Mathematical modeling on dynamics through ordinary differential equation

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Abstract

In this pack we discuss the mathematical terms like mathematical model, simple harmonic motion. Motion under gravity in a resisting medium have been defined and their expressions were discussed by making of ordinary differential equation and initial value problems were applied. At last the mathematical model was developed and tested by making use of these problems in this model.

Keywords: Mathematical model, simple harmonic motion, motion under gravity, ordinary differential equation ODE, initial value problems (IVP)

1. Introduction

A mathematical model is an abstract description of a concrete system using mathematical concepts and languages. The process of developing a mathematical model is known as mathematical modeling.

A dynamic model accounts for time dependent changes in the state of the system while a static model calculates the system in equilibrium and thus is time invariant dynamic models typically are represented by differential equations.

Simple harmonic motion can serve as a mathematical model, but it is typified by the oscillation of a mass on a spring when it is subject to the linear elastic restoring force given by Hook's law. The motion is sinusoidal in time and demonstrates a single resonant frequency.

Let a particle travels a distance 'x' in time 't' in a straight line, then its velocity 'v' is given by $\frac{dx}{dt}$ and its acceleration is given by:

$$\frac{dx}{dt} = \left(\frac{dx}{dt}\right)\left(\frac{dx}{dt}\right) = \frac{vdv}{dx}$$

$$\frac{d^2x}{dt^2}$$

1. Simple harmonic motion: Here a particle moves in a straight line in such a manner that its acceleration in always proportional to its distance from the origin and is always directed towards the origin, so that

$$V\frac{dv}{dx} = \lambda x \qquad (1)$$

Integration both sides we get

$$V^2 = \lambda (a^2 - x^2) \quad (2)$$

Where the particle is initially at rest at $x = a$. Then equation (1) gives

$$\frac{dx}{dt} = \bar{\sqrt{\lambda}} \sqrt{a^{2-} x^2} \quad (3)$$

We take the negative sign. Since velocity increases as *"x"* decreases as shown in figure (a)

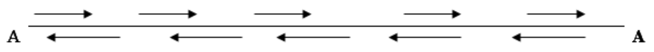

Fig (a)

Again integrating both sides and using condition that at t = 0, and x= a then

$$x(t) = a \cos \sqrt{\lambda}\ t \quad (4)$$

So that

$$V(t) = -a\sqrt{\lambda}\, sin\sqrt{\lambda}\, t \quad (5)$$

This is simple harmonic motion; both displacement and velocity are periodic functions with periods $2\pi/\sqrt{\lambda}$
The particle stands from "A" with zero velocity and moves towards zero with increasing velocity and reaches at zero time $Z/2\sqrt{\lambda}$ with velocity$\sqrt{\lambda a}$. It continue to move in the same direction but now with decreasing velocity till it reaches A' (oA = a) where its velocity is again zero. It them begins moving towards zero with increasing velocity and reaches at zero with velocity $\sqrt{\lambda a}$ and again comes to rest at A after a total time period $2Z/\sqrt{d}$. The periodic motion them repeats itself.
As an example of simple Harmonic motion, consider a particle of mass "m" attached to one end of a perfectly elastic string, the other end of which is attached to a fixed point "O". As shown in figure (b). The particle moves under gravity vaccume.

Let to be the natural length of string and let "a" be its
When the particle is in equilibrium. So that by Hoops law.

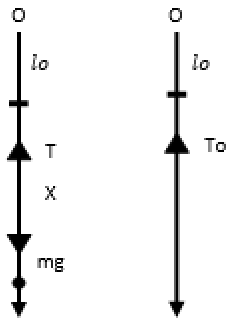

fig (b)

$$mg = To = \lambda \frac{a}{lo} \quad (6)$$

Where 'λ' is the coefficient of elastically. Now let the strong be further stretched a distance "C" and then mass be left free. The equation of motion which states that
Mass X Acceleration in any direction is equal to force on the particle in that direction, given

$$mv\frac{dv}{dx}\, mg - t = mg - \lambda \frac{a+x}{l_o} = \frac{\lambda s}{lo} \quad (7)$$

Which gives a simple harmonic motion with time period $2z\sqrt{\frac{a}{g}}$

2. Motion under gravity in a resisting medium: A particle falls under gravity in a medium in which the resistance is proportional to the velocity. The equation of motion is given by.

$$m\frac{dv}{di} = mg - mkv$$

Or

$$\frac{dv}{v-v} = kdt\ ;\ v = \frac{g}{k} \quad (8)$$

Integration equation (8) both sides we get.

$$\boldsymbol{V - V = Ve^{-kt}} \quad (9)$$

If the particle states from rest with zero velocity. The equation (7) gives

$$V = V\ (I - E^{-kt}) \quad (10)$$

So that the velocity goes on increasing and approaches the limiting velocity g/k as t→∞. Replacing 'V' by $\frac{dx}{dt}$ we get.

$$\frac{dx}{dt} = V\ (I - e^{-kt}) \quad (11)$$

Integrating both sides and using

$$x = 0 \text{ when } t = 0 \text{ we get}$$

$$X = Vt + \frac{Ve^{-kt}}{K} - \frac{V}{K} \quad (12)$$

3. Develop the Mathematical Model: The model includes those aspects of the application so that its solution will provide answers to the questions of interest. However inclusion of too which complexity may make the model unsolvable and unless. To develop the mathematical model we use laws that must be followed, diagram must be understand mathematical model. In this page our models are initial value problems (IVP's) for a first order ordinary differential equation that is rate equation. We choose *'t'* as our independent variable and start at *t = o.* for our one start variable problem. We use y and hence use the general first order ODE with an initial condition as our model. For specific applications, finding *f(x,y)* is a major part of the modeling process.
In mathematical language the general linear model may be written as.

$$ODE = \frac{dy}{dt} = f(t,y) \quad (13)$$

IVP
IC $y(o) = y_o$

For many of the applications we investigate the model is the simple linear autonomous model.

$$ODE = \frac{dy}{dt} = ky + r_o \quad (4)$$

IC y(o) = ye^{o}

The parameters *ro, k* and *yo* as well as the variable *y* and *t* are included in our nomenclature list.

y = quantity of state variable, *t* = time,
ro = rate of flow for the source
k = constant of proportionality
yo = the initial amount of our state variable

The model is generally in that we have not explicitly given the parameters *ro, k,* or *yo*. These parameters are either given or found using specific data. However their values need not be known to solve the linear model.

Conclusion

Analyzed system exhibits simple harmonic motion characterized by periodic displacement and velocity functions with a period of 2πλ\frac{2\pi}{\sqrt{\lambda}}λ 2π. The particle starts from equilibrium with zero velocity, moves towards the origin with increasing velocity, reaches zero velocity at Z2λ\frac{Z}{2\sqrt{\lambda}}2λZ, and returns to equilibrium. This cyclical motion repeats every 2Zλ\frac{2Z}{\sqrt{\lambda}}λ2Z. Additionally, the model illustrates different scenarios including motion under gravity in a resisting medium, characterized by a velocity approaching a limiting value as time progresses. These mathematical models, governed by differential equations, offer predictive tools essential for understanding and analyzing physical phenomena.

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