



## Non-GPS UAV positioning algorithm based on two ground reference point

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### Abstract

In this paper, a method to determine the coordinates of a UAV without using GPS and an altimeter is built. Different from conventional methods, which use GPS or/and IMU to determine UAV coordinates, as well as, the method of determining UAV coordinates from a ground reference point, it is necessary to use devices that have cumulative errors (INS) or are susceptible to jamming (GPS), or are affected by weather conditions (barometric altimeter measuring device). The proposed method only uses tilt sensors and magnetometers, which are not affected by external noise, along with a pan-tilt camera system with an integrated image processing system to determine UAV coordinates, according to the coordinates of two ground reference points. The proposed algorithm has been rigorously mathematically proven and illustrated and verified through simulation.

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### Introduction

Currently, UAVs have been widely applied for socio-economic and national security and defense purposes <sup>[1, 2, 3]</sup>. To ensure proper operation and mission accomplishment, the UAV needs information about its real coordinates in space. One of the basic problems of UAVs is to follow a preloaded trajectory in space. To accomplish this task the UAV must always be provided with accurate information about its coordinates. The coordinates of the UAV are usually determined by GPS. However, in reality there are areas where UAVs operate without GPS signals or GPS signals are distorted and faked... This affects the ability of the UAV to perform its tasks. To overcome this difficulty, the method of using cameras to observe markers with pre-loaded coordinates to calculate and determine the coordinates of the UAV to provide for the navigation system as an alternative to GPS has been proposed <sup>[4]</sup>. This is a highly effective method, but this method still uses a measuring device with high errors, depending on weather conditions, that is, using a barometer to determine the altitude of the UAV. When using this measuring device, fluctuations in atmospheric pressure will affect altitude errors, and also cause errors in the coordinates of the UAV in general. Cameras or image sensors have become popular in robotic applications thanks to the strong development of image sensing, image processing and machine vision technology. Digital image sensors and image processing systems are used to detect, track and collect target data <sup>[5, 6]</sup>, at the same time, it can also be used to identify ground reference points, thereby determining the coordinates of the UAV.

Currently, thanks to strongly developed remote sensing systems, ground reference points can be easily identified and the density of ground reference points can be large enough so that the camera can observe two reference points simultaneously. With two ground reference points, combined with the angle sensors of the pan-tilt camera and the sensors of the UAV, the coordinates of the UAV are completely determined without using the altitude value measured by the barometer. The algorithm for determining UAV coordinates from two ground reference points will be presented in detail below. The simulation results will show the effectiveness of the algorithm.

## Methodology

### A. Method of determining UAV coordinates from a ground reference point

Determining target coordinates from a ground reference point has been presented in detail in the work [8]. In this report, the contents presented in [8] will be briefly summarized to serve as a basis for further research.

To determine UAV coordinates, use the following coordinate systems:

- Fixed ground coordinate system, or inertial coordinate system, denoted  $O_iX_iY_iZ_i$ .
- The coordinate system attached to the UAV (UAV body frame), denoted  $O_bX_bY_bZ_b$ , has its center located at the center of gravity of the UAV, has Euler's angles  $(\phi \ \theta \ \psi)$  compared to the inertial coordinate system determined by tilt sensors and magnetometer.
- Camera coordinate system, denoted  $O_cX_cY_cZ_c$ , is determined by Euler's angles from the coordinate system attached to the UAV  $(\phi_b^c \ \theta_b^c \ \psi_b^c)$ ,  $\phi_b^c = 0 \ \theta_b^c = \varepsilon \ \psi_b^c = \beta$
- Extended camera coordinate system, is the coordinate system received from the camera coordinate system, such that the  $O_cX_{cr}$  axis passes through the ground reference point, has Euler's angles compared to the camera coordinate system are  $\theta_c^{cr} = \Delta\varepsilon, \psi_c^{cr} = \Delta\beta$ , where  $\Delta\beta, \Delta\varepsilon$  are, respectively, coordinate angle of the target in the frame along the horizontal and vertical axes of the frame. The angles are determined as follows:

To determine these Euler's angles, you first need to determine the focal distance in pixels:

$$f_{\text{pix}} = \frac{n_z}{\text{tg}\alpha_z} = \frac{n_y}{\text{tg}\alpha_y} \quad (1)$$

Where  $f_{\text{pix}}$  is the focal distance in pixels.

