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Using ethno-cultural pedagogical materials in teaching the concepts of diagonal and rectangular arithmetic to grade 3 Pupils

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Abstract
New semiotic approaches to elementary number theory have emerged in recent times in the forms of diagonal arithmetic (DA) and rectangular arithmetic (RA). These new forms of arithmetic which view numbers as entries in a matrix, were juxtaposed with the conventional arithmetic (CA) with a view to determining their effects on pupils' arithmetical achievement when treated with ethno-cultural pedagogical instruments. The study was a post-test, quasi experiment involving two groups of pupils in a population of 300 pupils. Out of this population a sample of 120 pupils was used for the study and for statistical analysis using descriptive and inferential statistics. The study showed that there are significant differences among the three semiotic approaches to arithmetic, with diagonal and rectangular arithmetic gaining the upper hand. The study also showed that the use of ethno-cultural pedagogy amplified the arithmetical achievement of pupils exposed to diagonal and rectangular arithmetic. The study further revealed that it is safe to adopt diagonal and rectangular arithmetic as independent parallel curricula alongside conventional arithmetic.

Keywords: Diagonal arithmetic, rectangular arithmetic, conventional arithmetic, ethno-cultural pedagogy

Introduction

Arithmetic actually began in ancient times with the invention of various semiotic systems of numeration. Semiotic in numeration is the creation and use of signs and symbols to foster the process of counting. The Various operations of counting such as addition, subtraction, multiplication, division etc. are generally what is known as arithmetic. In ancient times, various civilizations devised their own systems of numeration. These numeration systems evolved over time from one civilization to the other and eventually culminated in what we now recognized as the Hindu- Arabic numeration system. With much awareness today, the Hindu-Arabic numeration system has become widely accepted for arithmetic calculation and higher level of computation. It is important to note that this numeration system has brought about major transformations in mathematics as well as physics and social sciences. Much of it is actually owed to the Western Europe and especially to the very famous and prolific mathematician Leonardo of Pisa (also unknown as Fibonacci) who popularized them in his writings.

As stated earlier, arithmetic involves the basic operations of country; addition, subtraction, multiplication and division as well as squaring, roots, etc.

Inah (2022) [5] views it as elementary numbers theory based principally on the Hindu-Arabic numeral system which is a decimal system. This view stems from the fact that arithmetic came to its limelight with the dawn of these numerals. Arithmetical operations are thus performed today based on this decimal system of numerals having 0-9 as its principal digits or figures and this has gained world- wide acceptance already. Among the reasons adduced for the acceptance and adoption of this digits is the fact that they possess swift qualities making them easy and convenient to use in mathematics. Besides, they are seen to be very powerful and influential.

Arithmetical semiotics actually transcends the creation and utilization of signs and symbols. It goes further into the involvement of these signs and symbol in a well-ordered and swiftly patterned and systematically oriented arrangement that enables for ease of operation as well as for speedy finding of results. For several millennia and centuries now the form of arithmetical semiotics that has been in use is that in which the digits of numbers are systematically oriented so that the digits lie horizontally. Thus arithmetical operations are usually performed by arranging these digits of numbers in a horizontal pattern that systematically forms block numbers with one set lying above or below the other to constitute rectangular arrays. For example, 1234 is a systematic orientation that arranges digits horizontally. Hence it is a horizontal array. Again 1234+5678 is another horizontal array while

1 2 3 4
+5 6 7 8

is called a rectangular array. Semiotic method has been in use for centuries and millennia. Hence Inah (2022) [5] refers to it as conventional arithmetic.

Indeed, these digits of numbers can be treated as vectors or perhaps entries in a matrix, which of Course, is a rectangular array of numbers. This concept of arranging numbers systematically in a rectangular array probably existed subconsciously since the beginning of arithmetic before it become established as a field of study. It is proper to view numbers that way because all numbers having more than one digit are usually written as row vectors even though they are not usually seen that way.

1 0 = (1 0)

11 = (1 1)

12 = (1 2)

124 = (1 2 4), etc

A row vector, A, with only two entries is represented as A= ($a11$ $a12$). A row vector B=with three entries can be written as B= (b_{11} b_{12} b_{13}). Thus any number N, having two or more digits can be written in the following ways.

Compact		Form	Vector	Form
$d11$ $d11$	$d12$ $d12$	$d13$	($d1$ $d12$) ($d11$ $d12$ $d13$)	
$d11$	$d12$	$d13$ $d14$	($d11$ $d12$ $d13$	$d14$) etc,

dijrepresents the digit in the ith row and the jth column. Any arithmetical operation involving two numbers D=$d11$ $d12$ and E= $e11$ $e12$ such as addition, subtraction, multiplication etc. will require a systematic arrangement of the form shown below.

(a) T U
d_{11} d_{12}
e_{11} e_{12}

(b) T U
2 3
4 5

An example is given as shown in (b) above D = 23 and E = 45.

