



Modeling and optimal control of cocoa black pod disease in Ondo State, Nigeria

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Abstract

An epidemiological model gives basic insights into plant diseases, which enables an analysis of issues that may affect how to prevent and combat diseases that cause plant discomfort and reduce production. For cocoa black pod disease transmission dynamics and control, we present a mathematical model. A qualitative analysis of the developed model is conducted to find out more about its associated equilibria and the basic reproduction number. To determine the relative importance of model parameters to disease transmission, the epidemiological model is subjected to a sensitivity analysis and this reveals the parameters that are sensitive to the transmission and control of the disease. Based on the findings of the study, farmers whose cocoa plants are affected by black pod can reap the benefits of the research, as prevention and control of the epidemics can be achieved.

Keywords: Black pod, boundedness, sensitivity analysis, epidemiological, next generation matrix, reproduction number

1. Introduction

Essentially, a mathematical model represents what actually happens in real life. Infectious disease mathematical modeling aims to describe the transmission process, which is caused by the introduction of infectious individuals into a population of susceptible individuals (Li, 2017) ^[7], to estimate the spread of infectious diseases, a quantitative and qualitative study of epidemic dynamics is an invaluable tool. Taking into account the dynamic properties of population growth, infectious disease spread, and other social factors is essential for developing an accurate mathematical model of infectious diseases. The mathematical model can be useful in predicting the trajectory of communicable diseases.

Cocoa (*Theobroma cacao* L.) is one of the most widely grown perennial cash crops around the world. The beans of this plant are commonly harvested for use in cosmetic, pharmaceutical, and chocolate industry applications (Dumont & Tchuente, 2012) ^[4]. Unfortunately, approximately 20 to 30 percent of the annual cocoa harvest is lost to pests and diseases, which could result in significant economic losses for the cocoa farmers (Nembot *et al.*, 2018) ^[9]. A significant amount of this damage is by the Black Pod Disease (BPD), which is one of the most devastating cocoa diseases (Oduro *et al.*, 2020) ^[11]. BPD is caused by a Fungus that spreads quickly on the pods under condition of excessive rain and humidity, insufficient sunshine and temperature below 21°C (78°F).

Ali *et al.*, 2017 ^[1], Guest, 2007 ^[6] reported that a special species of *Phytophthora*, *Phytophthora megakarya* is the most damaging and aggressive pathogen, causing about 80 to 90 percent crop loss in West and Central Africa. Infected pods first show yellow spots that gradually turn brown and enlarge to a dark brown color within five days, indicating BPD, after eight days of infection, the lesion has spread to cover the entire pod. A pod infection does not only affect the surface of the pod, but also penetrates the interior of the pod, affecting the beans within (Guest, 2007) ^[6]. A healthy cacao pod may become infected from either an environmental reservoir or from an infected pod through air transmission (Vanegtern, 2015, Nembot *et al.*, 2018) ^[14,9]. According to Brasier, 1981, disease inoculums can survive in cocoa plantations for up to eighteen (18) months.

As cocoa pods develop at any stage of development, disease susceptibility and the risk of attack can be adversely affected, depending, among other factors, on the stage at which the cocoa pod is at its development (Takam-Soh, 2013, Efombagn, 2004)^[12, 5]. There has been a great deal of damage caused by this fungus disease to cocoa pods, causing farmers to struggle in dealing with it to a great extent. The most common means of controlling the black cocoa pod disease is by fumigating the cocoa pods, which is a highly effective and economical method. Alternatively, hybrid cultivars that are partially resistant or tolerant to the disease can also be used.

There are a number of models that have been developed, including those of (Brasier, 1981, Nembot *et al.*, 2018 Oduro *et al.*, 2020)^[9, 11], but very few studies have systematically examined how epidemics in cocoa have evolved over time. It is important to remember, however, that many unresolved questions remain regarding the importance of primary and secondary sources of inoculum, spore dispersal as well as their interaction, but mathematical modeling can provide insight into them.

The aim of this study is to develop a new epidemiological model to gain qualitative insight into the transmission and control of BPD. The notable feature of the model is the incorporation of three developmental stages of cocoa, two infectious pathways and approach to combat the disease at early stage.

2. Model Formulation

There is no way to distinguish an infected pod from an unaffected ones at the earlier stage of BPD development, but with the use of ordinary differential equations, the dynamics of the outbreaks of BPD can be described.