$n_z, n_y, \alpha_z, \alpha_y$  are in order the number of pixels along the z, y axis and the opening angle along the z and y axis.

Then, from (1) we have:

$$\begin{cases} \Delta\varepsilon = \text{arctg} \frac{n_z}{f_{\text{pix}}} = \text{arctg} \left( \frac{n_z}{n_z} \cdot \text{tg}\alpha_z \right) \\ \Delta\beta = \text{arctg} \frac{n_y}{f_{\text{pix}}} \cdot \cos\Delta\varepsilon = \text{arctg} \left( \frac{n_y}{n_y} \cdot \text{tg}\alpha_y \cdot \cos\Delta\varepsilon \right) \end{cases} \quad (2)$$

Using sensors provides status parameters of the UAV, which are the height h, the Ele angles of the coordinate system attached to the UAV.

Then, the coordinate system conversion matrices are determined as follows:

$$T_{NED}^B = \begin{bmatrix} c\theta.c\psi & c\theta.s\psi & -s\theta \\ -c\theta.s\psi + s\phi.s\theta.c\psi & c\phi.c\psi + s\phi.s\theta.s\psi & s\phi.c\theta \\ s\phi.s\psi + c\phi.s\theta.c\psi & -s\phi.c\psi + c\phi.s\theta.s\psi & c\phi.c\theta \end{bmatrix}; \quad (3)$$

$$T_B^C = \begin{bmatrix} s\varepsilon.c\beta & s\varepsilon.s\beta & c\varepsilon \\ -s\beta & c\beta & 0 \\ -c\varepsilon.c\beta & -c\varepsilon.s\beta & s\varepsilon \end{bmatrix}; T_C^{CE} = \begin{bmatrix} s\Delta\varepsilon.c\Delta\beta & c\Delta\varepsilon.s\Delta\beta & -s\Delta\varepsilon \\ -s\Delta\beta & c\Delta\beta & 0 \\ s\Delta\varepsilon.c\beta & s\Delta\varepsilon.s\Delta\beta & c\Delta\varepsilon \end{bmatrix} \quad (4)$$

$$T_{NED}^{CE} = T_C^{CE} \cdot T_B^C \cdot T_{NED}^B \quad (5)$$

Matrix  $T_{NED}^{CE}$  with elements denoted as follows:

$$T_{NED}^{CE} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (6)$$

Deviation of target coordinates compared to UAV coordinates:  $\Delta \mathbf{P} = \mathbf{P}_T - \mathbf{P}_U = [\Delta x \quad \Delta y \quad \Delta z]^T$  determined according to the following equations:

$$\begin{cases} a_{11} \cdot \Delta x + a_{12} \cdot \Delta y + a_{13} \cdot \Delta z = x^c \\ a_{21} \cdot \Delta x + a_{22} \cdot \Delta y + a_{23} \cdot \Delta z = 0 \\ a_{31} \cdot \Delta x + a_{32} \cdot \Delta y + a_{33} \cdot \Delta z = 0 \end{cases} \quad (7)$$

After some transformations, get:

$$\Delta x = - \frac{(a_{22} \cdot a_{33} - a_{32} \cdot a_{23}) \cdot \Delta z}{a_{22} \cdot a_{31} - a_{32} \cdot a_{21}} \quad (8)$$

$$\Delta y = \frac{(a_{21} \cdot a_{33} - a_{31} \cdot a_{23}) \cdot \Delta z}{a_{21} \cdot a_{32} - a_{31} \cdot a_{22}} \quad (9)$$

There,  $\Delta z = h_u - h_p$  is determined according to the UAV's height difference compared to the reference point.

UAV coordinates are determined from target coordinates and target coordinate deviation compared to UAV coordinates as follows:

$$\mathbf{P}_u = \mathbf{P}_T - \Delta \mathbf{P} \quad (10)$$

The algorithm for determining UAV coordinates from the coordinates of a reference point is performed based on equations (1) to (10) above.

