \quad \Delta z]^T$ is determined according to the equations (16), (17), (18).
- The UAV coordinates are determined from the target coordinates and the target coordinates deviation from the UAV coordinates as follows:

$$\mathbf{P}_u=\mathbf{P}_T-\Delta\mathbf{P} \quad (24)$$

The algorithm for determining UAV coordinates from the coordinates of a marker point is presented according to the algorithm flowchart in Figure 3.

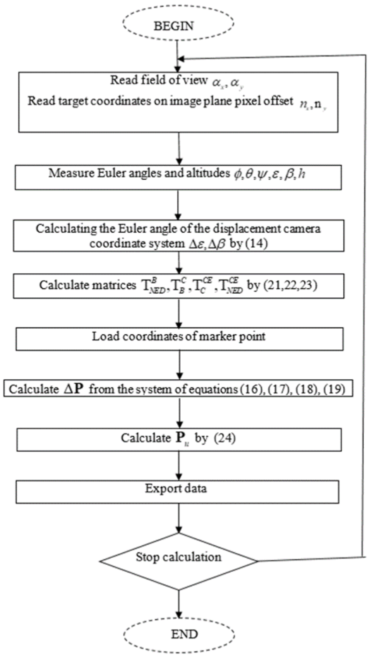

Fig 3: Flowchart of the algorithm to determine the coordinates of the UAV from the camera and the marker

B. Simulations

The UAV flies with speed $v=v_0=10m/s$, inclination angle $\phi=1/100$, initial direction angle $\psi=\psi_0=pi/2$, initial coordinates $x_0=1000, y_0=0, h=const=1000$.

The reference point has coordinates $x_c=400, y_c=100$;

The relative coordinates of the reference point relative to the UAV in the UAV body coordinate system are shown in the graph of Figure 4

The error in determining coordinates with camera angles measured with a random error of $\pm 1/2$ mrad is shown in Figures 5 and 6.

The error of determining coordinates with camera angles measured with a random error of $\pm 1/10$ mrad is shown in Figure 7 and Figure 8.

C. Results and discussion

The results show that, the larger the relative distance between the reference point and the UAV in each axis, the higher the UAV positioning error, and vice versa, the lower the relative distance between the reference point and the UAV in each axis, the lower the error.

In the case of random error of $\pm 1/2$ mrad, the maximum error in the x-axis is 11m when the relative distance between the reference point and the UAV in the x-axis is 1400m, the maximum error in the y-axis is 1m.

In the case of random error of $\pm 1/10$ mrad, the maximum error in the x-axis is 11m when the relative distance between the reference point and the UAV in the x-axis is 1400m, the maximum error in the y-axis is 0.3 m.

The second case is to simulate the UAV moving in a straight line with the trajectory starting from the origin in the x-axis. The reference point has coordinates of (1500, 50).

The relative coordinates of the reference point relative to the UAV in the body coordinate system are shown in the graph of Figure 9.

The error of determining coordinates with camera angles measured with a random error of $\pm 1/2$ mrad is shown in Figure 10 and Figure 11.

The error in determining coordinates with camera angles measured with a random error of $\pm 1/10$ mrad is shown in Figures 12 and 13.

The results show that, the larger the relative distance between the reference point and the UAV in each axis, the higher the UAV positioning error, on the contrary, the lower the relative distance between the reference point and the UAV in each axis, the lower the error.

In the case of a random error of $\pm 1/2$ mrad, the maximum error in the x-axis is 40m when the relative distance between the reference point and the UAV in the x-axis is 4800m, the maximum error in the y-axis is 3m.

In the case of random error of $\pm 1/10$ mrad, the maximum error in the x-axis is 8m when the relative distance between the reference point and the UAV in the x-axis is 4800m, the maximum error in the y-axis is 0.3m.

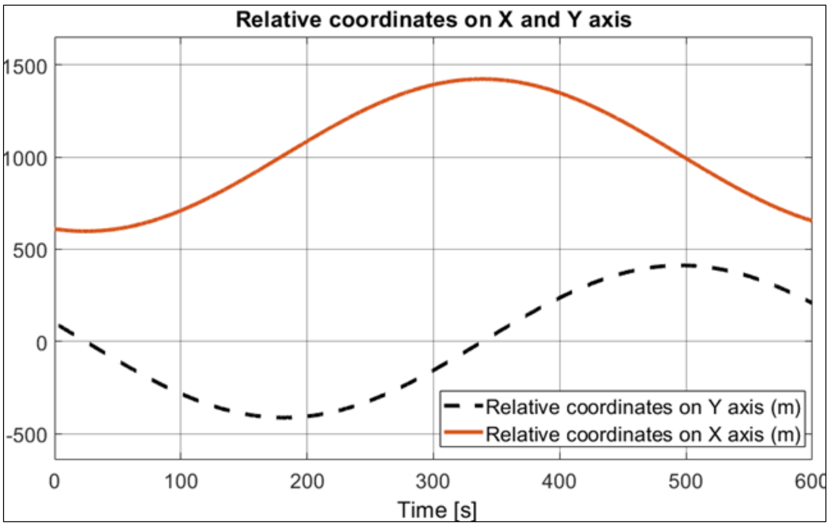

Fig 4: Relative coordinates of the reference point to the UAV in the UAV body coordinate system