In an epidemiological model describing the transmission dynamics of Cocoa Black Pod Disease, at time t , host populations (Cocoa Pods) are classified according to their developmental stages, in turn, infectious agents are classified according to their infectious pathways. The model assumptions are as follows;

1. There are 3 stages in which susceptible pods develop; Cherelles (floral and fruiting stages), young and mature stages, and ripe pod stages (Nembot *et al.*, 2018)^[9].
2. The infectious pod $I(t)$ has two sub classes which is according to its transmission path; Secondary Infections, i. e from Environment to pod transmission are spores produced by infected pods and released to the environment ($P_{p,e}(t)$) and Primary Infections i. e pod to pod transmission, which are spores produced by infected pod ($p_{p,i}(t)$) (Nembot *et al.*, 2018)^[9].
3. An inflow into the population (Cherelles) is at a constant rate δ_0 since the planting is seasonal.
4. Susceptible pods in contact with spores either through primary or secondary inoculum are usually contaminated at an infection rate λ_k (Nembot *et al.*, 2018)^[9], where

$$\lambda_K(P_{p,e}, P_{p,i}) = \beta_1^K \frac{p_{p,e}}{K_1 + P_{p,e}} + \beta_2^K \frac{p_{p,i}}{K_2 + P_{p,i}}, K = 1, 2. \quad (1)$$

- i. As *Phytophthora megakarya* infects pods, it takes 3-4 days for the first macroscopic symptoms to emerge after the pods have been exposed to the organism.
- ii. A pod that is infected cannot be distinguished from an infectious pod, so delay does not enter into the dynamics of the disease.
- iii. Each compartments have different death rate.
- iv. The only entry point is through cherelles.
- v. k_i is the rate of fungicide application, where $i=1,2$

Table 1: Description of parameter in the model

Parameters	Description
μ	Death rate
c	Cherelles recruitment rate
δ_1	Maturity rate from Cherelles to young and matured pods
δ_2	Maturity rate from young and matured pods to ripe pods
γ_f	harvesting rate of ripe pods
d_1	spores production rate
d_2	shedding rate of spores
β_1^K	releasing speed of spores $r_1 + r_2 < 1$
β_2^K	The rate at which spores are inactivated by infected pods due to parasitism.
K	Michaelis-Menten constant which represent the quantity of One specific inoculum has a 50% chance of infecting a pod of <i>Phytophthora megakarya</i> spores.

The model introduced consist of population whose entry point is through Cherelles, and in due time mature to young and mature pods which later is due for harvesting at ripe pod stage. Pod-to-pod and environment reservoir-to-pod transmission is both possible with the disease spreading from spores to spores or from reservoir to reservoir. After contact with a susceptible pod, the susceptible pod is moved to the infected class. Assuming Michaelis-Menten kinetics, the disease-causing organism responds to susceptible pods using Holling type II functional forms.

A schematic diagram of the model is shown in Figure 1, and the equations in equations 2 and 3 govern the model.

