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A solution to the risk-return conundrum: Return exposure and the nonparametric Bayesian approach

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Abstract
The topic of the risk-return relationship is of broad importance in the fields of finance and economics. It has been widely investigated on an international scale, especially by developed markets from as early as the 1950's, with the primary motive being to help market participants optimize their chance to earn higher returns. According to conventional economic theory, the relationship between risk and return is a positive and linear relationship – the higher the risk, the higher the return. However, there are many studies documented in literature which show a positive or negative or no relationship at all. As a result, due to the magnitude of conflicting results over the years, this has caused an ongoing debate to arise regarding the risk-return relationship. International studies have explored a number of theories and models to attempt resolving the inconclusive empirical backing of the risk-return relationship. A valuable contribution of this study is the introduction of the novel concept "returns exposure" which refers to the risk that arises from the asymmetric nature of returns. This measure has a certain level of uncertainty attached to it due to its latent and stochastic nature. As a result, it may be ineffectively accounted for by existing parametric methods such as regression analysis and GARCH type models which are prone to model misspecification. This motivates the use of a more robust method, namely, the nonparametric Bayesian approach. The Bayesian approach has the ability to average out sources of uncertainty and measurement errors and thus effectively account for "return risk" or "returns exposure." The Bayesian approach can be modelled within a parametric or nonparametric framework. The nonparametric approach is considered more robust as it relaxes modelling assumptions such as normality. Thus, in combination with the nonparametric approach, this provides a more robust estimation of the risk-return relationship.

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1. Introduction

According to literature, forecasting risk and returns still remains a fundamental problem in any financial market (Liu, 2019) [56]. In the investigation of the risk-return relationship, it is found that there is always some dynamic component to account for variability or asymmetry that arises from price data, in order to address the omitted variable bias (Kim and Kim, 2018) [50]. That's because the assumption that price data of an entire financial system follows a normal and symmetric distribution cannot be accepted (Li, 2018). Share prices are dynamic as they constantly change over time, thus, are stochastic or random in nature which means that they can be statistically analysed but not with certain precision (Harris, 2017) [34]. Therefore, it follows that the return distribution, which is derived from price data, follows an asymmetric distribution, as supported by Gyldberg and Bark (2019) [33].

This research makes a significant contribution to the ongoing debate about the magnitude of the risk-return relationship by introducing the novel concept "return risk" or "returns exposure" which refers to the risk that arises from the asymmetric nature of returns. This "return inherent risk" may be ineffectively accounted for by existing parametric methods due to its latent and stochastic nature (Jin, 2017) [42]. This is mainly due to the model's limitations and misspecifications to effectively estimate risk (Jensen and Maheu, 2018) [41]. For example, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is a common method used to investigate the risk-return relationship (Madaleno and Vieira, 2018; Savva and Theodossiou, 2018) [57, 74].

There has been a number of extensions of the standard GARCH model and various sources of price data variability to take into account (Cenesizoglu and Reeves, 2018) [14]. Extensions of the standard GARCH model include the EGARCH, GJR-GARCH and APARCH models, among others, which account for the asymmetric nature of volatility (Savva and Theodossiou, 2018) [74]. Further, sources of price data variability include volatility feedback, the leverage effect, skewness and behavioral biases. However, if a model can effectively estimate risk, there is no need for such model extensions, specifications and omitted variables biases (Demirer *et al*., 2019) [21].

A model that satisfies these conditions is the nonparametric Bayesian approach by Jensen and Maheu (2018) [41]. The nonparametric Bayesian approach is unique to existing literature that typically uses conventional parametric methods such as the GARCH approach and regression analysis (Jensen and Maheu, 2018) [41]. The application of the Bayesian approach in practical real-life situations demonstrates its usefulness and effectiveness. Given the recent pandemic of the Coronavirus disease (COVID-19), a number of studies applied the Bayesian approach and methods to contribute uncovering its properties (Linton *et al*., 2020; Nishiura *et al*., 2020) [55, 43].

The Bayesian approach stems from Bayes (1763) [8] theorem which is defined as the probability estimation of a relationship given prior information. In this case, the relationship is between risk and return and the prior information refers to the inherent properties of the financial data – which is nonlinear, asymmetric, volatile, stochastic and latent. The nonparametric approach is a "model free" approach where the data is estimated free from assumptions, nonnegativity or economic restraints (Jin, 2017) [42]. Studies highlight that the nonparametric approach relaxes the normality assumption, effectively accounting for asymmetric properties such as skewness, kurtosis and multiple modes (Apergis *et al*., 2018) [6]. By modelling data in a nonparametric framework, this allows "the data to speak for itself" solely based on its nature and not any predetermined assumptions or bias (Jensen and Maheu, 2018) [41]. The combination of the Bayesian approach and nonparametric approach produces a powerful method of data estimation. Thus, the use of a model such as the nonparametric Bayesian approach, that can fit the aforementioned properties of financial data effectively, improves forecast accuracy and ensures research progression (Demirer *et al*., 2019) [21].

This study is made up of four parts. First, the topic of the risk-return relationship, significant contribution of this research and nonparametric Bayesian approach is introduced. Second is the literature review, which is made up of a conceptual framework, empirical review and a critical analysis. The conceptual framework is on the risk-return variables and their relationship, the empirical review focuses on existing international literature and the critical analysis discusses the concept of return exposure. The third part covers the Bayesian approach, provides a framework when such a model is suitable and shows the quantification of the return variable with attention to the concept of return risk. Finally, the main points of this study is highlighted in conclusion.

