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# New non-singular terminal sliding mode control of DC motor

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## Article Info

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**Abstract**
Terminal Sliding Mode control (TSM) is created by equivalent control and discontinuous control, but when the states are close to zero the singularity may occur. To avoid this phenomenom Non-Singular Terminal Sliding Mode Control is introduced. But this technique also makes the convergence time longer. This article proposes a new Non-Singular Terminal Sliding Mode Control (NSTSM) to keep the convergence time short when avoiding sigularity. The process of synthesizing the control law is strictly mathematically guaranteed. Simulations in Matlab visually represent the research results.

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**Keywords:** DC motor, SMC, TSM, NTSM, convergence time

## 1. Introduction

The Sliding Mode Control is a popular method to control a plant which has matched disturbances or uncertainties [6]. But the convergence time of this method is undetermined, therefore TSM is introduced as a better version when it can determine the convergence time by initial conditions of states and choosing parameters [1-3]. But the sigularity occurs when the states of the system are close to zero. Then the NSTSM is introduced [4] to avoid this, but it also makes the convergence time of the sytem longer by increase the exponent in the sliding variable expressions. Other technique like Fast Terminal Sliding Mode [5] can only change convergence time but the singularity phenomenom is still there.
In this article, a new type of NSTSM is introduced when it can keep a short convergence time at the same time avoiding sigularity phenomenom. The dynamic model of a DC motor is also mentioned as an example to apply the introduced method when the proof of singularity free is taken by using an equivalent system. The uncertainty in the system of a DC motor is defined as the load which is changing during the time when the motor is in action.

## 2. Methodology

### 2.1. Dynamic model of a DC motor

Dynamic model of a DC motor is mentioned in many literatures [1]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -a_1 x_1 - a_2 x_2 + f(t) - bu \end{cases} \tag{1}$$

Where $a_1, a_2, b$ are positive parameters; $x_1, x_2$ are the velocity and acceleration errors of the motor rotation angle, respectively, and *f(t)* is the expressions of load placed on the motor shaft. Assume that *f(t)* is internal limitless load*:*

$$|f(t)| \leq M \tag{2}$$

Where M is a known positive value.

### 2.2. TSM control

To apply TSM control for system (10), choose sliding surface as follows:

$$S = cx_1^{\frac{p}{q}} + x_2 \quad (3)$$

Where $c$ >0 and $p$, $q$ are odd intergers.
Then, its derivative:

$$\dot{S} = c\frac{p}{q}x_1^{\frac{p-q}{q}}\dot{x}_1 + \dot{x}_2 \quad (4)$$

Combine (4) with (1) one can have:

$$\dot{\boldsymbol{S}} = \boldsymbol{c}\frac{\boldsymbol{p}}{\boldsymbol{q}}\boldsymbol{x}_1^{\frac{\boldsymbol{p}-\boldsymbol{q}}{\boldsymbol{q}}}\boldsymbol{x}_2 - \boldsymbol{a}_1\boldsymbol{x}_1 - \boldsymbol{a}_2\boldsymbol{x}_2 + \boldsymbol{f}(\boldsymbol{t}) - \boldsymbol{bu} \quad (5)$$

There are two parts of control input, equivalent part or continuous part will keep the sliding variable stay on the sliding surface and the discontinuous part that drag it to the sliding surface, or:

$$u = u_{eq} + u_d \quad (6)$$

Then apply $u_{eq}$ to (5) and one can get the equation:

$$\dot{S} = 0 \quad (7)$$

From what one can achieve $u_{eq}$:

$$u_{eq} = \frac{1}{b}\left[c\frac{p}{q}x_1^{\frac{p-q}{q}}x_2 - a_1x_1 - a_2x_2\right] \quad (8)$$

When discontinuous part has the form of:

$$u_d = -K\text{sign}(S) \quad (9)$$

Where K is a positive parameter such that: $b.K \geq M$ or:

$$\boldsymbol{b}.\boldsymbol{K} = \boldsymbol{M} + \boldsymbol{\sigma}, \boldsymbol{\sigma} \geq \boldsymbol{0} \quad (10)$$