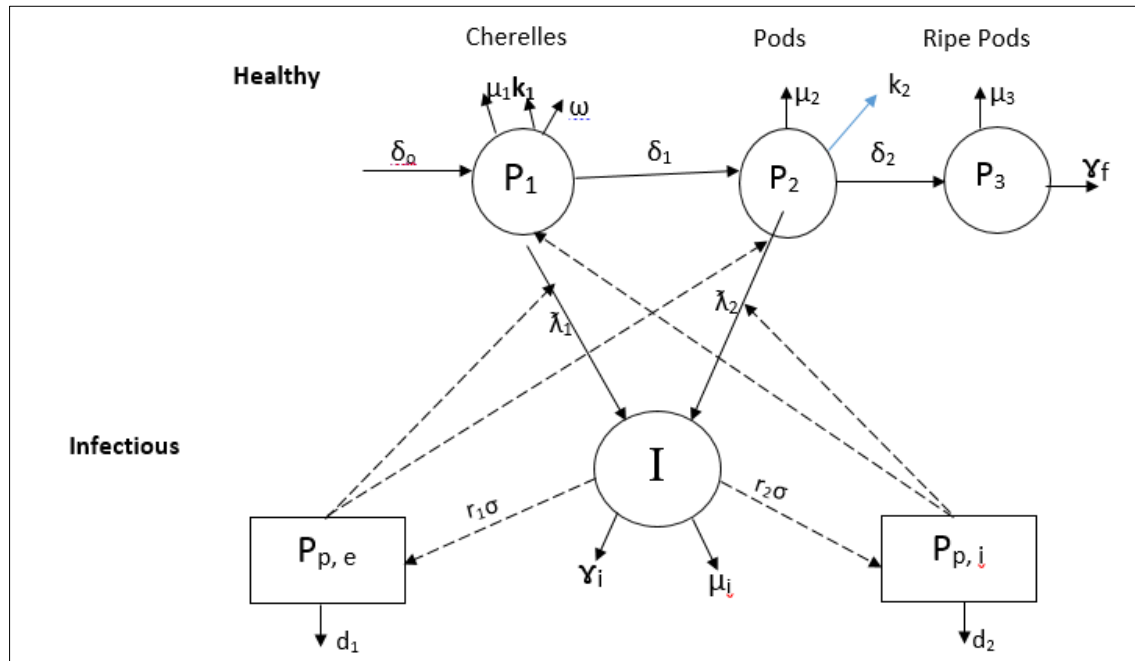


Fig 1: Schematic diagram of Cocoa black pod transmission

$$\frac{dP_1}{dt} = \delta_0 - \mu_1 P_1 - \delta_1 P_1 - \lambda_1 P_1 - \omega P_1 - k_1 P_1 \quad (2)$$

$$\frac{dP_2}{dt} = \delta_1 P_1 - \delta_2 P_2 - \mu_2 P_2 - K_2 P_2 - \lambda_2 P_2 \quad (3)$$

$$\frac{dP_3}{dt} = \delta_2 P_2 - \mu_3 P_3 - \gamma_f P_3 \quad (4)$$

$$\frac{dI}{dt} = \lambda_1 P_1 + \lambda_2 P_2 - \mu_i I \quad (5)$$

$$\frac{dP_{p,i}}{dt} = r_2 \sigma I - d_2 P_{p,i} \quad (6)$$

$$\frac{dP_{p,e}}{dt} = r_1 \sigma I - d_2 P_{p,e} \quad (7)$$

With initial conditions,

$$P_1(0) \geq 0, P_2(0) \geq 0, P_3(0) \geq 0, I(0) \geq 0, P_{p,i}(0) \geq 0, P_{p,e}(0) \geq 0. \quad (8)$$

Our proposed model take account of the fact that Susceptible pods are classified into 3 compartments according to their developmental stages; Cherelles $P_1(t)$ (Flowering and pod formation stage), Young and Mature stage $P_2(t)$ and the Ripe pod stage $P_3(t)$ (Nembot *et al.* 2018) ^[9]. The infectious pod $I(t)$ has two sub classes which is according to its transmission path; Secondary Infections, i. e from Environment to pod transmission are spores produced by infected pods and released to the environment ($P_{p,e}(t)$) and Primary Infections i. e pod to pod transmission, these are spores produced by infected pod ($P_{p,i}(t)$) (Nembot *et al.* 2018) ^[9], k_1 & k_2 are rate of fungicide application as a prevention against the disease. The proposed model differs from the model of Nembot *et al.* 2018 ^[9], as it consider application of fungicide at Cherelles and Young and Mature stages of development.

3. Ondo State and Cocoa Production

Ondo State is located in the southwestern part of Nigeria, it has a historical connection to cocoa production, and the crop has been a key component of its agricultural and economic landscape. However, like other cocoa-producing regions, the state faces challenges that require concerted efforts from both the government and the private sector to ensure the sustainability and growth of the cocoa industry.

Cocoa black pod disease, caused by the fungus *Phytophthora spp*, is a significant threat to cocoa production in in Ondo State. The disease affects cocoa pods, leading to the rotting of the pods and a subsequent decline in cocoa yields. The disease is particularly destructive during periods of high humidity and rainfall.

To access the menace of this disease, selected Cocoa farm in each local government in Ondo State were visited to access the damage caused by this disease. The figure below present the incidence of black pod disease in the state. 5 farms per local government in the state were visited and pods with visible signs of black pod disease were estimated.