2. Literature Review

2.1. Theoretical framework

2.1.1. Risk and return variables

Risk is defined as the possibility of a future event deviating from an expected outcome where the greater the possibility of deviation implies a greater level of risk (Kempers *et al*., 2019) [48]. For an investor, this is the possibility of failing to realize an expected rate of return for an investment venture (Hussain *et al*., 2019) [40]. Further, the probabilities of possible future outcomes can be estimated given prior information (Aliu *et al*., 2017) [4]. This means that risk allows an individual to have some probability of knowledge, whereas in contrast, uncertainty does not (Aliu *et al*., 2017) [4]. An understanding of risk is vital to all market participants in a financial system, especially in the decision-making process (Kempers *et al*., 2019) [48].

According to the Capital Asset Pricing Model (CAPM) framework by Sharpe (1964) [77], there are two types of risk, namely, systematic and unsystematic risk, which makes up total risk. Systematic risk is also known as undiversifiable risk, market risk or volatility and is a market inherent risk. This means that the entire market and all the securities within it are exposed to risk factors that arise from the market such as the interest rate, currency rate and monetary policy (Gyldberg and Bark, 2019) [33]. Although an investor may not completely keep clear of systematic risk by means of diversification, it can be managed by hedging or by a proper security allocation strategy (Aliu *et al*., 2017) [4]. On the other hand, unsystematic risk is also known as diversifiable risk, specific risk or residual risk and is a company or industry inherent risk (Charles and Okoro, 2019). This means that the securities that an individual invests in are exposed to risk factors associated with the firm or industry such as a change in management or regulation, respectively (Gyldberg and Bark, 2019) [33]. However, unsystematic risk can be reduced by means of diversification (Aliu *et al*., 2017) [4].

The standard measures used to quantify risk are often captured by beta for total systematic risk, which is specifically characterized by the CAPM, and standard deviation or variance for total risk. Theoretically, variance is an appropriate risk estimator only when the return distribution is normal; however, empirically this is not always the case (Sehgal and Pandey, 2018) [75]. Thus, the quantification of risk can pose a challenge to researchers (Chiang and Zhang, 2018) [17]. As a result, a certain criterion is often set to support their approach or why studies tailor a risk estimator to cater for such statistical conditions which may be stochastic in nature. For example, in the context of the risk-return relationship, the four standard types of risk measures that are typically used are historical, implied, conditional and realized variance (Jin, 2017) [42]. Historical and realized variance which are computed from historical data are considered to be inflexible, have limited forecast ability and low explanatory power (Park *et al*., 2017) [16].

Implied variance is proposed as a better risk measure due to its ability to capture investor behavior and future firm prospects (Bekiros *et al*., 2017) [9].

However, from a financial perspective, implied variance is limited in that it does not account for the risk that arises from macroeconomic fundamentals. On the other hand, many studies document conditional variance as their risk measure as characterized by the GARCH approach (Madaleno and Vieira, 2018) [57]. However, the use of conditional variance may require certain assumptions and constraints to be imposed to the data (Kim and Kim, 2018) [50]. In contrast, realized variance is a data driven measure due to its random, stochastic nature and better forecast ability (Maneemaroj *et al*., 2019; Noguchi *et al*., 2016; Zhang and Lan, 2014) [60, 66, 89]. Hence, the realized variance risk measure is used in models that are able to incorporate such properties, unlike a normal-type GARCH model (Chiang and Zhang, 2018) [17, 89]. Realized variance is also a popular choice in nonparametric Bayesian modelling due to being in line with a model free approach where it does not make any assumptions about the data (Jensen and Maheu, 2018) [41].

Financial market returns are used to determine whether or not a trading strategy is profitable (Gyldberg and Bark, 2019) [33]. Investors often use the CAPM to determine a rate of return to compensate for a level of risk taken which originates from the trade-off theory, the idea of higher the risk the higher the return. Under the framework of the CAPM, there exists a direct relationship between expected returns and systematic risk (Sharpe, 1964) [77]. Financial securities that do not correspond to this relationship act as a source of price data variability (Gyldberg and Bark, 2019) [33]. Further, returns are subject to risk often as a result of uncertainty in the market (Apergis *et al*., 2018) [6]. This calls the validity of the Efficient Market Hypothesis (EMH) by Fama (1970) [25] into question. The EMH states that in an efficient market, prices contain all available information. Consequently, no securities are mispriced under the EMH, making excess returns impossible to realize consistently (Lehoczky and Schervish, 2018) [52]. As a result, this causes a more realistic approach to strategies and models in the estimation of risk and return (Apergis *et al*., 2018) [6].

According to Li (2018), returns of the entire financial market follow a normal distribution for two reasons. First, the Central Limit Theorem by de Moivre (1733) [20], states that for a sample drawn from a distribution with a finite mean and variance, for a sufficiently large sample, tends to a normal distribution. Second, market stability arises from investor sentiment and individual risk preferences, which follow a positively skewed distribution. This area of the distribution is favored by investors due to being able to achieve higher payoffs. According to Casella and Gulen (2018) [13], there exists a substantial amount of evidence in literature that financial market returns can be forecast. However, forecasting returns by time series and behavioral models are subject to a number of factors that cause returns to deviate from a normal distribution (Cenesizoglu and Reeves, 2018; Casella and Gulen, 2018) [13, 14]. To name some contributing factors that affect returns, include volatility feedback, the leverage effect, inefficient information, behavioral biases and different investor sentiment. All of which, in turn, affect the risk-return relationship.