#### A. Algorithm to determine target coordinates from two ground reference points

Using a barometer to measure UAV altitude is a simple and effective solution. However, when it is necessary to accurately determine altitude, this method is not suitable because it cannot determine air pressure at zero altitude (sea level). To overcome this drawback, the following method proposes to determine UAV coordinates from two ground reference points.

Suppose there are two predetermined reference points located in the camera's field of view with deviations from the image plane center in Oz and Oy axes in pixels as  $n_{z1}, n_{y1}, n_{z2}, n_{y2}$  with the camera's field of view and the number of pixels in each axis is

$\alpha_z, n_z, \alpha_y, n_y$ .

$$\left\{ \begin{array}{l} \Delta \varepsilon_1 = \arctg \frac{n_{z1r}}{f_{pix}} = \arctg \left( \frac{n_{z1r}}{n_{z1}} \cdot tg \alpha_{z1} \right) \\ \Delta \beta_1 = \arctg \frac{n_{y1r}}{f_{pix}} \cdot c \cos \Delta \varepsilon_1 = \arctg \left( \frac{n_{y1r}}{n_{y1}} \cdot tg \alpha_{y1} \cdot c \cos \Delta \varepsilon_1 \right) \\ \Delta \varepsilon_2 = \arctg \frac{n_{z2r}}{f_{pix}} = \arctg \left( \frac{n_{z2r}}{n_{z2}} \cdot tg \alpha_{z2} \right) \\ \Delta \beta_2 = \arctg \frac{n_{y2r}}{f_{pix}} \cdot c \cos \Delta \varepsilon_2 = \arctg \left( \frac{n_{y2r}}{n_{y2}} \cdot tg \alpha_{y2} \cdot c \cos \Delta \varepsilon_2 \right) \end{array} \right. \quad (11)$$

With Euler's angles from the inertial coordinate system to the attached coordinate system  $\phi, \theta, \psi$  and Euler's angles from the attached coordinate system to the camera coordinate system  $\alpha, \beta$ , the orientation cosine matrices from the inertial coordinate system to the displacement camera coordinate system for reference point 1 and reference point 2 are determined according to formula (12):

$$\begin{cases} T_{NED}^{CE1} = T_C^{CE1} \cdot T_B^C \cdot T_{NED}^B = \begin{bmatrix} {}^1a_{11} & {}^1a_{12} & {}^1a_{13} \\ {}^1a_{21} & {}^1a_{22} & {}^1a_{23} \\ {}^1a_{31} & {}^1a_{32} & {}^1a_{33} \end{bmatrix} \\ T_{NED}^{CE2} = T_C^{CE2} \cdot T_B^C \cdot T_{NED}^B = \begin{bmatrix} {}^2a_{11} & {}^2a_{12} & {}^2a_{13} \\ {}^2a_{21} & {}^2a_{22} & {}^2a_{23} \\ {}^2a_{31} & {}^2a_{32} & {}^2a_{33} \end{bmatrix} \end{cases} \quad (12)$$

And the system of equations showing the relationship between the coordinate deviations of reference point 1 and reference point 2 with the UAV is expressed according to (13) and (14):

$$\begin{cases} {}^1a_{21} \cdot \Delta x_1 + {}^1a_{22} \cdot \Delta y_1 + {}^1a_{23} \cdot \Delta z_1 = 0 \\ {}^1a_{31} \cdot \Delta x_1 + {}^1a_{32} \cdot \Delta y_1 + {}^1a_{33} \cdot \Delta z_1 = 0 \end{cases} \quad (13)$$

$$\begin{cases} {}^2a_{21} \cdot \Delta x_1 + {}^2a_{22} \cdot \Delta y_1 + {}^2a_{23} \cdot \Delta z_2 = 0 \\ {}^2a_{31} \cdot \Delta x_1 + {}^2a_{32} \cdot \Delta y_1 + {}^2a_{33} \cdot \Delta z_2 = 0 \end{cases} \quad (14)$$