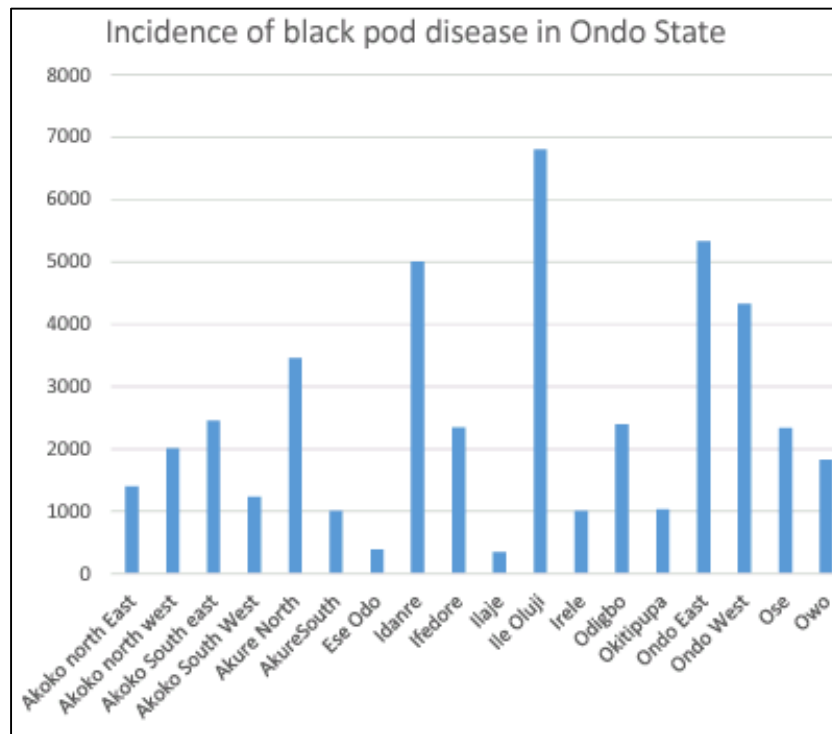


Fig 2: Showing cases of black pod disease in some selected Cocoa farmland

4. Stability Analysis

4.1 Basic properties of the model

Here we present the basic results of the model by considering its basic properties such as existence, uniqueness, positivity and boundedness. We will also compute the existence and stability of the equilibrium in this section.

Theorem 1: The solution $(P_1(t), P_2(t), P_3(t), I(t), P_{p,i}(t), P_{p,e}(t))$ of the system (2-7) exists and is bounded for all time $t \geq 0$ under the initial conditions (8). Moreover, the solution is positive for all $t > 0$ and the region

$$\Omega = \{(P_1(t), P_2(t), P_3(t), I(t), P_{p,i}(t), P_{p,e}(t)) \in \mathbb{R}_+^6: P_1 + P_2 + P_3 + I + P_{p,i} + P_{p,e} \leq N\}$$

$$\text{Where } N \leq \frac{\delta_0}{h}$$

Proof: It is important to show that for any non-negative initial data, the system has a unique solution at $t \geq 0$ that lies in the region defined in Ω . For boundedness of solution, we prove that the total population of cocoa pods N are bounded at time $t \geq 0$ and has a solution.

$$\frac{dN}{dt} = \frac{dP_1(t)}{dt} + \frac{dP_2(t)}{dt} + \frac{dP_3(t)}{dt} + \frac{dI(t)}{dt} + \frac{dP_{p,i}(t)}{dt} + \frac{dP_{p,e}(t)}{dt} \quad (9)$$

$$\frac{dN(t)}{dt} + hN \leq \delta_0 \quad (10)$$

$$\text{Where } h = \min\{\mu_1, \mu_2, \mu_3, \mu_4, \omega, \gamma_f, r_1, r_2, d_1, d_2, \sigma, k_1, k_2\}$$

And its solution is given as

$$\frac{dN(t)}{dt} \leq \frac{\delta_0}{h}(1 - e^{-ht}) + N(0)e^{-ht}, \quad (11)$$

as $t \rightarrow \infty$, we arrive at

$$\frac{dN(t)}{dt} \leq \frac{\delta_0}{h} \quad (12)$$

In other words, the solution is bounded

$$0 \leq N(t) \leq \frac{\delta_0}{h}. \quad (13)$$

Thus all solutions of the model [2-7] in $\mathfrak{R}^6$ are restricted in the region,