2.1.2. The risk-return relationship

According to the Modern Portfolio Theory (MPT), the variables of the risk-return relationship explain the construction of an efficient portfolio. Steyn and Theart (2019) [78] emphasizes the importance of the MPT in portfolio and risk management where it provides a framework to quantify and understand the variables, risk and return as well as their relationship. Specifically, that risk can be reduced by means of diversification and higher returns can only be attained by higher risk. Further developments of the MPT by Sharpe (1964) [77], Lintner (1965) [54] and Mossin (1966) [65], led to the CAPM which provides a simplified explanation of the MPT. First, the CAPM introduced the two types of risk, systematic and unsystematic risk, as discussed, to provide a practical understanding of risk. Second, following the MPT, unsystematic risk should be diversified away, leaving an investor with an opportunity to a higher return from systematic risk (Rutterford and Sotiropoulos, 2016) [73].

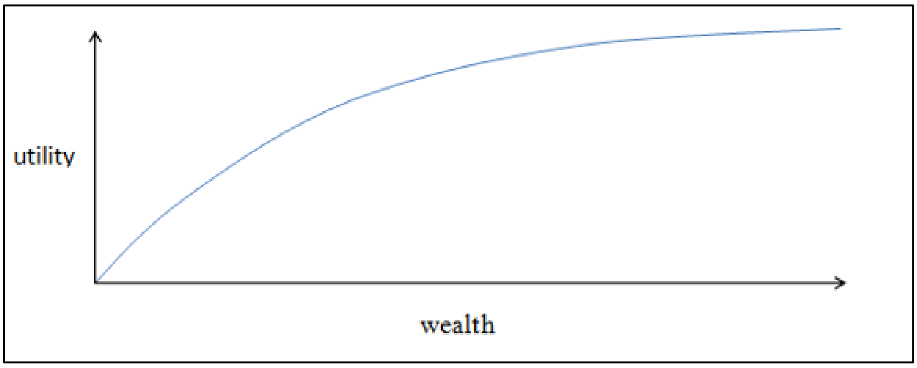
According to Steyn and Theart (2019) [78], the CAPM explains the risk-return relationship by an equilibrium in which there is a linear relationship. According to the CAPM, the expectation of excess returns in a portfolio is a function of systematic risk and market excess returns. That is, by understanding the risk-return relationship in a market, an investor has insight as to whether they have an opportunity to optimize their chance of earning a superior rate of return. Essentially, from both the MPT and CAPM, the risk-return relationship demonstrates the traditional risk-return trade-off in which an investor can only earn a superior return if they undertake a higher level of risk. By following this theory and understanding the empirical risk-return relationship in the market, an investor can construct an efficient portfolio in order to meet their desired risk profile and expected rate of return (Rutterford and Sotiropoulos, 2016; Steyn and Theart, 2019) [73, 78].

Forecasting the risk-return relationship is of paramount importance as it forms the basis of a number of strategies by investors, financial institutions, asset pricing models and policy frameworks (Vo *et al*., 2019). There are a number of theories and models built upon it (Liu, 2019) [56]. Such theories include the underlying idea that the risk-return relationship is a requirement in the modelling of valuation techniques such as the Discounted Cashflow Model and the Contingency Claims Approach to name a few (Sehgal and Pandey, 2018) [75]. Financial institutions are able to determine and implement proper cash flow strategies in terms of borrowing and lending (Liu, 2019) [56]. When predicted for a specific market, it can help determine profitable strategies and curb market risk. It assists policy makers in the construction of regulatory and policy frameworks (Mandimika and Chinzara, 2012) [59]. Further, it can also be used as an indicator of investor behavior in terms of risk profiling (Dicle, 2018) [22]. Finally, the estimation of the risk-return relationship can assist in the prediction of a potential financial crisis, according to Sehgal and Pandey (2018) [75].

Based on literature, there are four existing types of the empirical risk-return relationship which are positive, nonlinear or curvilinear, negative and none (Savva and Theodossiou, 2018) [74]. The underlying theories and graphs are briefly explained for each type of relationship.

2.1.2.1 Positive

For a positive risk-return relationship, a rational investor has the ability to choose their risk-return preference based on a wide array of investment choices (Dicle, 2018) [22]. The trade-off theory suggests that an investor is risk averse whereby a low level of risk undertaken results in a corresponding level of return and vice versa for a risk-taking investor. This is supported by the expected utility theory which states that an investor makes a choice that maximizes utility, which is similar to a measure of satisfaction, and minimizes loss (Rutterford and Sotiropoulos, 2016) [73]. Figure 1 shows the expected utility function.