An arithmetical operation involving D = $d11$ $d12$ $d13$ and F = $e11$ $e12$ $e13$is given as shown below. The example is also given below

(a) T U (b) T U
$d12$ $d11$ 2 3
$e12$ $e11$ 4 5

D=$d12$ $d11$ E=$e12$ $e11$

From the example in (d) above, it can seen that D=241 and E=317

Note

The digits usually appear according to place values, with the first being in the unit place. Thus, it is fitting to reverse the order of arrangement as follows:

(a) T U
d_{12} d_{11}
e_{12} e_{11}
D=d_{12} d_{11} E=e_{12} e_{11}

(b) T U
2 3
4 5

The last digit obviously appears in the last place value. Since the Hindu-Arabic numeral system is a base ten number system, all arithmetical operations are performed in 'base ten' and they are based on the concept of renaming. No place is expected to have more than the digit 9, since any sum or difference or product giving a result of more than 9 will have to be renamed and written approximately. For example, 23+45 is performed as follows by adding terms in the same column from right to left.

T U
2 3
+ 4 5
6 8

conventional addition

Again, 45+28 is performed as follows by adding corresponding terms in the same column from right to left

T U
2 3
+ 4 5
6 8

conventional addition

Notice that 5+8 =13, 13 is renamed as 10+3, 3 is then written in the unit place while 10 is transferred as 1 to the tens place and written at the left top of 4 then 1+4+2 =7 which is written below. This form of arithmetic is what this study consider as "Conventional rectangular arithmetic" or simply "Conventional arithmetic". It has the advantage that is concise to write and avoids for many details. It's major disadvantages is that it is clumsy and thus hides details of renaming. This makes arithmetic difficult to comprehend especially among early school learners. The same difficulty is experienced by learners with subtraction, Multiplication and Division (Inah, 2022) [5]. The Foregoing, therefore, gives rise to a new semiotic system of arithmetization called the "Modified rectangular arithmetic" or simply "rectangular arithmetic". It is modified from the conventional arithmetic to make renaming more descriptive and detailed and thus clearer to understand. In the case, all digits of numbers are viewed as entries in a matrix. This allows for two digits or more to rename temporarily within a particular place value before renaming is carried out to remove the digit which has a higher place value to its rightful place. Using this semiotic method 45+28 will be performed as follows:

4	5	4	5	4	5
+ (2	8) ⟶	+ (2	8) ⟶	+ (2	8)
6	13	16	3	7	3

Notice here that this method of arithmetic operation is much easier to understand but more elaborate and time consuming to write. Besides, it occupies more space. However, the fact that renaming is much clearer makes it interesting. This semiotic method of arithmetic was devised by the researchers in this study with the sole aim of improving arithmetization.

In the case of subtraction and multiplication, a comparison can be made between the following two methods of semiotic arithmetization.

T	U
3	2
- 2	8
0	4

(conventional subtraction, renaming not clear)

3 2 → [1]2 2 → 2 12
− (2 8) ⟶ - (2 8) ⟶ (2 8)
0 4

(rectangular subtraction ,renaming clear enough)

  2 3
× 4 2
  4 6

In the case of division, we can also compare between the following semitic methods.

75÷5 = 75/5 =5 (conventional division, renaming not clear)

(7 5 / 5) ⟶ (75 5 / 5) ⟶ (5 25 / 5) = (1 5) = 15 {Rectangular division, renaming clear enough}

Diagonal arithmetic is another innovation in arithmetical semiotic created by Inah (2022) [5] from the concept of matrices just like rectangular arithmetic. There are two ways of viewing any number having more than one digit horizontally and vertically. For example, the number 71521 can be viewed in two ways;

Horizontally view:

71521 = 70000 + 1000 + 500 + 20 + 1

Vertical view:

$$71521 = \begin{pmatrix} 7 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 5 & 0 & 0 \\ & & & 2 & 0 \\ & & & & 1 \end{pmatrix}$$

The second representation appears to provide a semiotic representation similar to the canonical form of a matrix. Ignoring the addition sign, (+), and adding 0s below gives a complete diagonal matrix as shown below.

71521	=					
		7	0	0	0	0
		0	1	0	0	0
		0	0	5	0	0
		0	0	0	2	0
		0	0	0	0	1

The result is a 5× 5 diagonal matrix. Notice that the digits re-appear in the leading diagonal of the resulting matrix. Inah (2022) [5], therefore, proposed that all numbers having one or more digits can be represented as n×n diagonal matrices with the digits falling in the leading diagonal.

More examples can be seen as shown below;

In this matrix, the numeration of the rows must be in the reverse order so that every digit corresponds to its own place value. Thus the last row in the usual matrix becomes the first row and the last column becomes the first column. Thus by letting d=digit, dii (i.e, the digit in the ith row and the ith column of its corresponding matrix). Suppose, therefore, that N is any given number then N= dkk…d33 d22 d11, where 1 ≤ i ≤ k Thus d11 is the unit digit, d22 the ten digit , d33 the hundreds digit, etc. The diagonal matrix form of N is given as shown below.

$$N = \begin{pmatrix} d_{kk} & . & . & . & 0 & 0 & 0 & 0 \\ . & . & . & . & . & 0 & 0 & 0 \\ . & . & . & . & . & . & 0 & 0 \\ 0 & . & . & . & . & . & . & 0 \\ 0 & 0 & . & . & . & d_{33} & . & 0 \\ 0 & 0 & 0 & . & . & . & d_{22} & 0 \\ 0 & 0 & 0 & 0 & . & . & . & d_{11} \end{pmatrix}$$

