



International Journal of Multidisciplinary Research and Growth Evaluation.

The main function of the Simpson method

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Article Info

ISSN (online): 2582-7138

Volume: 05

Issue: 03

May-June 2024

Received: 13-03-2024

Accepted: 20-04-2024

Page No: 347-349

Abstract

Integral calculus, an important component of the university curriculum, requires effective methods of numerical integration. The Simpson method is one of the most accurate and widely used methods for approximating certain integrals of functions with high accuracy. In the context of the university program, students of mathematical and natural sciences study this method as a tool for solving a variety of integral problems. They conduct research comparing Simpson's method with other numerical integration methods, such as the rectangle method or the trapezoid method, to determine its effectiveness and scope. As a result of the analysis, the importance of the Simpson method for achieving accurate numerical results in solving integral problems is deduced, which makes it an integral part of academic education in mathematics and natural sciences.

Keywords: Simpson's method, numerical integration, computational accuracy, comparative analysis

Introduction

The parabola method, also known as Simpson's method, is a numerical integration method that uses the approximation of an integral function by parabolas. It represents an improvement on the rectangle method and the trapezoid method, allowing for more accurate results when calculating certain integrals.

The parabola method is based on the fact that the integral function is approximated by a parabola on each segment of integration, which makes it possible to more accurately approximate the area under the curve and, consequently, the value of a certain integral.

Compared to the trapezoid method, which approximates the integral function by straight line segments, and the rectangle method, which uses rectangles, the parabola method allows you to get more accurate results by using parabolas to approximate the function.

The results of the study: The formula of the Simpson method for approximating a certain integral of the function $f(x)$ on the segment $[a, b]$ is as follows

$$\int_a^b f(x) dx \approx \frac{h}{3} [f(a) + 4f(2a+b) + f(b)]$$

Where $h=2b-a$ is the integration step.

The parabola method has quadratic accuracy, which means that it gives an accurate result for polynomials of degree no higher than the second. However, for functions other than polynomials, the error of the method can be calculated using the error estimation formula ^[1].

Research results: The parabola method is widely used for numerical integration in various fields of science and engineering, such as physics, engineering calculations, economics and others.

In the school curriculum, the Simpson method, as a numerical integration method, can be used to understand the basic principles of numerical integration and approximation of certain integrals, and further improve in the university curriculum. Here are a few ways to use it:

1. Integral calculus

As part of the study of integral calculus, Simpson's method can be used to demonstrate ways to numerically solve certain integrals. This allows students to better understand the concept of integration and various methods of approximate calculation of certain integrals. Let's consider an example of using the Simpson method to calculate a certain integral of the function $f(x) = x^3$ on the segment $[0, 1]$

Step 1: Divide the interval $[0, 1]$ into several sub-sections. The Simpson method uses an even number of sub-sections, usually 2, 4, 6, etc.

In this example, we take the step $h=21-0=21$ P_, that is, we divide the interval into two equal sub-sections: $[0, 0.5]$ and $[0.5, 1]$.

Step 2: For each sub-section, we approximate the function $f(x) = x^3$ with a parabola.

In the first sub-section $[0, 0.5]$, we use the formula of the Simpson method:

$$\begin{aligned} \int_{0.5}^1 x^3 dx &\approx \frac{h}{3} [f(0) + 4f(0.5) + f(1)] \\ &= \frac{1}{23} [(0)^3 + 4(0.5)^3 + (1)^3] = \frac{1}{23} [(0)^3 + 4(20+0.5) + (1)^3] \\ &= \frac{1}{23} [0 + 4(18) + 18] = \frac{1}{23} [0 + 4(81) + 81] \\ &= \frac{1}{23} [0 + 12 + 18] = \frac{1}{23} [0 + 21 + 81] \\ &= \frac{1}{23} \cdot 58 = 24 = 31 \cdot 85 = 245 \end{aligned}$$

In the second sub-section $[0.5, 1]$, we perform similar calculations and get the same result $5 \div 24$

Step 3: Summarize the calculation results for each sub-section:

$$\int_0^1 x^3 dx \approx 245 + 245 = 2410 = 125$$

Thus, we used the Simpson method to approximate the integral $\int_0^1 x^3 dx$ and got the result $512^{[2]}$.