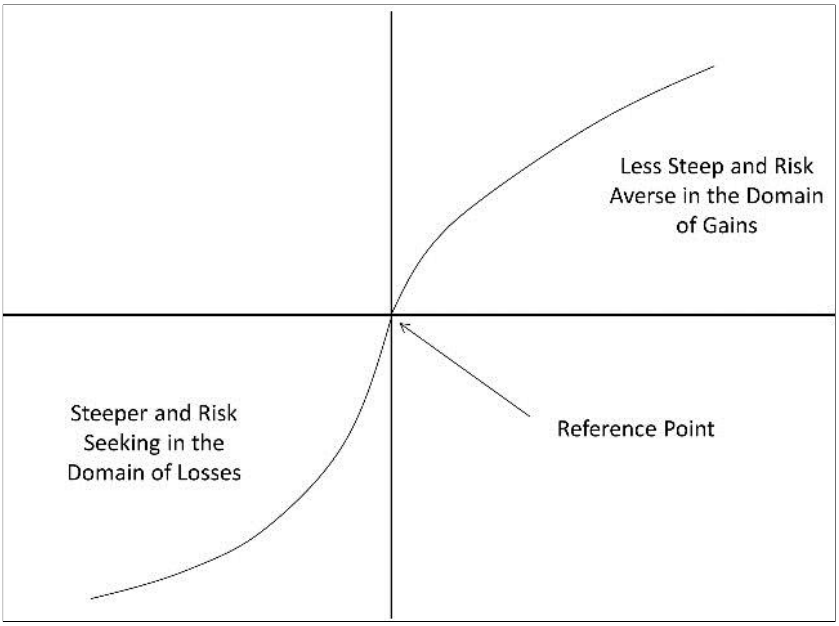
Source: du Preez (2011)

Fig 1: Expected utility function

From Figure 1, the conventional utility function has a concave shape which has a positive slope. The shape of the function demonstrates an investor's preference to a higher rate of returns, in comparison to a lower rate. However, there is also a diminishing marginal utility which means that an investor's preference for higher returns increases but at a decreasing rate (Rutterford and Sotiropoulos, 2016) [73]. Since the graph is measured over total wealth, this suggests an investor's risk averse behavior is symmetric for both gains and losses (Dicle, 2018) [22].

2.1.2.2 Negative

On the other hand, a nonlinear risk-return relationship is explained by the prospect theory. The prospect theory by Kahneman and Tversky (1979) [44], states that investors are risk seeking in unstable market conditions but risk averse in stable conditions. This is because the prospect of gain outweighs the prospect of loss and an investor makes a decision to ensure maximum gain and minimum loss. The prospect theory is essentially where an investor is more likely to take on a higher level of risk to avoid losses and ensure gains (Kahneman and Tversky, 1979) [44]. Figure 2 shows a utility function.



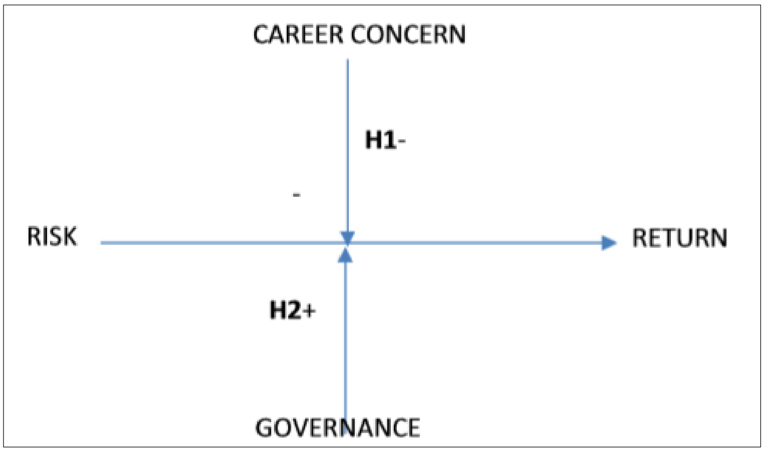
Source: Phillips and Pohl (2017)

Fig 2: Utility function

From Figure 2, the utility function is s-shaped and the positive part of the function is concave in the region of gains. This is similar to the expected utility function in Figure 1 above. From Figure 2, the negative part of the function is convex in the region of losses reflecting risk averse behavior of an investor. Since the graph is measured over both losses and gains, this suggests an investor's risk averse behavior is asymmetric where gains are favored over losses (Dicle, 2018) [22].

Highlighted that a negative risk-return relationship is considered as paradoxical based on traditional theoretical literature. This is because a negative risk-return relationship is contrary to expectations founded on conventional economic theory and traditional literature. Thus, it is also known as Bowman's (1980) Paradox which is explained by the prospect, behavioral and agency theory.

According to, in a firm setting, managers take on projects that do not match their risk profiles. Specifically, they take on risky projects when the firm is performing negatively, to advance their careers by improving their reputation in the labor market. Their actions are not necessarily in the interest of improving the value of the firm and increasing the wealth of the shareholders. But rather in line with their own self-interest because their actions suggest the ability to take on risks which can lead to a positive effect on their careers. Managers could simply pay out dividends if they cannot find profitable projects or investments. However, if they take on risky projects and if such risky projects consistently fail to meet expected returns, the value of the firm is negatively affected. Thus, problems arise between managers and shareholders if actual returns fail to meet the expected returns of shareholders. It consequently leads to a conflict of interest known as the agency theory developed by Mitnick (1973) [64] and Ross (1973) [72]. Figure 3 explains Bowman's paradox by testing hypotheses that consolidate the theory of the negative risk-return relationship.