3.2. Disease Free Equilibrium

In the absence of the disease, we assume

$$I(t) = P_{p,i}(t) = P_{p,e}(t) = 0, \quad (14)$$

Solving for $P_1(t)$, $P_2(t)$ and $P_3(t)$ in model at equations (2-4),

$$P_1^* = \frac{\delta_0}{\mu_1 + \omega + \delta_1 + \lambda_1 + k_1} \quad (15)$$

$$P_2^* = \frac{\delta_1 P_1}{(\mu_2 + \delta_2 + \lambda_2 + k_2)} \quad (16)$$

$$P_3^* = \frac{\delta_2 P_2}{(\mu_2 + \psi_j)} \quad (17)$$

Accordingly, the equilibrium point for diseases free of disease is

$$E_0 = (P_1^*, P_2^*, P_3^*, 0, 0, 0) \quad (18)$$

3.3. Computation of Reproduction number ($\mathcal{R}_0$)

In order to compute $\mathcal{R}$, we used the next generation matrix method with these matrices.

$$F_i = \begin{pmatrix} \lambda_1 S_1 + \lambda_2 S_2 \\ r_2 \sigma I \\ r_1 \sigma I \end{pmatrix} \text{ and } V_i = \begin{pmatrix} \mu_i I \\ d_2 S_{p,i} \\ d_2 S_{p,e} \end{pmatrix} \quad (19)$$

Evaluating the Jacobian of F_i and V_i at E_0 gives F and V matrices. Then we obtained the spectral radius $\rho(FV^{-1})$ as the basic reproduction number as:

$$R_0 = \frac{(\delta_2 + \mu_2) \mu_i k_2 \delta_0 d_2 (\omega + \delta_1 + \mu_1) ((\beta_2^1 \delta_1 + \beta_1^1 (\delta_2 + \mu_2) k_2 k_2 (\sigma r_1) + \sigma r_2 (\beta_2^2 \delta_1 + \beta_1^2 (\delta_2 + \mu_2) k_1 d_1) k_1 d_1)}{\mu_i d_1 (\delta_2 + \mu_2) (\omega + \delta_1 + \mu_1) k_1 k_2 d_2} \quad (20)$$

Thus, we claim the following.

Theorem 2: Disease free equilibrium (DFE), E_0 , is locally asymptotically stable (LAS) when $R_0 < 1$ and unstable when $R_0 > 1$.

Proof 2: Consider the DFE, $E_0 = (P_1^*, P_2^*, P_3^*, 0, 0, 0)$. We compute the Jacobian of system (2),

$$J(E_0) = \begin{pmatrix} -(\mu_1 + \omega + \lambda_1 + \delta_1 + k_1) & 0 & 0 & 0 & 0 & 0 \\ \delta_1 & -(\mu_2 + \lambda_2 + \delta_2 + k_2) & 0 & 0 & 0 & 0 \\ 0 & \delta_2 & -(\mu_3 + \gamma_j) & 0 & 0 & 0 \\ \lambda_1 & \lambda_2 & -\mu_i & 0 & 0 & 0 \\ 0 & 0 & 0 & r_1 \sigma & d_1 & 0 \\ 0 & 0 & 0 & r_2 \sigma & 0 & d_2 \end{pmatrix}$$

at the E_0 , and taking the characteristics equation $|\lambda - A| = 0$ leads to the linearized system. Obtaining the eigenvalue of the linearized System and applying the Routh-Hurwitz criterion, we established that all the eigenvalue of the system are negative ($\lambda < 0$).

4. Sensitivity Analysis

In order to assess the relative impact of various factors on the stability of a model under uncertain data, a Sensitivity Analysis is used. The purpose of the analysis is to determine which parameter is critical. Based on both the local method for calculating the sensitivity indexes of the basic reproduction number to the parameters in the model, we conduct the analysis. Normalized forward sensitivity index is used to analyze local sensitivity. The sensitivity index of R_0 with respect to the parameters in our model are derived in Wang *et al.* (2019).

$$F_P^{R_0} = \frac{\partial R_0}{\partial P} \times \frac{P}{R_0}, \quad (21)$$

Where P is the parameter value displayed in Table 2

Table 2: Model Parameters

Parameters	Values	Source	Sensitivity Index
μ_1	0:05	[9]	-0.7724
μ_2	0:00469	[9]	-0.7714
μ_3	0:05	Estimated	-0.4724
ω	0:1	[12]	-0.2743
d_2	0:02	[9]	-0.2324
d_1	0:4	[9]	-0.1754
σ	574200	[9]	+0.3727
k_2	10^8	[9]	-0.3218
k_1	2×10^9	[9]	-0.3217
r_2	0.4	[9]	+0.3541
r_1	0.4	[9]	+0.3548
β_1^1	0:05	[9]	0.3511
β_1^2	0:05	[9]	0.3511
β_2^1	0.02	[9]	0.3448
β_2^2	0.02	[9]	0.3241
γ_f	0:2	Estimated	-0.2549
δ_0	20	[2]	+0.3541
δ_1	0:05	[2]	+0.3541
δ_2	0:027	[9]	+0.3541

5. Numerical Simulation

Numerical simulations were carried out to illustrate the developed model in equation (2-7) and (8) using some parameters obtained from the literature and some estimated from ecological observations, the numerical simulations were carried out with the aid of Maple software.