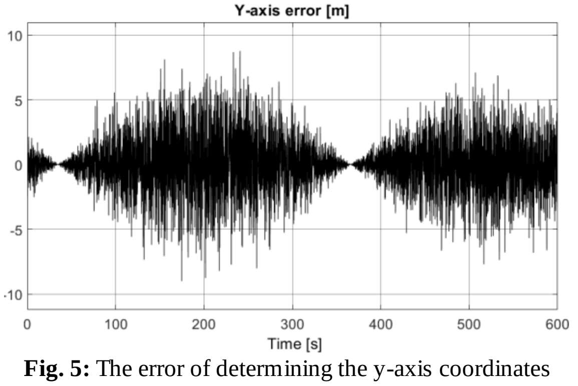

Fig. 5: The error of determining the y-axis coordinates when the camera angles are measured with a random error of $\pm 1/2$ mrad

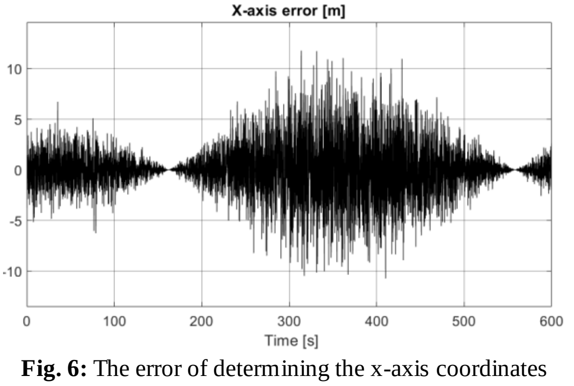

Fig. 6: The error of determining the x-axis coordinates when the camera angles are measured with a random error of $\pm 1/2$ mrad

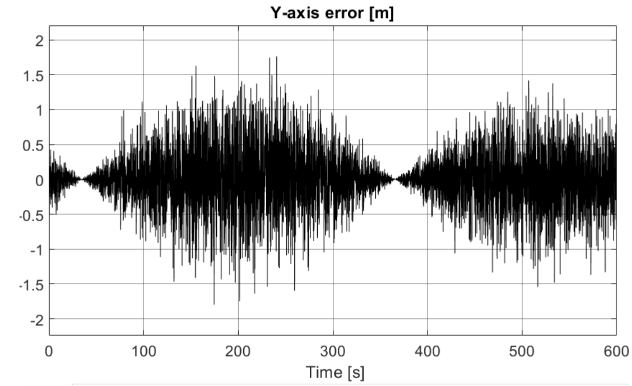

Fig. 7: The error of determining the y-axis coordinates when the camera angles are measured with a random error of $\pm 1/10$ mrad

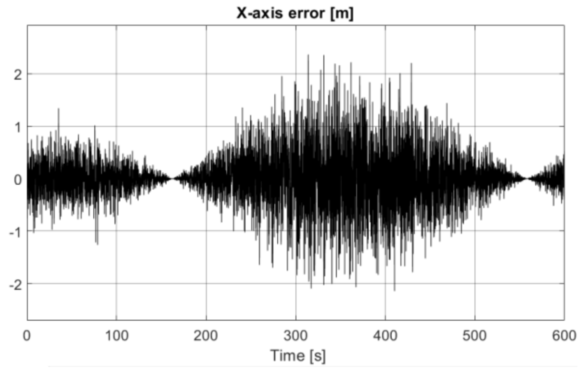

Fig. 8: The error of determining the x-axis coordinates when the camera angles are measured with a random error of $\pm 1/10$ mrad

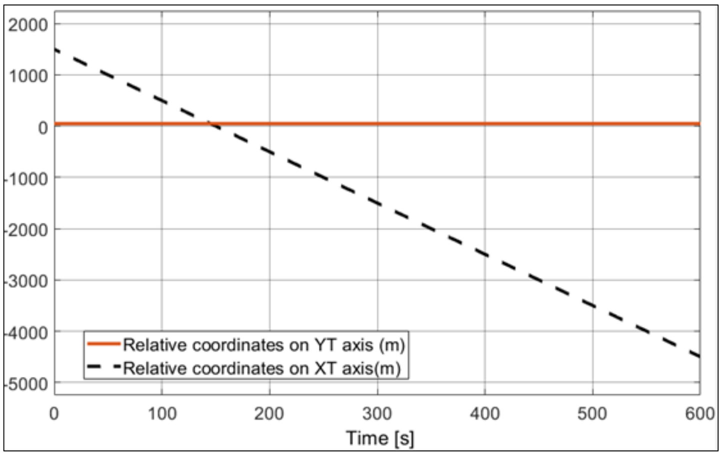

Fig 9: Relative coordinates of the reference point relative to the UAV in the UAV body coordinate system when the trajectory is a straight line.