Source: Chari *et al*. (2018)

Fig 3. Theoretical model

From Figure 3, Hypothesis 1 (H1-) states that the negative risk-return relationship is intensified by a manager's risk-taking behavior. This is in order to improve his professional reputation and not add value to the firm or enhance the wealth of shareholders. Hypothesis 2 (H2+) encapsulates a number of solutions to counteract the negative risk-return relationship and align the interests of managers and shareholders by means of governance. Hence, the risk-return paradox can be resolved by the establishment of governance mechanisms such as the market monitoring corporate control, establishing incentives and having organizational owners.

2.1.2.3 No relationship

When there is no risk-return relationship, this indicates that the risk-return relationship is insignificant (Apergis *et al*., 2018) [6]. This often occurs in international studies that incorporate a number of countries which have different market return characteristics that can affect the final result of the risk-return relationship (Savva and Theodossiou, 2018) [74]. The returns of emerging markets are characterized as having higher levels of volatility, heavy tails and better forecast ability, in comparison to developed markets (Herbert *et al*., 2018) [38]. Therefore, it is considered more useful to investigate countries with similar market return characteristics such as BRICS which consists of emerging markets only (Adu, *et al*., 2015) [2]. The analysis of similar markets allows for better statistical inference and comparative analysis between them (Sultan, 2018) [79]. Similarly, when investigating the risk-return relationship on an aggregate market level, it is important to consider the sectors within the market (Khan *et al*., 2016) [49]. This is because the heterogeneity among firms, in terms of firm characteristics such as leverage and capital structure, can affect risk estimation (Horpestad *et al*., 2019) [39].

2.2. Empirical review

According to Chou (1988) [18], the US market is the largest market in the world from which market participants have been seeking various ways to earn superior returns from as early as the 1950's. From 1958 onwards, the nature of the share market became a central focus due to the negative impact on returns earned by investors (Liu, 2019) [56]. To investigate volatility, the procedure used by previous studies at the time was a two-stage Ordinary Least Squares (OLS) method as documented by French *et al*. (1987) [28], Pindyck (1984) [70] and Pagan and Ullah (1988) [67]. However, this is essentially a linear parametric model which does not adequately account for higher moment asymmetric properties (Madaleno and Vieira, 2018) [57]. Thus, the GARCH-M model in conjunction with maximum likelihood estimation (MLE) instead proves to be a more robust method as it addresses the drawback of OLS (Chou, 1988) [18]. The MLE method estimates parameters from the actual data allowing for nonlinear parameters (Madaleno and Vieira, 2018) [57].

In an early US study conducted by Chou (1988) [18], the returns from the New York Stock Exchange (NYSE) index for the sample period July 1962 to December 1985 is analyzed at a weekly frequency. A GARCH (1, 1) MLE and linear AR (1) method is used, where the correlation coefficient in AR (1) acts as the parameter ($\alpha+\beta$) found in the standard GARCH model, to capture the persistence of volatility. A plot of the NYSE returns shows the clustering nature of volatility where high returns follow high returns and low returns follow low returns. The AR (1) method explicitly captures volatility clustering; however, the GARCH method provides more robust parameter estimates. The AR (1) method gives less robust parameter estimates due to its inability to capture a high level of volatility persistence over time. Chou (1988) [18] concludes that the relationship between risk and return is time varying whereby the relationship changes over time.

More importantly, the study by Chou (1988) [18], highlights the importance of using nonlinear models to capture market return characteristics since variance does have implications on returns earned. Specifically, the parameters reflect a strong impact of variance on the market resulting in negative returns earned. This finding is in contrast to Poterba and Summers (1986) [71], who states that volatility is temporary and has a negligible effect. Chou (1988) [18] further states the parameter estimates are found to be sensitive to data frequency. That is, the reason for the finding by Poterba and Summers (1986) [71], is because of the use of monthly data instead of weekly which holds stronger for persistent levels of volatility. This is essentially due to the MLE method which has an improved ability to capture volatile properties, in comparison to the previous documented studies by French *et al*. (1987) [28], Pindyck (1984) [70] and Pagan and Ullah (1988) [67].

The OLS method has been shown to be inadequate in capturing nonlinear properties by the model parameters, in contrast to MLE which is a nonlinear method by Chou (1988) [18]. Likewise, the AR (1) and VAR model which are parametric models are not designed to fit higher moment properties effectively (Demirer *et al*., 2019) [21]. A parametric model has a set number of parameters with respect to the sample size (Jin, 2017) [42]. Consequently, it is limited in its ability to account for every possible risk-return relationship that can hold (Demirer *et al*., 2019) [21]. This includes asymmetric forms with properties such as skewness, kurtosis and multiple modes (Apergis *et al*., 2018) [6]. A model cannot be effective in modelling data with properties it is not designed or specified to take into account (Jensen and Maheu, 2018) [41]. This is especially relevant to OLS since higher moment properties lie outside of its design parameters which can lead to biased parameter estimates (William and Ligori, 2016) [86].