Table 2 below displays the model parameters, value and literature where found.

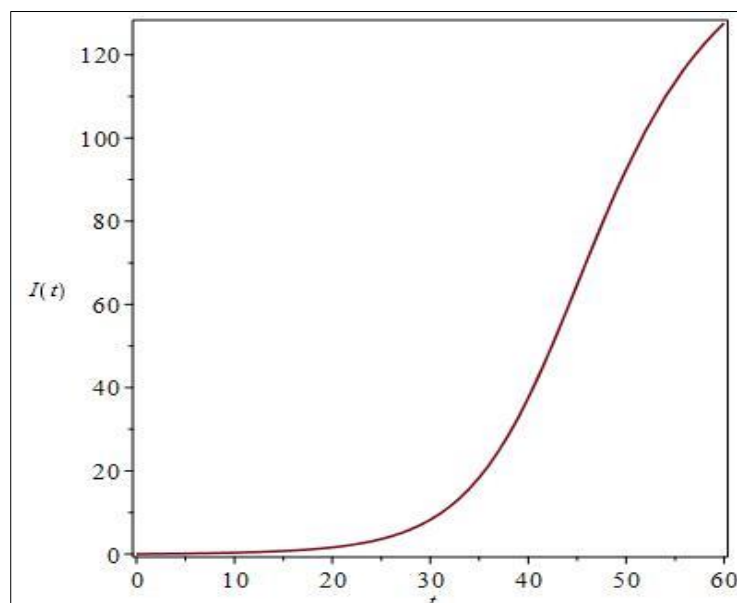


Fig 2: Graph of I (t) without control against t

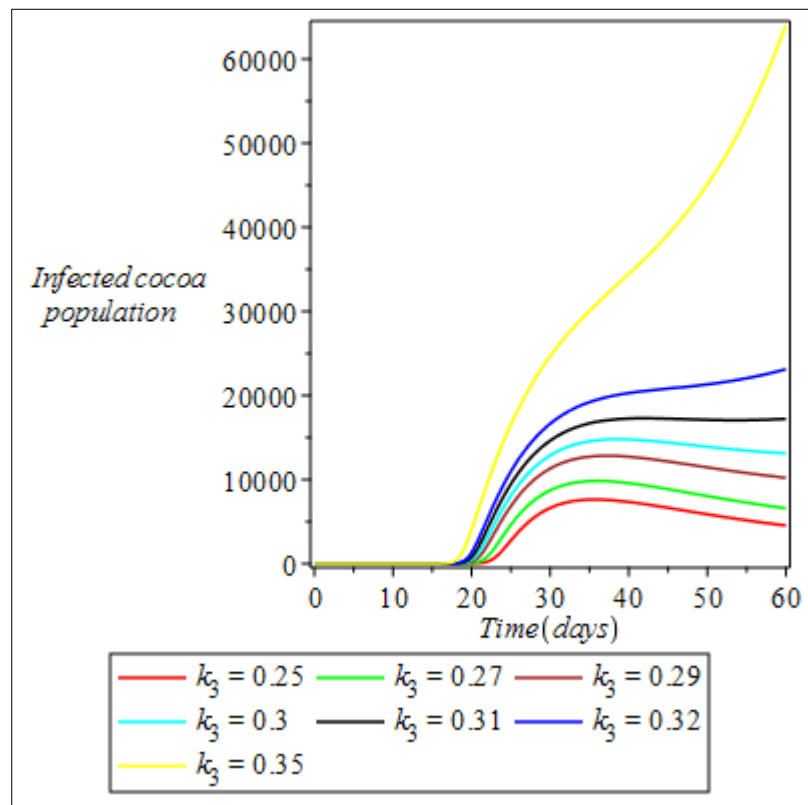


Fig 3: Graph of $I(t)$ with control against t

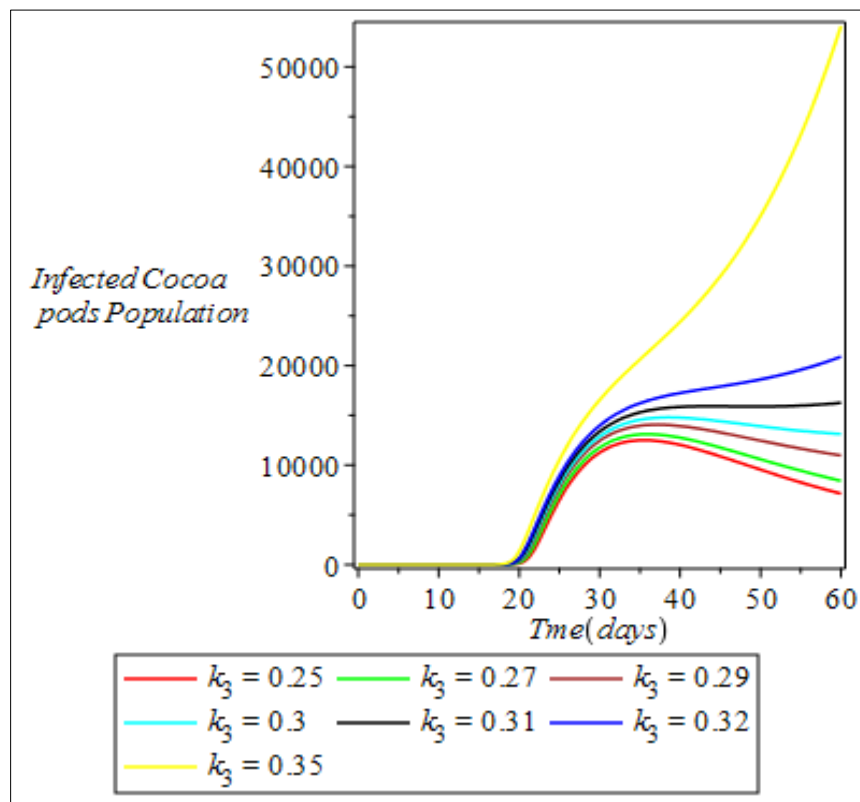


Fig 4: Graph of $I(t)$ with control against t (varied application 1)

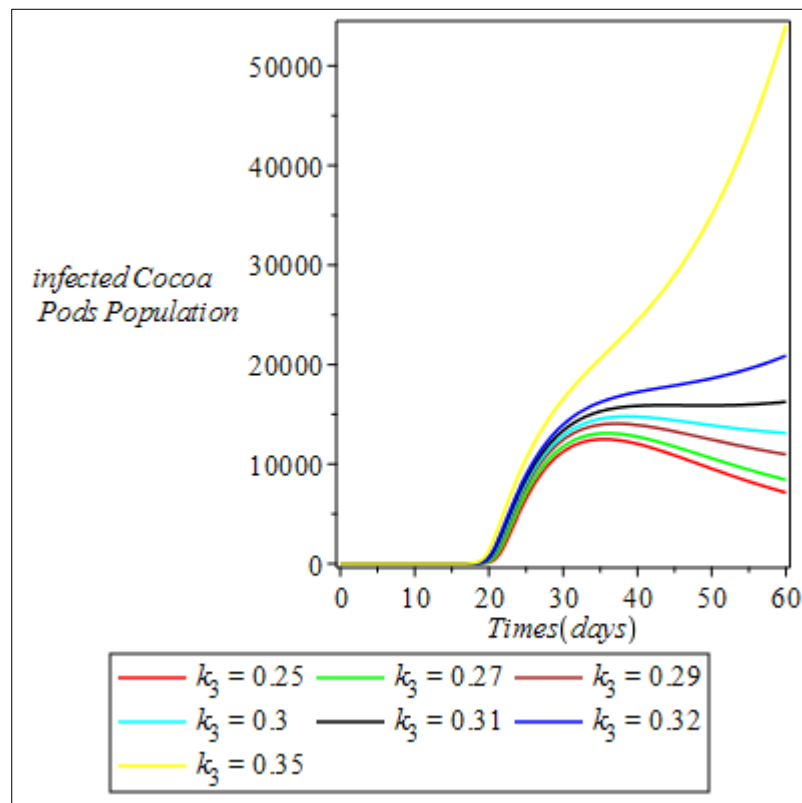


Fig 5: Graph of $I(t)$ with control against t (varied application 2)

5.1. Discussion

The study considers the dynamics transmission and management strategies to combat Cocoa black pod disease. The model equation was shown to be biologically feasible by establishing its positivity and boundedness of the solution. The stability of the disease equilibrium depends on the threshold number (R_0) called the Reproduction number. Whenever the $R_0 < 1$, the disease free equilibrium is both locally and asymptotically stable and unstable otherwise, this implies that the black pod disease fade out of the cocoa population.