Using (8), from (13) and (14) get:

$$\begin{cases} \Delta x_1 = \frac{({}^1a_{22} \cdot {}^1a_{33} - {}^1a_{32} \cdot {}^1a_{23}) \cdot \Delta z_1}{{}^1a_{22} \cdot {}^1a_{31} - {}^1a_{32} \cdot {}^1a_{21}} = c_1 \cdot \Delta z_1 \\ \Delta x_2 = \frac{({}^2a_{22} \cdot {}^2a_{33} - {}^2a_{32} \cdot {}^2a_{23}) \cdot \Delta z_1}{{}^2a_{22} \cdot {}^2a_{31} - {}^2a_{32} \cdot {}^2a_{21}} = c_2 \cdot \Delta z_2 \end{cases} \quad (15)$$

$$\text{Where } \begin{cases} \Delta z_1 = -(h_u - h_1) \\ \Delta z_2 = -(h_u - h_2) \end{cases} \quad (16)$$

$$\text{Then from (16): } \Delta z_2 = \Delta z_1 - (h_1 - h_2) \quad (17)$$

$$\text{And } \begin{cases} \Delta x_1 = x_1 - x_u \\ \Delta x_2 = x_2 - x_u \end{cases} \quad (18)$$

Substitute (17) and (19) to (15):

$$\Delta x_1 = (x_1 - x_2) + c_2 \cdot \Delta z_1 - c_2 \cdot (h_1 - h_2) \quad (20)$$

$$(x_1 - x_2) + c_2 \cdot \Delta z_1 - c_2 \cdot (h_1 - h_2) = c_1 \cdot \Delta z_1 \quad (21)$$

$$\Delta z_1 = \frac{(x_1 - x_2) - c_2 \cdot (h_1 - h_2)}{c_1 - c_2} \quad (22)$$

And the altitude received by the UAV will be:

$$h_u = h_1 - \Delta z_1 \tag{23}$$

Then the different coordinate offsets are determined according to:

$$\begin{cases} \Delta x_1 = \frac{\begin{pmatrix} {}^1a_{22} \cdot {}^1a_{33} - {}^1a_{32} \cdot {}^1a_{23} \end{pmatrix} \cdot \Delta z_1}{{}^1a_{32} \cdot {}^1a_{21} - {}^1a_{31} \cdot {}^1a_{22}} \\ \Delta y_1 = \frac{\begin{pmatrix} {}^1a_{21} \cdot {}^1a_{33} - {}^1a_{31} \cdot {}^1a_{23} \end{pmatrix} \cdot \Delta z_1}{{}^1a_{22} \cdot {}^1a_{31} - {}^1a_{21} \cdot {}^1a_{32}} \end{cases} \tag{24}$$

And the coordinates of the UAV are determined according to (10):