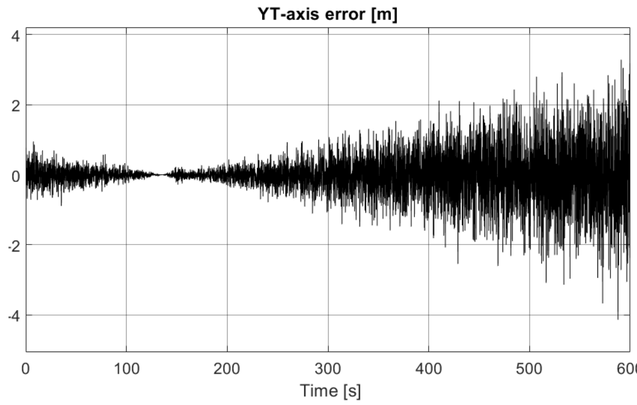

Fig. 10: The error of determining the YT-axis coordinates when the camera angles are measured with a random error of $\pm 1/2$ mrad

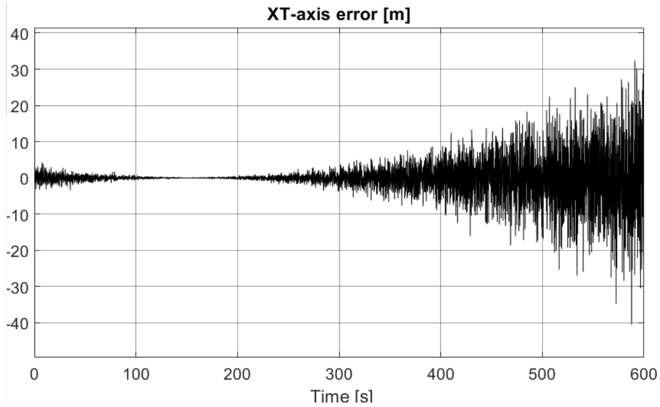

Fig. 11: The error of determining the XT-axis coordinates when the camera angles are measured with a random error of $\pm 1/2$ mrad

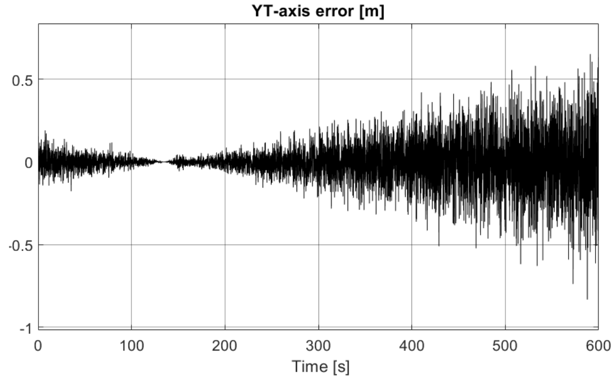

Fig 12: The error of determining the YT-axis coordinates when the camera angles are measured with a random error of $\pm 1/10$ mrad

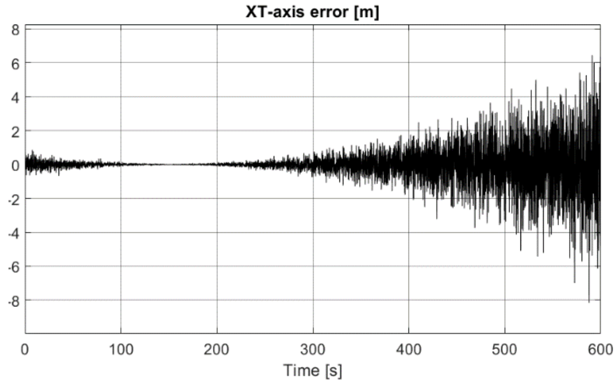

Fig. 13: The error of determining the XT-axis coordinates when the camera angles are measured with a random error of $\pm 1/10$ mrad

Conclusions

The proposed algorithm for determining the coordinates of the UAV allows to determine the coordinates of the UAV when there is no GPS signal. This ensures the stability of the navigation systems for the UAV under all conditions. These results can be applied in inertial navigation systems using multi-sensors, thereby providing full components of the UAV's state vector as the basis for UAV control. In order to increase accuracy when determining coordinates according to the reference point, it is necessary to increase the number of reference points so that the distance from the UAV to the reference point is not too large.

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