A suitable suggestion about this proposition is that the 0s should be discarded while the brackets be replaced with a symbol such as asingle left-sided brace, {, thus we have the following

$$71521 = \begin{cases} 7 \\ \quad 1 \\ \qquad 5 \\ \qquad\quad 2 \\ \qquad\qquad 1 \end{cases} \qquad 25 = \begin{cases} 2 \\ \quad 5 \end{cases} \qquad 217 = \begin{cases} 2 \\ \quad 1 \\ \qquad 7 \end{cases}$$

In general,

$$N = \begin{cases} d_{kk} \\ \quad . \\ \qquad . \\ \qquad\quad . \\ \qquad\qquad d_{33} \\ \qquad\qquad\quad d_{22} \\ \qquad\qquad\qquad d_{11} \end{cases}$$

This view of numbers and their digits allows for a new form of arithmetical semiotics in which arithmetical operations are diagonalized. Hence the name "diagonal arithmetic". Diagonal arithmetic gives more details of about renaming the conventional arithmetic. Its major challenge, however, is that it occupies more space just like rectangular arithmetic. It could also consume time during operation. For example, 428 + 384 can be performed diagonally as follows:

$$428 + 384 = \begin{cases} 4 \\ \quad 2 \\ \qquad 8 \end{cases} + \begin{cases} 3 \\ \quad 8 \\ \qquad 4 \end{cases} = \begin{cases} 7 \\ \quad 10 \\ \qquad 12 \end{cases} = \begin{cases} {}^{1}7 \\ \quad {}^{1}0 \\ \qquad 2 \end{cases}$$

8 1 2

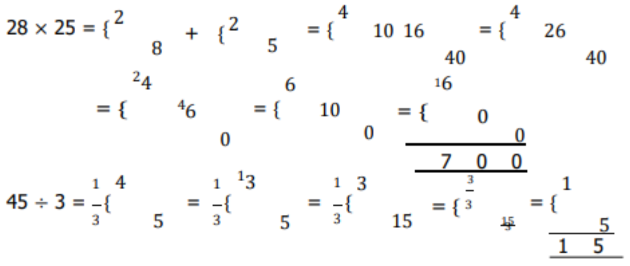
Again, 428 – 384 can be performed as follows diagonally;

$$428 - 384 = \begin{cases} 4 \\ \quad 2 \\ \qquad 8 \end{cases} - \begin{cases} 3 \\ \quad 8 \\ \qquad 4 \end{cases} = \begin{cases} 1 \\ \quad 2-8 \\ \qquad 4 \end{cases} = \begin{cases} {}^{1}0 \\ \quad 2-8 \\ \qquad 4 \end{cases}$$

$$= \begin{cases} 0 \\ \quad 12-8 \\ \qquad 4 \end{cases} = \begin{cases} 4 \\ \quad 4 \end{cases}$$

4 4

Multiplication and division are exemplified with the examples below.



The use of ethno-cultural pedagogical approaches in the teaching and learning of mathematics is an ethno-mathematical perspective that has been zealously advocated by some researchers since the dawn of ethno-mathematics (D'Ambrosio, 1985; Khotinets, 2014; Steponova, 2014; Sunzuma, Zerekwa, Gwizangwe and Zinyeka, 2021; Omere, Ogedengbe, 2022) [2, 6, 8]. For instance, D'Ambrosio (1985) [2] was the proponent of ethno-mathematics in which case he views it as the relationship between mathematics and culture. According to him, ethno-mathematics has a key role to play in the history and pedagogy of mathematics. Khotinets (2014) [6], stated that one of the most pressing problems in the system of education is the task of forming the ethnic self-awareness of new generations. Education must be seen to be a national venture permeating its people only if it is ensured that its content embodies the peoples' spirituality and ethnic self-awareness, when it incorporates the ethnic culture, psychology, way of life, and spiritual-moral values of its community. Sunzuma *et al.* (2021) [8] opined that ethno-cultural pedagogy not only motivates the learners of mathematics but also arouses their interest in the subject and improves their understanding of it. They had a study done that compared the effectiveness of ethno-mathematics and traditional lecture approaches in teaching consumer arithmetic. The study sample was about 90 learners and two teachers from one secondary school in Zimbabwe. The findings of the study revealed that learners taught using an ethno-mathematics approach achieved significantly higher in the test than their counterparts who were taught consumer arithmetic using the traditional approach. The study also revealed that teachers appreciated the use of ethno-mathematics approach in the teaching of consumer arithmetic as their learners were motivated, actively involved and interested in learning the concepts. The study found, in addition, that learners understanding and retention of arithmetic concepts were drastically improved.

Omere (2022) [7] goes further to buttress this assertion by stating that ethno-mathematics teaching method offers learners opportunity not only to learn of their culture and mathematics but also increases their motivation and hence their achievement in the subject. This view is shared by Unudiaku (2013) who posited that ethno-mathematics teaching materials enhanced students' achievement in mathematics.

Since rectangular and diagonal arithmetic are new concepts springing into research, the conception of the impact of combining ethno-cultural pedagogy with these new semiotic adjustments on arithmetic is no doubt an interesting one. This study seeks to achieve this objective through the use of ethno-cultural pedagogical materials.