When apply $u$ to (5) then the equivalent part will neutralize most of the terms of $\dot{S}$:

$$\dot{S} = -bK\text{sign}(S) + f(t) \quad (11)$$

Then, by choosing Lyapunov function $V = \frac{1}{2}S^2$, one can obtain:

$$\dot{V} = S\dot{S} = bK|S| + S.f(t) \quad (12)$$

When $f(t)$ meet condition (2), then one can have:

$$\dot{V} < -bK|S| + |S|.M = (-bK + M)|S| = -\sigma|S| \leq 0 \quad (13)$$

Then with the control input as in (6), (8) and (9) the system is stable according to Lyapunov.
To calculate convergence time of the system, let the sliding variable be zero:

$$S = cx_1^{\frac{p}{q}} + x_2 = 0 \quad (14)$$

Taking one term to the other side:

$$cx_1^{p/q} = -\dot{x}_2 = -\frac{dx_1}{dt} \quad (15)$$

Transform (15) to a more convenient form:

$$-x_1^{-\frac{p}{q}}dx_1 = cdt \quad (16)$$

Intergrate both sides over $t$ and $x_1$:

$$\frac{q}{q-p}\int_{x_1(t_0)}^{0} dx_1^{\frac{q-p}{q}} = -c\int_{t_0}^{t_0+t_r} dt \quad (17)$$

In which, $t_0$ is the time to bring $S$ from the initial value $S_0$ at the initial time to zero value.
Then convergence time of the system states can be determined:

$$t_r = \frac{q}{c(q-p)}x_1(t_0)^{\frac{q-p}{q}} \quad (18)$$

Similarly, one can determine the time that takes to bring S from the initial value $S_0$ at the initial time to zero value:

$$t_0 = \frac{|S_0|}{bK} \quad (19)$$

Then the total convergence time of the system can be calculated as:

$$t_{total} = t_0 + t_r \quad (20)$$

But when apply the control inut in (6), (8), (9) to sysyem (1) one can achieve the equivalent system:

$$\begin{cases} \dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2 \\ \dot{\boldsymbol{x}}_2 = \boldsymbol{f}(\boldsymbol{t}) - \boldsymbol{bK}\textbf{sign}(\boldsymbol{S}) + \boldsymbol{c}\frac{\boldsymbol{p}}{\boldsymbol{q}}\boldsymbol{x}_1^{\frac{\boldsymbol{p}-\boldsymbol{q}}{\boldsymbol{q}}}\boldsymbol{x}_2 \end{cases} \quad (21)$$

There is a singularity phenomenom when $x_1 \rightarrow 0$ and $x_2 \neq 0$ because the expressions $c\frac{p}{q}x_1^{\frac{p-q}{q}}x_2$ is infinity.

### 2.3. New NSTSM control

The proposed NSTSM is totally similar to a conventional TSM with the same sliding variable:

$$S = cx_1^{\frac{p}{q}} + x_2 \quad (22)$$

Where $c$ >0 and $p$, $q$ are odd intergers.
Then, its derivative:

$$\dot{S} = c\frac{p}{q}x_1^{\frac{p-q}{q}}\dot{x}_1 + \dot{x}_2 \quad (23)$$

Combine (23) with (1) one can have:

$$\dot{\boldsymbol{S}} = \boldsymbol{c}\frac{\boldsymbol{p}}{\boldsymbol{q}}\boldsymbol{x}_1^{\frac{\boldsymbol{p}-\boldsymbol{q}}{\boldsymbol{q}}}\boldsymbol{x}_2 - \boldsymbol{a}_1\boldsymbol{x}_1 - \boldsymbol{a}_2\boldsymbol{x}_2 + \boldsymbol{f}(\boldsymbol{t}) - \boldsymbol{bu} \quad (24)$$

There are two parts of control input, equivalent part or continuous part will keep the sliding variable stay on the sliding surface and the discontinuous part that drag it to the sliding surface, or:

$$u = u_{eq} + u_d \qquad (25)$$

Choose $u_{eq}$:

$$u_{eq} = \frac{1}{b}[-a_1x_1 - a_2x_2] \qquad (26)$$

When discontinuous part has the form of:

$$u_d = -K\text{sign}(S) \qquad (27)$$

Apply (25), (26), (27) to (24):

$$\dot{S} = -bK\text{sign}(S) + c\frac{p}{q}x_1^{\frac{p-q}{q}}x_2 \qquad (28)$$

Then, by choosing Lyapunov function $V = \frac{1}{2}S^2$, one can obtain:

$$\dot{V} = S\dot{S} = -bK|S| + S.f(t) + c\frac{p}{q}x_1^{\frac{p-q}{q}}x_2S \qquad (29)$$

From (29), similarly to (13), there is always $\dot{V} \le 0$ when $|c\frac{p}{q}x_1^{\frac{p-q}{q}}x_2| < \sigma$, or the system is stable according to Lyapunov.

Because the sliding variable expressions is unchanged, then the convergence time of the system is the same as was shown in (18), (19), (20).

To prove the system is singularity free, one can apply control input expressions (25), (26), (27) to the system equation (1) to get the equivalent system.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(t) - bK\text{sign}(S) \end{cases} \qquad (30)$$

In equations (30), there is no singularity factor $c\frac{p}{q}x_1^{\frac{p-q}{q}}x_2$ in $\dot{x}_2$ expressions then the system in this case is singularity free.