However, the drawbacks of the OLS method can mainly be attributed to the assumptions that they are based on. According to Conradt *et al*. (2015) [19], the dependent variables are assumed to be constant and normally distributed. Additionally, the innovations are assumed to be normally distributed as well as have a constant variance. According to William and Ligori (2016) [86], while improvements have been made, such improvements still involve imposing constraints to the parameters or omitting some of the parameters. It is further noted that a more favorable approach would be to retain all the parameters without omission but such an approach has not been explored as yet. While William and Ligori (2016) [86] note such an unexplored approach in the context of OLS, Conradt *et al*. (2015) [19] highlight that the nonparametric approach overcomes these issues. The nonparametric approach is a model free approach and models the data free from assumptions and restrictions (Jin, 2017; Conradt *et al*., 2015) [42, 19].

Thus, Umutlu (2019) [80] employed a parametric and nonparametric VAR model to investigate the relationship between market returns and idiosyncratic volatility. Idiosyncratic volatility is similar to unsystematic risk where risk exposure arises from the firm or industry such as a change in management or regulation, respectively (Gyldberg and Bark, 2019) [33]. Umutlu (2019) analyzed monthly data consisting of nineteen local indexes of thirty-seven nations on an international aggregate level for the period 1973 to 2015. Measures of volatility are model dependent and independent as well as a further four sub samples are analyzed to investigate the possibility of a nonlinear risk-return relationship. Despite the use of subsamples, a parametric and nonparametric approach, results still reveal no risk-return relationship with the VAR model. However, the study did conclude strong support for the presence of volatility feedback during recessionary and high volatility periods.

Chakrabarti and Kumar (2017) [15] analyzed daily data sets for the period 3 March 2008 to 31 August 2015. The risk and return variables were obtained from the National Stock Exchange (NSE). Returns were obtained from Nifty which is the Indian share market index and consisted of over twenty-three sectors. Implied volatility is the risk measure used due to its popularity in recent studies based on developed markets, according to Chakrabarti and Kumar (2017) [15]. This risk measure is a forward-looking value that captures investor behavior and future prospects of a firm (Bekiros *et al*., 2017) [9]. It is in contrast to realized variance which is a recent preferred risk measure due to its ability to capture the stochastic nature of risk (Maneemaroj *et al*., 2019) [60].

Chakrabarti and Kumar (2017) [15] employed a VAR model and Granger causality tests where volatility feedback was found to have the strongest explanatory power, in comparison to the other two theories. However, the VAR result and model was concluded to be ineffective because of its inability to account for extreme values and asymmetric properties. The Granger causality test is supported by a nonparametric framework where results reveal weak evidence for volatility feedback explaining the risk-return relationship. Thus, a quantile regression analysis is applied which is effective in accounting for extreme values, in comparison to the VAR model and OLS method. The quantile regression found the behavioral theory to be the dominant factor in explaining the risk-return relationship.

However, according to Waldmann (2018) [84], the determination of the parameters for quantile regression is more difficult, in comparison to other regression types such as normal or generalized. In the context of model implementation, specific to the R software, the number of iterations has to be chosen in order to have optimal parameter estimates. This refers to the process of repetitive resampling by trial and error which can be tedious in nature (Karabatsos, 2017; Waldmann, 2018) [47, 84]. Waldmann (2018) [84] further recommends a Bayesian approach to enhance the method of interest; however, the priors should be as noninformative as possible. In other words, the prior should be objective and not guided by a source of subjectivity which overcomes the main limitation of the Bayesian approach (Bartlett and Keogh, 2016) [7]. The Bayesian method is often recommended to aid other methods because it has the ability to average out uncertainty affecting parameters (Kang 2014; Chang *et al*., 2017; Waldmann, 2018) [46, 84].

The Granger causality test is another parametric model that is often employed to investigate the risk-return relationship (Apergis *et al*., 2018) [6]. Similar to the Bayesian approach, the nonparametric framework is often used in conjunction with other methods such as the VAR model in order to aid capturing nonlinear properties (Umutlu, 2019; Demirer *et al*.,

2019) [80, 21]. This is shown in the previous study by Chakrabarti and Kumar (2017) [15], who applies the nonparametric approach in the context of Granger causality tests. Similarly, Apergis *et al*. (2018) [6], employs the same method of nonparametric Granger causality tests to the monthly data sets of twenty-four international defence firms. The variables analyzed are the geopolitical risk index, realized variance and returns. Realized variance is used since it is in line with the nonparametric approach in being model free, thus, aiding capturing asymmetric properties (Noguchi *et al*., 2016) [66]. According to Apergis *et al*. (2018) [6], the nonparametric approach is used to account for nonlinearity in price data before applying the causality test. However, despite the application of the nonparametric approach, the results indicate no risk-return relationship. The study concludes the causality approach is unreliable and highlights the importance of accounting for nonlinearity before establishing the risk-return relationship to avoid model misspecification.

In contrast, to the result found by Apergis *et al*. (2018) [6], Demirer *et al*. (2019) [21] finds a significant relationship between risk and return in the US market. Demirer *et al*. (2019) [21] applies a number of models in order to investigate the relationship between equity return dispersion and share market volatility. Linear and nonlinear as well as bivariate and multivariate causality tests are employed to the sample July 1963 to February 2017. The share and market returns are obtained from the Center for Research in Security Prices (CRSP) value-weighted index return. The one-month Treasury bill (T-bill) rate is used to proxy the risk-free rate for calculating excess returns. The nonlinear bivariate and multivariate tests are found to be more robust as these models were able to account for the causal impact of return variance on returns and volatility. The study concludes that by accounting for the variance in returns, this improves risk estimation and contributes to the improvement of volatility models in predicting the risk-return relationship. However, this can only be performed by nonlinear models with the necessary model specifications to account for asymmetry.