$$P_u = P_1 - \Delta P_1 \tag{25}$$

Where  $\Delta P_1 = [\Delta x_1 \quad \Delta y_1 \quad \Delta z_1]^T$

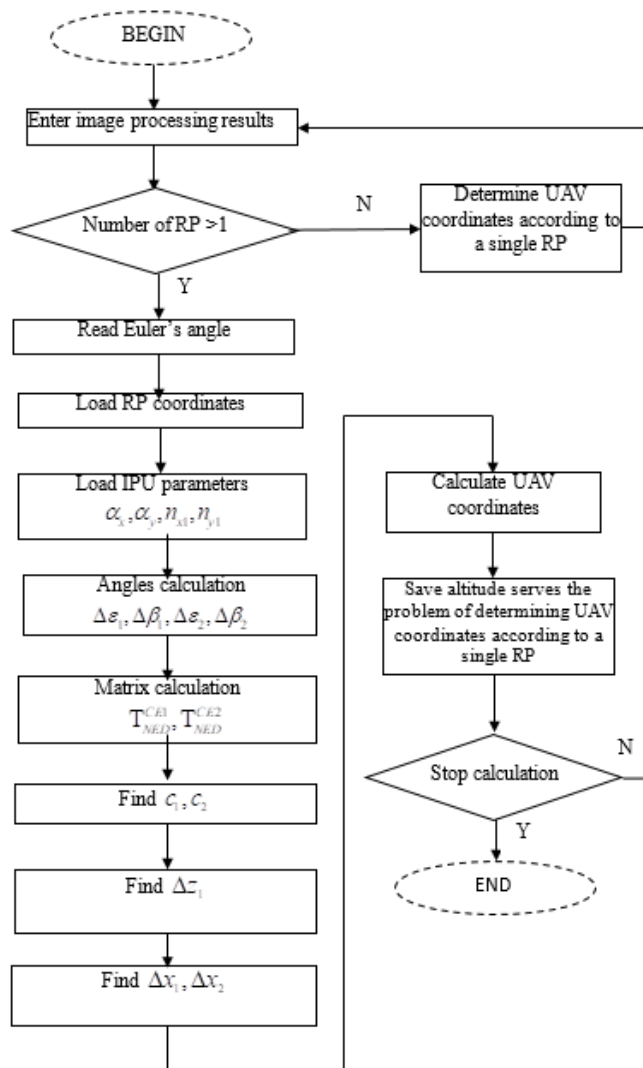
Thus, the algorithm for determining UAV coordinates and adjusting UAV altitude is performed as in Fig.1.

**B. Simulations**

Let the UAV fly straight with a constant altitude of 400m, two reference points with distances to the projection of the UAV trajectory and altitudes of 500m, 70m and 300m, 30m respectively.

The measurement error of the angles is 0.1mrad. The random error of measuring altitude using a barometer is 10m. The error of the altimeter measurement system using a barometer is simulated in two cases: error equal to 0 and error equal to 20m.

The simulation process is conducted using Matlab-Simulink software.