As regards semiotics and arithmetical achievement, studies have shown that relationship exist between semiotic representations and mathematics achievement. For instance, Dahiana, Herman, Nurlaelah and Pereira (2023) conducted a study recently in which they used a qualitative, descriptive-interpretive approach to examine students' semiotic representation skills when solving mathematical problems. The research was done in a year-9 classroom in a public school in Bandung, Indonasia, with 30 participants divided into high, middle and low- ability groups based on their level of mathematical ability. The results indicated that students in the high and middle ability groups had adequate skills in algebraic treatment and in geometry and verbal expressions skills for constructing algebraic expressions and converting verbal statements into mathematical equation. In contrast, the low-ability group demonstrated a lack of semiotic representation skills in problem solving. These findings highlighted the importance of transformation and conversion skills in mathematical problem solving activities and can be valuable information for teachers and observers of mathematics education.

Dyrvold (2016) [3] applied statistical methods in the investigation of the potential meaning that the presence and co-occurrence of semiotic resources have for how demanding a mathematical task is to read and solve. The results reveal that the number of different semiotic resources in a mathematical task is not related to difficulty, but that difficulty is related to the particular combinations of semiotic resources where pictorial images are one of the resources. The results also showed that the difficulty related to these semiotic characteristics is not related to an unnecessary reading demand. In another study by Viseu, Pires, Menezes and Costa (2021) [10] a teaching experiment for the learning of rational numbers by grade 2 students was conducted in order to determine the manner in which semiotic representations contribute to the learning of rational numbers, particularly with concern to unit fractions. The study showed that there was a greater use of the pictorial representations register compared to the other types. Students' main difficulties in learning rational numbers were related to the pictorial representation of unit fractions and to an understanding of the concept of fraction itself. The difficulties, according to the study, resulted from errors such as the misrepresentation of unit fractions in case of the pictorial register, the association of the concept ''half'' with multiple unit fractions, the absence of the fraction bar when it comes to the symbolic register, the use of everyday terms to represent fractions when students rely on the natural language register, and the misrepresentation of rational numbers when the graphic register was used.

It is apparent from the foregoing that research in the area of semiotics and mathematical or arithmetical achievement has not received widespread attention. Even available studies tend to duel more on how learners conceptualize semiotic representations on their own without reasonable support from the teachers in terms of adjustments of semiotic patterns of representation. Available researches tend to give emphasis to problem solving rather than emphasizing the semiotic patterns of representations of the counting operations performed during problem solving. The need, therefore, for a study that deals specifically with semiotic patterns of representations in counting where learners seem to have serious problems that build up gradually into phobia and hatred has become imperative. Thisthis study is intended to tackle the challenge.

Statement of the problem

There is indeed no gainsaying that the Hindu-Arabic numeral system is one of the most remarkable achievements of the human race. No doubt, this has led to great achievements in science and technology due to the developments in the mathematical theories emerging from these numbers. Nations that have developed significantly in science and technology depend heavily on mathematics, which makes extensive use of these numerals for axioms, propositions, theorems, lemmas, laws and theories, having a wide range of applicability. It does in fact seem like if we all had appreciable knowledge of Mathematics we all would make the world a much better place to live in. Unfortunately, this is not the case. Quite a number of us do not possess adequate number sense. Number sense itself simply means having the knowledge of what numbers truly represent in counting i.e., addition, subtraction, multiplication and division. This could as well be referred to as arithmetical knowledge. It helps in dealing with the daily issues of life and it grows from simple to complex and then to what is called advanced mathematics. As a teacher of mathematics at various levels of learning, with several years of experience in the subject, personal experience has revealed that one of the reasons for inadequate arithmetical knowledge is that learners at the elementary school stage usually encounter learning challenges resulting from poor understanding of the concept of renaming while carrying out the operations of arithmetic. These elementary challenges usually accumulate into bigger ones as they progress to higher levels thus leading to phobia or hatred for the subject. The fact that many people do not possess adequate knowledge of number numeration poses a serious challenge to national development especially in underdeveloped nations. Number sense and numeration are together an integral part of arithmetic (what is commonly referred to as elementary number theory). The challenges of learners in number and numeration stem from their difficulties during the course of learning. These can be categorized into two: the first category emanating from the pedagogical methods, while the second category emanates from the semiotic methods. Pedagogical methods here refer to methods of teaching arithmetic, appropriate strategies for each method based on the learners' behavioural characteristics. Semiotic methods, on the other hand, refer to methods of solution to arithmetical problems. In other words, they have to do with the way signs and numbers are arranged during arithmetical operations such as addition, subtraction, multiplication, etc. (Inah, 2022) [5]. Perhaps a third category to the challenges of learners of mathematics is the cultural context in which arithmetic is being studied. In this wise, arithmetic is viewed from an ethno-mathematics perspective rather than from a general one. The rationale here is that clarity is improved when arithmetic is studied with examples, illustrations and instructional materials that are culture-centred. Children tend to be confused when examples and illustrations given to them relate more to a foreign culture (D' Ambrosio, 1985). Besides, infusion of ethno-cultural pedagogy is believed to help learners gain ethnic self-awareness as well as ethno-spiritual and moral values of their culture (Khotinets, 2014) [6].