Park *et al*. (2017) [16] applied a DCC-MGARCH model to the daily data sets of the Korean market for the period 2004 to 2013. The study analyzed the variables KOSPI200 returns, VKOSPI implied volatility measure and four macroeconomic variables – risk-free rates, term spreads, credit spreads and exchange rates. The use of the asymmetric GARCH type model is confirmed by a sign and size bias which shows that the standard GARCH model has not adequately captured risk. The final findings of the study by Park *et al*. (2017) [16], reveal mixed results where return in relation to the macroeconomic factors vary based on the type of regression analysis and specifications. Further, the high correlation between the macroeconomic variables can also result in the problem of multicollinearity. Multicollinearity refers to a state in which several independent variables exhibit a high level of linear correlation, which can affect model fit and results (Khan *et al*., 2016) [49]. Additionally, the choice of macroeconomic variables analyzed are often guided by an underlying subjective approach (Messis *et al*., 2019) [63]. This suggests a bias in the chosen macroeconomic fundamentals in explaining the risk-return relationship (Park *et al*., 2017) [16]. With respect to the DCC-MGARCH model, it is a complex model which forms part of the multivariate GARCH family to detect transmissions of volatility from one market or sector to another (Savva and Theodossiou, 2018) [74]. However, it is still a parametric model which is subject to the limitations of the univariate GARCH approach (Jin, 2017) [42]. This includes the nonnegativity constraints and the inability to effectively account for asymmetric properties (Jin, 2017; Demirer *et al*., 2019) [42, 21]. Another extension of the standard GARCH model is the GJR-GARCH model which has an additional term to capture possible asymmetries (Maneemaroj *et al*., 2019) [60]. Specifically, in response to news which is a source of volatility and where the type of news has an asymmetric effect on volatility (Hussain *et al*., 2019) [40]. However, this asymmetric effect is a given empirical regularity that has been systematically proven over time (Yu *et al*., 2018). Maneemaroj *et al*. (2019) [60] applies the GJR-GARCH model to a sample of ten years for twenty-four stocks of the Thailand market. The study highlights the importance of variable choice in leading to the final result of the risk-return relationship. The risk measure is realized variance because it is found to increase the predictive power of the test, according to Zhang and Lan (2014) [89].

Following the theory of Koutmos (2012) [51], Maneemaroj *et al*. (2019) [60], argues the proxy for the return variable, where expected returns cannot equal historical returns. Therefore, a CAPM model is used to generate the expected return values. However, like capital structure, expected returns might have a negligible effect on volatility as opposed to a negative effect (Horpestad *et al*., 2019; Aboura and Chevallier, 2018) [1, 39]. Nonetheless, Maneemaroj *et al*. (2019) [60] finds a negative risk-return relationship when historical returns are used and a positive relationship when expected returns are used. However, this study does not account for a source of price data variability, creating an omitted variable bias, as pointed out by Kim and Kim (2018) [50]. On the other hand, Savva and Theodossiou (2018) [74] accounts for the omitted variable bias by taking into account skewness in their study, a measure of asymmetry found in price data. The authors found that skewness is found to be the main reason for the varying results regarding the risk-return relationship found in the US market.

Due to the magnitude of risk-return relationship results, Savva and Theodossiou (2018) [74] documented an international review of existing literature at the time, in an attempt to explain the ongoing debate. The Q-GARCH, GJR-GARCH and EGARCH type models were applied to the data sets of forty-eight global share markets at varying frequencies – daily, weekly and monthly. Standard returns are analyzed against time varying volatility, instead of excess returns due to the inaccessibility and unavailability of a high frequency risk-free rate. Since the results of QGARCH and EGARCH are similar, the results focus on the GJR-GARCH model. The total risk-return relationship is estimated by the combined effect of a pure and skewed risk premium. For the total forty-eight markets included in their sample, the following majority of markets show no risk-return relationship at their corresponding data frequency, respectively: Forty-three markets at daily frequency, forty-two at weekly and thirty-seven at monthly. Overall results indicate no relationship between risk and return; however, the risk-return relationship is shown to be stronger for monthly data.

In contrast, the study by Liu (2019) [56] found daily data to be the most effective in capturing the risk-return relationship. The study uses a GARCH-M model along with in-sampling and out-sampling to investigate the risk-return relationship of the Chinese market. The in-sample refers to a forecast made based on the same set of data from which the parameters are estimated. Whereas, an out-sample refers to using a smaller dataset by excluding some of the observations. Liu (2019) [56] took into account lagged returns since returns are subject to delays in response to new information. Both the Shanghai and Shenzen Stock Exchange indices of the aggregate Chinese stock market are analyzed for the sample period 4 January 2000 to 21 May 2018. Varying data set frequencies are taken into account, namely, intraday, 2-day, 3-day, 5-day,10-day,15-day and 20-days. Results reveal a risk-return relationship that changes over time and concludes intraday data as the most robust, in comparison to the other frequencies.