## 3. Simulation and discussion

The article chooses to simulate two cases: when the ititial angle velocity is zero and when it is not.

The parameters of the system are:

$$\begin{cases} a_1 = 1 \\ a_2 = \frac{1}{100} \\ b = 10 \\ c = 10 \\ K = 10 \\ p = 1001 \\ q = 2001 \end{cases} \qquad (31)$$

First case when ititial conditions are:

$$\begin{cases} x_{10} = 1 \\ x_{20} = 1 \end{cases} \qquad (32)$$

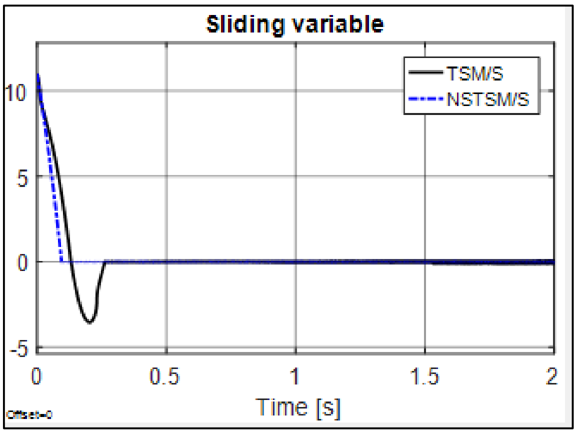


**Fig 1:** Sliding variable

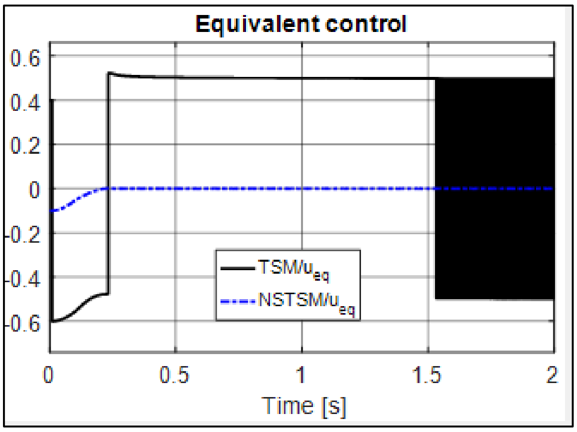


**Fig 2:** Equivalent control

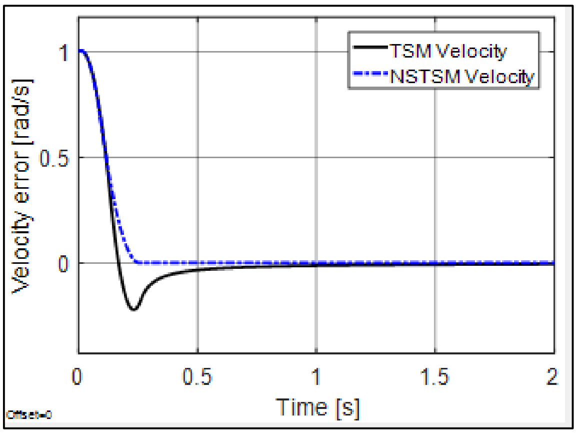


**Fig. 3.** Error of angle velocity

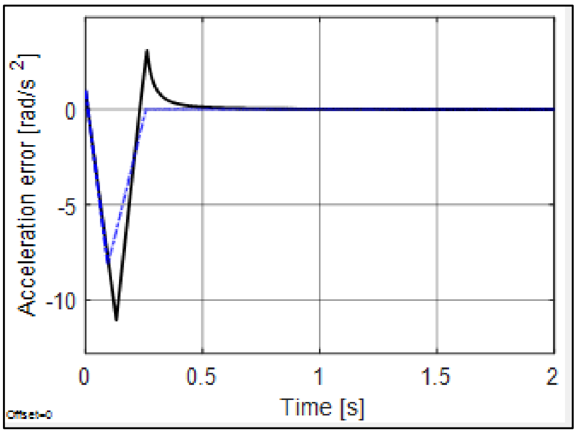


**Fig 4:** Error of acceleration angle

The results were shown in Fig. 1 to Fig. 4 where the convergence time of the sys tem in the new NSTSM is shorter than in conventional TSM at the same time equivalent control of the NSTSM also more stable.

Second case whenititial conditions are:

$$\begin{cases} x_{10} = 0 \\ x_{20} = 1 \end{cases} \quad (33)$$

And the results were shown in Fig. 5 to Fig. 8 where con can see that the sliding variable of the new NSTSM is more stable than the conventional TSM. The same situation is with equivalent control input of the two controler, when conventional TSM has the chatterring effect even in equivalent control input. In the end the new NSTSM has the smaller error compared to the conventional TSM when angle velocity error of NSTSM is close to zero but in conventional TSM the error is about $5*10^{-3}$ rad/s.