In the light of the above, the problems of learning arithmetic can be said to have three dimensions: the pedagogical, the semiotic and the ethno-cultural. Over the years, research has focused on the pedagogical methods to the neglect of the semiotic methods. Thus research in the area of semiotics and arithmetic has been grossly inadequate. Even the available studies do not seem to be dealing with the right target. They seem to focus more on learners' conceptual abilities of various semiotic representations in problem solving rather than focusing on the adjustments of the patterns of semiotic representations with a view to facilitating the process of conceptualization of the representations. This study, therefore, seeks to find both semiotics and ethno-cultural solutions to the challenges of learners by fielding two different semiotic methods of arithmetical knowledge against the conventional arithmetical method in a detailed comparative juxtaposition.

Purpose of the study

The purpose of the study is to investigate the effects of using ethno-cultural pedagogical materials in the teaching of diagonal and rectangular arithmetic on pupils' arithmetical achievement. In particular, the study we will focus on:

1. Determining the effect of three semiotic methods: diagonal arithmetic, rectangular arithmetic and conventional arithmetic, on pupils' arithmetical achievement;
2. Determining the effect of using pedagogical materials in teaching diagonal arithmetic, rectangular arithmetic and conventional arithmetic on pupils' arithmetical achievement.

Research Question

The research questions under considerations in this study are stated thus;

1. To what extent does each of these semiotic methods: diagonal arithmetic, rectangular arithmetic and conventional arithmetic, affect pupils' achievement in arithmetic?
2. To what extent does the use of ethno-cultural pedagogical materials in teaching diagonal arithmetic, rectangular arithmetic and conventional arithmetic affect pupils' arithmetical achievement?

Research hypotheses

The null hypothesis in relation to the main variable are hereby stated as follows:

Ho1: The effects of diagonal arithmetic, rectangular arithmetic and conventional arithmetic on pupils' arithmetical achievements are NOT significant;

Ho2: The effects of using ethno-cultural pedagogy in teaching diagonal arithmetic, rectangular arithmetic and conventional arithmetic on pupils' arithmetical achievement are not significant.

Research methods Research design

This research is a post – test quasi – experiment involving two groups of subjects A and B. the first group, A, were exposed to the three semiotic methods of arithmetic namely; diagonal arithmetic, rectangular arithmetic and conventional arithmetic. The instruction involved the use of non-ethno-cultural pedagogical materials (i.e. instruction materials that are general in nature). The second group, B were taught the same three semiotic methods of arithmetic namely; diagonal arithmetic, rectangular arithmetic and conventional arithmetic. This time, ethno – cultural pedagogical materials were used for the instruction. These materials are culture-centred. Both groups were taught independently, and the three semiotic methods were implemented simultaneously for a period that spanned ten (10) months. Group B was the experimental group while the other group A was the control.

Area of the study

The study area was Akamkpa Local Government Area, Cross River State, Nigeria. The study was sited in two communities within the local government: Akamkpa town which is the local government headquarters and Awi community which also lies adjacent to the main town. The choices are informed by proximity and educational disadvantage.

The people living in this area exhibit a very high degree of social heterogeneity, but they are linguistically, religiously, politically and culturally homogeneous. Their homogeneity patterns actually determine to a large extent decent and biological connections among them.

They are predominantly Christians and basically subsistence farmers. The people living in this area are also highly educationally disadvantaged.

Population of study

The population or the study comprised pupils in elementary or primary schools within the selected local government area. The class of the subject was grade 3. There were about six (6) public primary schools within these two communities and each grade 3 class in these schools had a range of about 50 – 60 children. Thus the population of the study was estimated at over 300 pupils.

Sample and sampling technique

The sample for the study was obtained by simple random sampling technique. Two schools were selected from the six schools within the two communities mentioned above. The selection was done by simple random sampling technique. The names of the schools were collated and written in scraps of papers and folded and then put in a bowl. The bowl was thoroughly shaken to give a fair chance for each selection. The selection was effected with the eyes closed and repeated in the same manner until the two schools were selected. The total sample obtained from the technique described above was about 120 grade 3 pupils overall. This sample gave a 40% representation of the population of pupils used for the study. Each of the two schools had about 60 grade 3 pupils in it

Instrumentation and validation

The treatment instruments for the study include the following;

- Instructional Materials: These were abacuses designed locally using Rods, Wooden Planks and beads. They were meant for counting and place values arithmetical operations there were abacuses purchases from shops. Thus they were of two types; The locally designed culture centered ones and the general abacuses.
- Diagonal Arithmetic Handout (DAH)
  This was designed to contain arithmetical topics such as Addition, Subtraction, Multiplication, and Division. The topics were graded adequately to meet the level of the subjects in line with their scheme of work.
- Rectangular Arithmetical Handout (RAH)
  This was also made of arithmetical topics such as Addition, Subtraction, Multiplication, and Division. All topics treated here were based on the pupils scheme of work and therefore, graded to meet their level.
- Conventional Arithmetic Handout (CAH)
  This also contained topics as in the other two cases. However, the topics treated here were handled in the conventional manner and at the level of the scheme.
  The test instrument for the study comprised the following;

1. Test of Rectangular arithmetical achievement (T R A A)
2. Test of Diagonal arithmetical achievement (T D A A)
3. Test of Conventional arithmetical achievement (T C A A)

These tests were applied as post-test. Each test comprised 20 multiple choice items.