The result of the sensitivity analysis reveals the rate of fungicide application is the most sensitive parameter to the Reproduction number, with a negative index, thus, an increase in this parameter reduces the value of the reproduction number, which signifies that the diseases can be eradicated at certain varying range of application of fungicide on $P_1(t)$ and $P_2(t)$, thus, it is therefore recommended that farmers should intensify the effort of adopting the use of Fungicide application in the control of yam mosaic virus disease.

From figure 2, the solution plot of the modeled equation reveals the behavior of the infected pollution in the absence of control measures. The populations of infected cocoa pods increases drastically in the absence of control measures. The infected cocoa pod population is stable in the first 20 days, after which it increases,. This increment can be attributed to the lack of control measures.

Figure 3 is the plot of infected cocoa pod population with time at various rate of fungicide application, here same rate is applied on $P_1(t)$ and $P_2(t)$, and it reveals that the application has no effect on the first 20 days, after which there is reduction in the number of the infected population,. It implies that the best optimal recommended rate of application is between 0.3 to 0.25.

Figure 4 is the plot of infected cocoa pod population with time at various rate of fungicide application, here constant rate of 0.3 is applied on $P_1(t)$ is pend at 0.3 and with varying application for $P_2(t)$, and it reveals that the application has no effect on the first 20 days, there is an increment of number of infected pods between day 20 and 30, after which there is drastic reduction in the number of the infected population, it implies that the best optimal recommended rate of application is between 0.29 to 0.27.

Figure 5 is the plot of infected cocoa pod population with time at various rate of fungicide application, here constant rate of 0.3 is applied on $P_2(t)$ and with varying application on $P_1(t)$, and it reveals that the application has no effect on the first 20 days, there is an increment of number of infected pods between day 20 and 35, after which there is drastic reduction in the number of the infected population, thus under this condition, the best optimal recommended rate of application is between 0.30 to 0.25.

6. Conclusion

The study develop a mathematical model to clarify the dynamic of black pod cocoa disease and control with fungicide application to combat the menace of the disease at $P_1(t)$ and $P_2(t)$ stages. The study showed that the threshold value, R_0 must be less than unity in order to eliminate the disease. To achieve this, the application of fungicide on cocoa pod population should be encouraged by farmers as a control measure at the specified spraying rate for better yield of the economical plant in Ondo State, Nigeria.

The mathematical modeling study of black pod disease has provided valuable insights into the dynamics and spread of the disease, as well as the effectiveness of various control strategies. Our analysis suggests that the model is epidemiologically and

mathematically well-posed. The model's associated equilibrium's stability (both Local and Global) was analyzed qualitatively and well established. The reproduction number was subjected to a sensitivity analysis to ascertain the relative significance of the various variables responsible for disease transmission and control, and this reveals that rate of fungicide application and infected pod remover are two factors that can be altered to reduce the spread of the disease and for potential long time control, regular harvest ripe pods and maintaining a healthy environment plays a significant factor in the eradicating the disease.

The incorporation of optimal control strategies into our analysis has yielded actionable recommendations for cocoa farmers and policymakers alike. By identifying parameters that minimize disease prevalence while considering economic constraints and ecological sustainability, we have strived to bridge the gap between theoretical modeling and practical application. This research underscores the importance of adaptive and sustainable approaches, recognizing that the battle against cocoa black pod disease requires a dynamic and evolving strategy.

However, it is important to note that our model is based on certain assumptions and limitations, and there is still much that is not fully understood about the disease. Future research could incorporate effect of environmental factors into the model development.

Overall, the amalgamation of mathematical modeling and optimal control strategies offers a promising pathway towards a more robust and sustainable cocoa industry. The ongoing collaboration between researchers, farmers, and policymakers is essential for translating these findings into on-the-ground practices that effectively combat cocoa black pod disease, thereby preserving the integrity and longevity of cocoa cultivation worldwide.

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