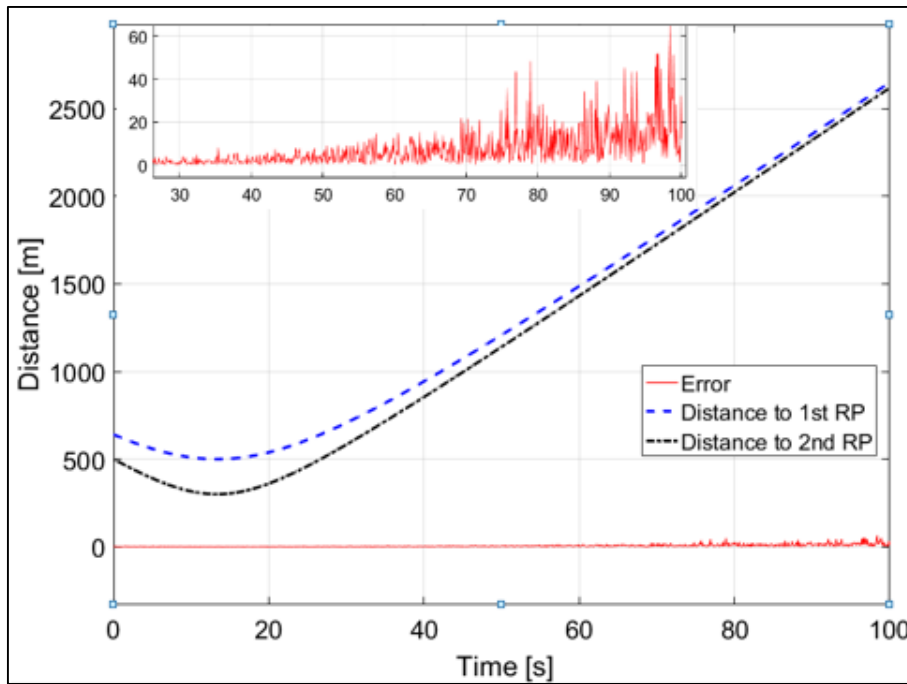


**Fig 1:** Algorithm flow chart for determining UAV coordinates and adjusting UAV altitude

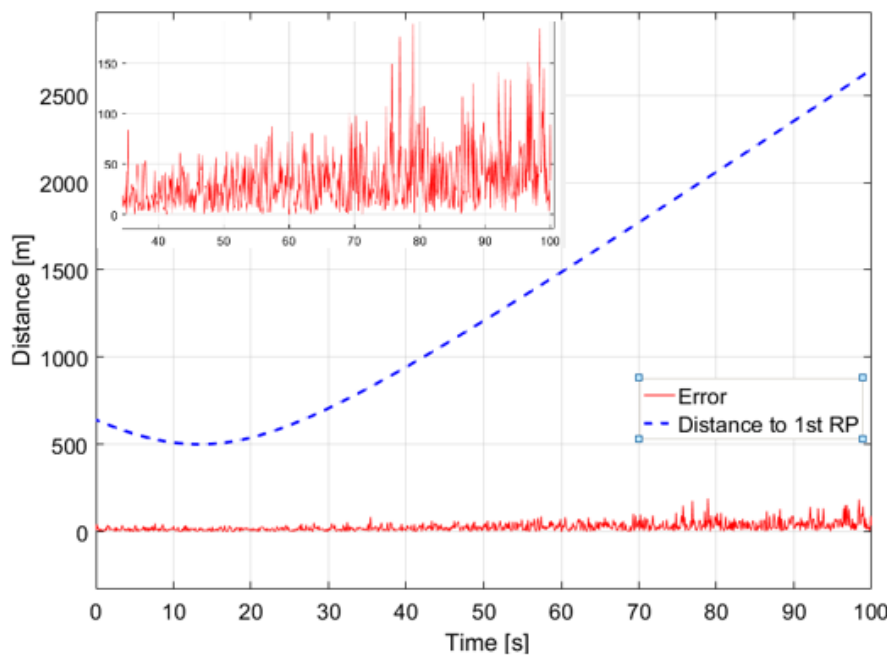
**Results and Discussion**

Simulation results determine the euclidean standard of the UAV's position error.

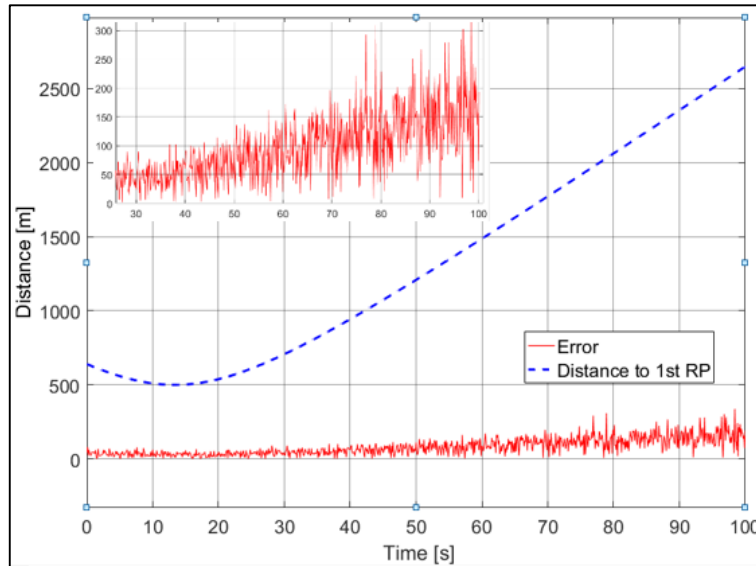
Figure 2 is a graph of the distance from the UAV to two reference points and the error in determining the coordinates of the UAV using the two reference point's method.