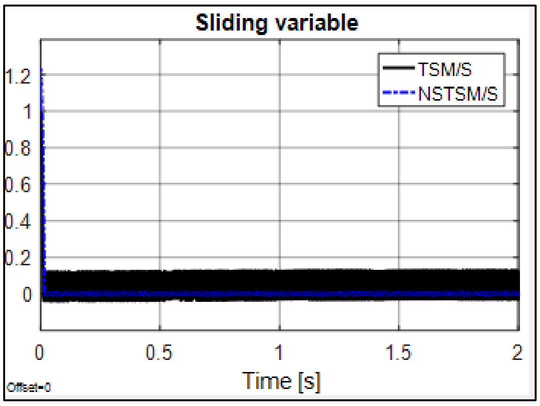


**Fig 5:** Sliding variable

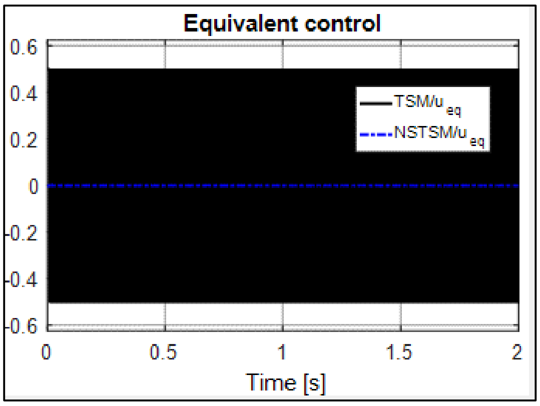


**Fig 6:** Equivalent control

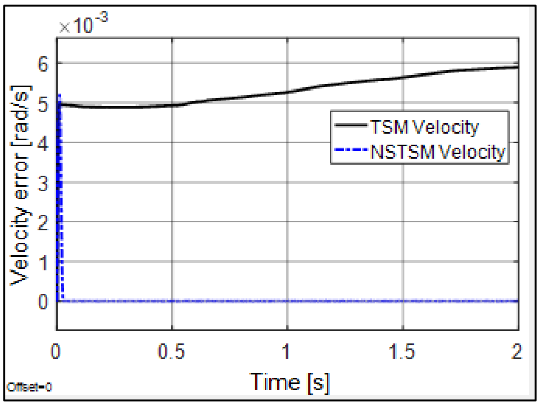


**Fig. 7.** Error of angle velocity

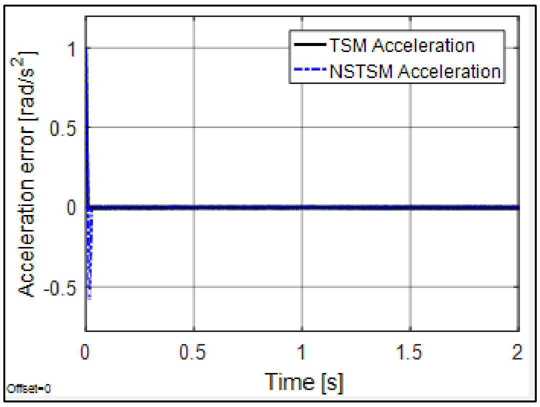


**Fig 8:** Error of acceleration angle

## 4. Conclusion

The article has presented a new NSTSM control to control a DC motor, whereby the quality of trajectory tracking is improved compared to conventional TSM, while ensuring the robustness of the system against uncertainty from changing load, at the same time avoids singularity phenomenom. With that result, this method can be applied to other plants, ensuring high efficiency. Research results are rigorously mathematically proven and visualized by simulation.

## 5. References

1. Utkin VI. Sliding mode control design principles and applications to electric drives, IEEE Transactions on Industrial Electronics. 1993; 40(1):(23-36), ISSN 1042-296X.
2. M Zak. Terminal attractors for addressable memory in neural network, Physics Letters A. 1988; 1333(1/2):18-22.
3. S Venkcataraman, S Gulati. Control of nonlinear systems using terminal sliding modes, in Proc. American Control Conf., Chicago, IL, USA. 1992; 24-26:891-893. Method.
4. Y Feng, X Yu, Z Man. Non-singular adaptive terminal sliding mode control of rigid manipulators, Automatica. 2002; 38(12):2159-2167.
5. X Yu, Z Man. Fast terminal sliding-mode control design for nonlinear dynamical systems, IEEE Trans. Circuits Syst. I, Reg. Papers. 2002; 49(2):261-264.
6. VI Utkin. Variable structure systems with sliding modes, IEEE Trans. Autom. Contr. 1977; 22(2):212-222.