Two uninvolved experts from the departments of Mathematics and Educational Tests and Measurement of different tertiary institutions were deployed to verify and certify the content of the instruments. The scope, clarity and /or ambiguity was also tested for the instrument. The difficulty level and discriminating indices of the test instrument were ascertained through item-by-item analysis to ascertain their scope, simplicity of presentation of facts and examples as well as determined inter-rater reliability by calculating the correlation of its scores with those obtained from another school not involved in the study.

Research procedure

The pupils were invited to give details of their personal information, then they were divided into two groups. Each group was independent of the other and is slated as much as possible to avoid contact. The Participants were notified and given details of commencement of the treatment through a well – articulated timetable. The treatment lasted for about ten (10) Months after which the Post – Test was administered according to the schedule of events in the timetable. Group A was the control group while group B was the experimental group. Each group was treated with the three semiotic methods of arithmetic and hence had three tests.

Data collection

Each item of the test was awarded 1mark so that the total numbers of marks equal 20. The scores were collected and tabulated according to the groups.

Result and discussion

The scores obtained from the post – test after the treatment are provided as shown in the table below.

Table 1: Summary of post – test scores by group

GROUP	Semiotic Method	N	$\bar{X}$	S^2
A(CTRL)	R A	58	16.733	6.097
	D A	58	17.267	4.640
	C A	58	13.600	4.515
B(EXP)	R A E P	60	16.733	6.097
	D A E P	60	17.350	4.604
	C A E P	60	13.600	4.515

Note: CTRL		Control	
	EXP	=	Experimental
	RA	=	Rectangular Arithmetic
	DA	=	Diagonal Arithmetic
	CA	=	Conventional Arithmetic
	RAEP	=	Rectangular Arithmetic with Ethno-cultural Pedagogy
DAEP		=	Diagonal Arithmetic with Ethno-cultural Pedagogy
CAEP		=	Conventional Arithmetic with Ethno-cultural Pedagogy

From table1 above, it can be seen that group A had 58 subjects while group B had 60 subjects. Each group took the three tests Group A which served as the control group, had mean scores of $\bar{X}$(RA) = 16.733, $\bar{X}$(DA) = 17.267 and $\bar{X}$(CA) = 13.600 of the three tests with variances of$S2$(RA) = 6.097, $S2$(DA) = 4.640 and $S2$(CA) = 4.515 respectively. Group B, which served as the experimental group for the study, had mean scores of $\bar{X}$(RAEP) = 16.733, $\bar{X}$(DAEP) = 17.350 and $\bar{X}$(CAEP) = 13.600 in each of the tests taken with variances of$S2$ (RAEP) = 6.097, $S2$ (DAEP) = 4.004 and $S2$ (CAEP) = 4.515 respectively. From the scores actually, both groups performed well in all three semiotic methods to which they were exposed. However, this does not imply that there are no differences in their performances. It remains, therefore, for us to establish whether or not their achievements had differences that could be considered significant.

Testing of research hypotheses

Ho1: The effects of diagonal arithmetic (DA), rectangular arithmetic (RA) and conventional arithmetic (CA) on pupil's arithmetic achievement are rot significant.

μCTRL (RA) = μCTRL (DA) = μCTRL (CA)
And
μEXP (RAEP) = μEXP (DAEP) = μEXP (CAEP)

Table 2: Within group analysis of the means and variances of participants in the three semiotic methods of arithmetic

group	Semiotic method		$\bar{X}$	$S2$	F	Fa	P (a=0.05)
A	RA	58	16.733	6.097	46.315	3.047	0.00
	DA	58	17.267	4.840			
	CA	58	13.600	4.515			
B	RAEP	60	16.733	6.097	47.830	3.047	0.000
	DAEP	60	17.350	4.604			
	CAEP	60	13.600	4.515			

Table 2 shows the analysis of the means and variances of participants in the three semiotic methods of treatment within each group. From the table above, one way ANOVA shows that the test statistic for group A is F (A) = 46.315 with a critical value of Fα = 3.047. The significance at α=0.05 is P =0.000. Since F > 3.047 and P<0.05, it follows that the null hypothesis Ho1 is to be rejected for group A at □ = 0.05 level of significance. This shows that there are significant differences between the mean scores of the pupils in the three tests taken in group A. Thus we conclude that the affects of diagonal arithmetic, rectangular arithmetic and conventional arithmetic on pupils arithmetical achievement in group A is significant.

From table2, we can also see that test statistic for group B is given for F (B) = 47.833 with the critical value still standing at Fα = 23.047.

The significance of α = 0.03 is P = 0.000. Now, since F > 3.047 and P < 0.05, it follows also that the null hypothesis Ho1 is to be rejected for group B at □ = 0.05. This again shows that there are significant differences between the mean scores of the pupils in the three tests taken by group B. Thus we conclude again that the effects of diagonal arithmetic, rectangular arithmetic and conventional arithmetic on pupils' arithmetical achievement in group B is significant.

In other to show which directions the differences take, the Tukey post-hoc analysis was done for groups A and B. Table 3 and table 4 below show the Tukey post-hoc analysis for each of the groups.