**Fig 2:** Determining the coordinates of the UAV using the two reference points method

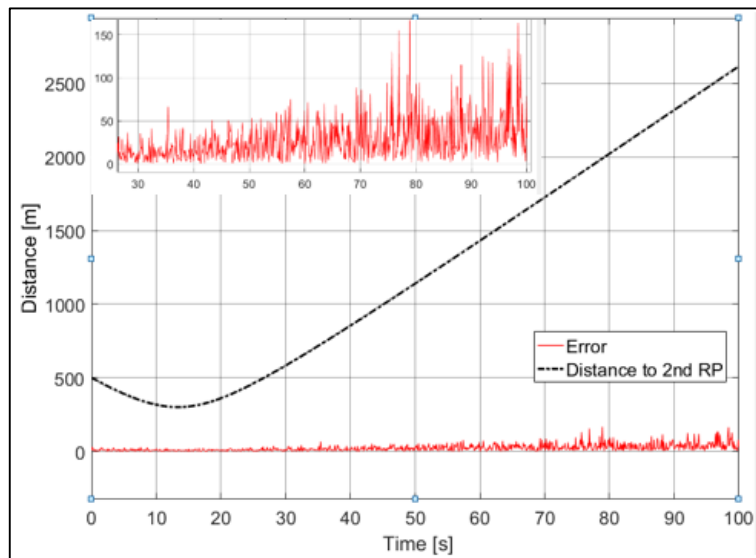


**Fig 3:** Error using only first reference point (no systematic error)

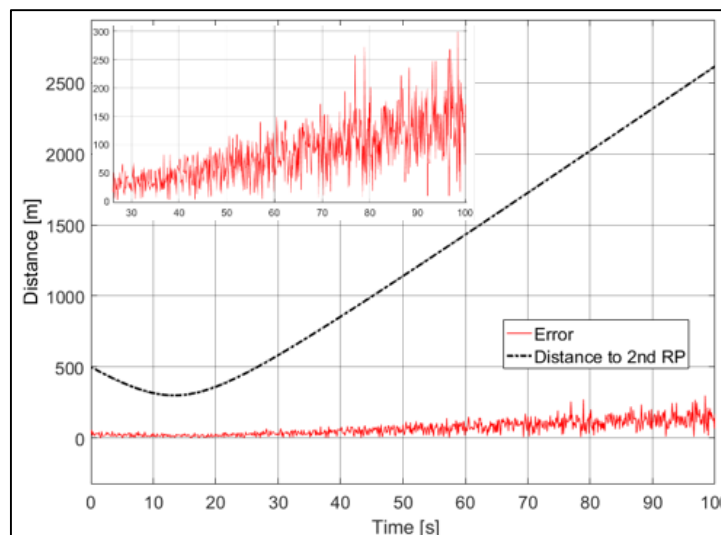


**Fig 4:** Error using only first reference point (systematic error is 20m)

Figure 3 is a graph of the distance from the UAV to the first reference point and the error in determining the UAV coordinates when the barometer has a random error of 10m, a systematic error of 0 m, when the air pressure at sea level altitude is the nominal pressure.



**Fig 5:** Error using only second reference point (no systematic error)



**Fig 6:** Error using only second reference point (systematic error is 20m)

Figure 4 is a graph of the distance from the UAV to the first reference point and the error in determining the UAV coordinates when the barometer has a random error of 10m, a systematic error of 20m, when the air pressure at sea level altitude is the nominal pressure.

Figure 5 is a graph of the distance from the UAV to the second reference point and the error in determining the UAV coordinates when the barometer has a random error of 10m, a systematic error of 0, when the air pressure at sea level altitude is the nominal pressure.

Figure 6 is a graph of the distance from the UAV to the second reference point and the error in determining the UAV coordinates when the barometer has a random error of 10m, a systematic error of 20m, when the air pressure at sea level altitude has the nominal pressure difference.

**Comment:** When determining UAV coordinates from a reference point, due to the random error of the barometer, the UAV coordinate error is high in both cases - when there is no systematic error as well as when there is systematic error of the barometer. The aggregate error is still large and with distances to the reference point greater than 1700m, the error will exceed 100m. In case of systematic error, the error in determining UAV coordinates also increases and is proportional to the distance from the UAV to the reference point.

## Conclusions

The method of determining UAV coordinates based on ground reference points can provide UAV coordinates for the UAV to complete its flight mission. Different from the method of determining UAV coordinates based on a ground reference point, subject to random errors and systematic errors of the barometer, the method of determining UAV coordinates from two ground reference points is not affected by these errors, it is only affected by the measurement error of the range angle and direction angle of the pan-tilt camera installed on the UAV. Therefore, when there is enough information about ground reference points, using this method will be more effective. The algorithm presented in this article allows determining UAV coordinates from two ground reference points and is easily applied to real missions.

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