2. Practical examples

Mathematics teachers can offer students to solve practical problems in which it is necessary to calculate certain integrals. Using the Simpson method in such problems allows students to understand how numerical methods can be applied to solve real-world problems.

The application of the Simpson method in practical tasks can shed light on its significance and applicability in real-world scenarios ^[5].

They can be used in Engineering calculations. In engineering disciplines, students may face tasks that require calculating surface areas, body volumes, or calculating the physical characteristics of systems. For example, when designing machine parts, students can use the Simpson method to calculate moments of inertia or centers of mass ^[6].

Physical simulations: In the physical sciences, the Simpson method can be used to simulate the movement of bodies under the influence of various forces or changes in the parameters of physical systems over time. For example, when studying the vibrations of a spring or the motion of a material point, students can numerically integrate equations of motion to obtain quantitative results.

Economic analyses: In economics, students can apply the Simpson method to analyze changes in economic models or evaluate integral indicators such as total profit or output. For example, when modeling supply and demand in a market, numerical integration can help in predicting long-term trends. **Medical Research:** In the medical sciences, students can apply the Simpson method to analyze data on the

concentration of drugs in the body or changes in health indicators over time. For example, when studying drug kinetics or modeling disease dynamics, numerical integration can be an important analysis tool.

Thus, the application of the Simpson method in practical tasks demonstrates its effectiveness and versatility in various fields of knowledge, from engineering to medicine, and allows students to see its application in real-world scenarios.

3. Programming

In the context of learning programming, students can write programs in programming languages such as Python or MATLAB to implement the Simpson method for numerical integration. This will not only help them better understand the method itself, but also develop programming skills.

4. Research projects

Students can use the Simpson method as a tool to explore various mathematical and physical phenomena. For example, they can investigate the area under the curve, the rate of change of quantities, or other phenomena requiring the integration of functions.

5. Comparative analysis of numerical integration methods, such as the Simpson method, the rectangle method and the trapezoid method, is an important aspect of university education.

The Simpson method usually provides more accurate results than rectangle and trapezoid methods, especially for smooth functions. Comparative analysis can show that the approximate value of the integral obtained by the Simpson method is closer to the exact value than using other methods. The Simpson method can converge faster to the exact value of the integral than the rectangle and trapezoid methods. This is due to the fact that Simpson's method uses parabola approximation, which allows it to more accurately approximate the shape of the integrand.

Comparative analysis can also assess the computational complexity of each method. In some cases, for example, for functions with oscillations or discontinuities, the Simpson method may require more computational resources than other methods.

An important aspect of comparative analysis is the stability of methods when working with different types of functions and integration intervals. The Simpson method may be more stable and predictable in such cases ^[3].

In addition, comparative analysis can help students understand in which situations each method is most effectively used. For example, the Simpson method may be preferable for integrating functions with a high degree of smoothness or over long intervals.

Thus, a comparative analysis of numerical integration methods allows students to gain a deep understanding of each method, their advantages and limitations, which contributes to the development of their analysis and decision-making skills in real engineering and scientific problems.

Conclusions

The application of the Simpson method in the school curriculum, as well as its further in-depth study at the university, can contribute to a deeper understanding of integral calculus, the development of programming skills and analytical thinking abilities among students.

It is important to note that the parabola method cannot always provide sufficient accuracy when integrating complex

functions or on segments with features such as discontinuities or oscillations.

In general, the parabola method is a powerful tool for numerical integration, which can be effectively applied to solve a wide range of problems requiring the calculation of certain integrals.

The parabola method is a powerful tool for approximate calculation of certain integrals, especially when other methods are insufficiently accurate or ineffective. However, it is necessary to keep in mind its limitations and take into account possible errors in its application.

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