Table 3: Turkey post-hoc analysis for group A

(I) (J)	Mean Diff (I-J)	Std Error	sig (a=0.05)	95% confidence	
				Lower bound	Upper bound
RA DA	-0.534	0.473	0.544	-1.511	0.339
CA	3.133	0.444	0.000	0.922	2.664
DA RA	0.534	0.473	0.544	-0.339	1.511
CA	3.667	0.422	0.007	1.552	3.206
CA RA	-3.133	0.444	0.000	-2.664	-0.922
DA	-3.667	0.422	0.000	-3.206	-1.552

Table 4: Turkey post-hoe analysis for group B

(I) (J)	Mean Diff (I-J)	Std Error	Sig (C20.05)	95% confidence	
				Lower bound	Upper bound
RAEP DAEP	-0.617	0.440	0.500	-1.345	0.378
CAEP	3.133	0.421	0.002	2.309	3.958
DAEP RAEP	0.617	0.422	0.500	-0.211	1.445
CAEP	3.750	0.393	0.000	3.028	4.565
CAEP RAEP	-3.133	0.418	0.000	-3.853	-2.214
DAEP	-3.750	0.390	0.000	-4.514	-2.986

From table3, it can be seen that the mean difference between the arithmetic scores of rectangular arithmetic and diagonal arithmetic is ±0.534. This difference was tested at 95% confidence interval and found to have a significance of P = 0.544. Since P> 0.05, it follows that the difference is NOT significant. The difference between the mean arithmetical achievements of rectangular arithmetic (RA) and conventional arithmetic (CA) is ±3.133. This difference is tested at 95% confidence interval and found to have a significance of P = 0.000. This shows that P< 0.05 which of course means that the difference between the mean arithmetical achievements of rectangular arithmetic (RA) and conventional arithmetic (CA) IS significant. The mean difference between arithmetical achievements of diagonal arithmetic (DA) and conventional arithmetic (CA) is ±3.667. Testing this difference at 95% confidence interval, we find that P = 0.000 which again shows that P< 0.05. Hence, it is clear that the mean difference between the arithmetical achievements of diagonal arithmetic (DA) and conventional arithmetic (CA) IS significant.

Table 4 also shows that the mean difference between the arithmetical achievements of pupils exposed to rectangular arithmetic with ethno-cultural pedagogy (RAEP) and diagonal arithmetic with ethno-cultural pedagogy (RAEP) is ±0.617. Testing this difference at 95% confidence interval, we find that P = 0.500. Since P > 0.05 it follows that this difference is NOT significant. From table4, it can also be seen that the mean difference between the arithmetical achievements of rectangular arithmetic with ethno-cultural pedagogy (RAEP) and conventional arithmetic with ethno-cultural pedagogy (CAEP) is ±3.133. This difference, tested at 95% confidence interval, shows that P = 0.002. This indicates that P < 0.05 hence we conclude that difference IS significant. Again, in table 4, it can be seen that the mean difference between the arithmetical achievements of pupils in diagonal arithmetic with ethno-cultural pedagogy and conventional arithmetic with ethno-cultural pedagogy is ± 3.750. By testing the difference at 95% confidence interval, it is observed that P = 0.00. Since P < 0.05, it follows that this difference IS also significant. Table5 below shows a summary of the turkey post-hoc analysis by subsets.

Table 5: Tukey HSD Post-Hoc for Group A and B

Semiotic method	N	Subsets for a = 0.05		
		1	2	3
Group A				
RA	58	16.733		16.733
DA	58	17.267	17.267	
CA Sig	58	0.544	13.600 0.000	13.600 0.000
Group B				
RAEP	60	16.733		16.733
DAEP	60	17.350	17.350	
CAEP sig	60	0.500	13.600 0.000	13.600 0.000

From table 5 above, the following conclusions can be drawn.

1. The mean difference between the arithmetical achievements of pupils in rectangular arithmetic and diagonal arithmetic is NOT significant; (Ho1 is accepted),
2. The mean difference between the arithmetical achievement in rectangular arithmetic and conventional arithmetic IS significant (Ho1 is rejected), and
3. The mean difference between the arithmetical achievements of pupils in diagonal arithmetic and conventional arithmetic is significant. (Ho1 is rejected).
4. Thus we can see that the effects of diagonal arithmetic and rectangular arithmetic on pupil's arithmetical achievement are more significant than the effect of conventional arithmetic on their achievement.

μCTRL (RA) = μCTRL (DA) > μCTRL (CA)
μEXP (RAEP) = μEXP (DAEP) > μEXP (CAEP)

Ho2: The effects of using ethno-cultural pedagogy in teaching diagonal arithmetic, rectangular arithmetic and conventional arithmetic on pupils' arithmetical achievement are NOT significant.

Table 6: Between group analyses of the post-test mean scores of pupils in group A and group B.

Group	Semiotic method	N	$\bar{X}$	$S2$	F	Fa	P(a=0.05)
A B	RA RAEP	58 60	14.707 16.733	7.578 6.097	18.492	3.923	0.000
A B	DA DAEP	58 60	15.203 17.350	5.892 4.604	23.818	3.923	0.00
A B	CA CAEP	58 60	12.914 13.617	4.431 4.478	3.271	3.923	0.073

Table6 shows the analysis of the mean arithmetical achievement between pupils in group A and pupils in group B. from table6, it can be seen that analysis between the RA and RAEP pupils gave a test statistic of F=18.492 while the critical value was F□ = 23.923 and P =0.000. Since F > 3.923, it implies that Ho2 must be rejected for this pair. Therefore, we conclude that there is a significant difference between the mean arithmetical achievements of the two groups. The difference in this case favours the RAEP pupils. Table6 also shows that the test statistic for the analysis between the DA and DAEP pupils is F = 23.818 while the critical value still stands at F□ = 3.923 and P =0.000. The fact that F > 3.923 and P<0.05 shows that Ho2 must be rejected for this pair of groups. As a result we conclude that there is a significant difference between the mean arithmetical achievements of the two groups in these semiotic methods. The test statistic for the analysis between the CA and CAEP pupils shows that F = 3.271 and P = 0.073 while the critical value still stands at F = 3.923 clearly, F< 3.923 and P > 0.05 which implies that Ho2 must be accepted for this pair. Thus, we conclude that there is no significant difference between the arithmetical achievements of the CA and CAEP pupils between the two groups. From the foregoing, it is safe to say that the effects of using ethno-cultural pedagogy in teaching rectangular arithmetic's and diagonal arithmetic IS significant, whereas the effect of using ethno-cultural pedagogy in teaching conventional arithmetic is NOT significant.

Discussion of findings

The first objective of the study was to determine the effects of three semiotic methods, rectangular arithmetic, diagonal arithmetic and conventional arithmetic, on pupil's achievements in arithmetic. The results obtained from the study showed that, within the groups, pupil's exposure to rectangular arithmetic and diagonal arithmetic actually increases their achievement in arithmetic much more than exposure to conventional arithmetic. The results also showed that rectangular arithmetic and diagonal arithmetic can be taught independently alongside the conventional arithmetic as though they were different subjects.

The second objective of the study was to determine the effects of using ethno- cultural pedagogy in teaching diagonal arithmetic, rectangular arithmetic and conventional arithmetic on achievement in arithmetic. The results of the study showed that exposure to ethno-cultural pedagogy actually favours the pupils in rectangular arithmetic and diagonal arithmetic much more than it affects the pupils in conventional arithmetic. This indicates that ethno-cultural pedagogy can be safely utilized in the teaching of arithmetic with the motive of improving achievement in the subject.

Conclusion

Going by the findings of this study, the conclusions of the study are hereby deduced as follows:

1. Rectangular arithmetic and diagonal arithmetic have more significant positive effects on pupil's achievement in arithmetic than conventional arithmetic.
2. The use of ethno-cultural pedagogical materials in teaching the three semiotic methods amplifies the arithmetical achievement of both pupil's exposed to rectangular arithmetic and diagonal arithmetic much more than it does to conventional arithmetic.

Recommendations

Rectangular and diagonal arithmetic are now being thrown into the space of educational and mathematical research for further exploration. These two semiotic methods of arithmetic no doubt present the world with new and inexhaustible topics to explore. Although rectangular and diagonal arithmetic may have other implications for many other researchers, their educational benefits are considered to be of utmost priority to the authors of the study. Even though, sometimes research outcomes are not being taken seriously and are often left to remain in the realm of research, it is important to mention that the outcomes of this study should not be treated with levity. They should be given priority and judicious attention. This is because the authors believe that these findings hold the potential for transforming the teaching and learning of mathematics. Thus against the backdrop of the foregoing, the following recommendations are hereby projected:

1) Curriculum design

The designers of national curricula at the primary and secondary school levels should do well to incorporate rectangular arithmetic and diagonal arithmetic into the curriculum design programme. The advocacy here is that while conventional arithmetic, which is already established among these two levels of education as 'general mathematics', will continue to maintain its place as a separate curriculum, additional parallel curricula should be created to cater for rectangular and diagonal arithmetic respectively. Either one of them could be adopted and applied or both are made to run simultaneously and independently in parallel with the conventional arithmetic curriculum. These three curricula will be treating the same concepts at each level but in their different and respective approaches. The programme for rectangular and diagonal arithmetic should be implemented in stages, with the first stage coming as a pilot study. Experts and teachers in the field of diagonal and rectangular arithmetic should be deployed to write textbooks matching the programmes at various levels.

2) Design and supply of ethno-cultural pedagogical materials

A platform should be created for the design, construction and supply of ethno-cultural materials based on the cultural context of the learners. By this, the learners will gain the ability to mathematize reality within their cultural context. In so doing, they will be able to advance their local technologies without looking to import ideas from foreign sources.

3) Teachers

Various training programmes such as seminars and workshops should be organized for teachers to help them not only to acquire the knowledge of rectangular and diagonal arithmetic but to gain mastery enough to be able to apply them in diverse forms during classroom interactions. In so doing, they will be able to diversify the teaching and learning of mathematics to include rectangular and diagonal semiotic methods of arithmetization.

4) Students and Pupils

Exposition to rectangular and/or diagonal arithmetic no doubt arouses the learners' interest and gives them a sense of appreciation of the beauty of divergence in reasoning. Doing so in an ethno-cultural context amplifies these feelings and (D'Ambrosio, 1985) [2] thus creates better understanding, grasping, retention and reproducibility. This pulls away mathematical phobia and creates options for choice of mathematical inclinations based mainly on the students'/pupils' ability to grasp arithmetical and mathematical concepts. All these account for why their achievement is boosted reasonably.

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