A comparative analysis reveals the model taking into account the lagged returns is more robust than the one without. However, this is not the only means to account for the lagged nature of returns since Khan *et al*. (2016) [49] uses monthly data to overcome this problem. Liu (2019) [56] further concludes that although out-sampling improves prediction precision, this method is not better than using historical price data. Liu (2019) [56] uses a standard GARCH model which is limited in its ability to account for asymmetric effects. Thus, a hybrid GARCH model which is a combination of some or all the GARCH type models is more useful to capture a number of volatility characteristics at a time. More complex models can also be tailored, such as ADCC-EGARCH, which forms part of the multivariate GARCH family to detect transmissions of volatility from one market or sector to another (Sultan, 2018) [79]. However, the use of hybrid and/or complex models may be time consuming, computationally intensive and complicated.

However, the main problem with a GARCH model is the linear function of volatility. It presents an issue which is similar to one of the main drawbacks of the parametric GARCH approach where the parameters are subject to a constraint of nonnegativity (Jin, 2017) [42]. Thus, if the parameters do not meet this restriction, some adjustment has to be imposed to the data (Demirer *et al*., 2019) [21]. Returns cannot be a linear function of volatility because empirically, both volatility and returns are not linear in nature (Gyldberg and Bark, 2019) [33]. The phenomenon asymmetric volatility describes the asymmetric nature of volatility where volatility has the tendency to increase more for negative returns than positive returns, or vice versa, for the same magnitude. “Return exposure” describes the risk that arises from the asymmetric nature of returns. The asymmetric nature of returns is due to the dynamic nature of price data which constantly changes over time (Harris, 2017) [34]. Hence, it follows that a return distribution is asymmetric since returns are derived from price data, as supported by Gyldberg and Bark (2019) [33].

This is further in line with Maneemaroj *et al*. (2019) [60], who states that returns follow an asymmetric and heavy-tailed distribution. The heavy-tailed distribution is a characteristic of an emerging market due to being subject to higher levels of volatility (Herbert *et al*., 2018) [38]. In parametric approaches, it is common practice to impose assumptions and constraints on data (Apergis *et al*., 2017) [5]. In contrast, a Bayesian approach is where parameters are treated as random variables with no constraints imposed when introduced into the model (Agilan and Umamahesh, 2017) [3]. This means that the parameters are treated in accordance to the probability of an outcome based on the method used and not adjusted to fit a certain or fixed result (Kim and Kim, 2018) [50]. Random sampling methods are the most effective in producing unbiased estimates since the outcome is based on equal chance. Essentially, a model free approach allows for more flexibility in the estimation of complex data with nonlinear and asymmetric properties (Demirer *et al*., 2019) [21].

Jensen and Maheu (2018) [41] applied a nonparametric Bayesian approach to the US market for the period January 1885 to December 2011, a sample of 126 years. In contrast, Maneemaroj *et al*. (2019) [60] advocates a sample of at least 200 years to represent the expected returns variable when, to the best of the authors knowledge, the longest sample in a study of this nature has been the data set of 126 years as used by Jensen and Maheu (2018) [41]. Further, the choice of sample size is also dependent on the availability of data, which can be overly restrictive in most cases, especially for international studies. Thus, the 200-year sample advocated by Maneemaroj *et al*. (2019) [60], can be considered impractical in reality.

Jensen and Maheu (2018) [41] analyzed monthly excess returns with a bias-adjusted realized variance. The study account for volatility feedback which is considered an important source of asymmetry which affects risk estimation. Like Demirer *et al*. (2019) [21], the study recommends moving away from linearity to include densities with higher moment properties such as skewness, kurtosis and multiple modes. Once it is taken into account, the study found a positive and nonlinear risk-return relationship.

Kim and Kim (2018) [50] also investigated volatility feedback by employing a unified framework which is a generalization of a number of sub models to the US market. The data sets analysed were monthly for a sample period January 1959 to May 2014. The variables of interest were excess returns and macroeconomic fundamentals to account for risk. In the study by Kim and Kim (2018) [50], the authors found a positive risk-return relationship for their study in line with traditional theoretical expectations.

2.3. Critical analysis: Return exposure

According to existing documented literature, studies show a magnitude of conflicting results with respect to the risk-return relationship (Savva and Theodossiou, 2018; Maneemaroj *et al*., 2019) [74, 60]. From a broad perspective, results vary from study to study as a result of different choices such as data frequency, sample period and model specification (Savva and Theodossiou, 2018) [74]. To narrow it down, the magnitude of the empirical risk-relationship can be explained by two respective theories by Maneemaroj *et al*. (2019) [60] and Jensen and Maheu (2018) [41].

Maneemaroj *et al*. (2019) [60] notes that there are five areas of concern that give rise to the different empirical outcomes regarding the risk-return relationship. The first is the type of frequency of return data, particularly high frequency data which is a source of unaccountable noise (Khan *et al*., 2016) [49]. Second, the proxy for expected returns cannot be equivalent to historical returns (Koutmos, 2012) [51]. Third, the use of historical returns to represent expected returns should contain a sample period that is at least 200 years (Maneemaroj *et al*., 2019) [60]. Fourth, the risk associated with return is due to information, and the reaction of investors in response to good and bad news are not the same (Yu *et al*., 2018). Finally, the return distribution is asymmetric and heavy tailed (Herbert *et al*., 2018) [38]. The first three factors surround the returns variable and the latter two are with respect to model specifications in capturing risk (Maneemaroj *et al*., 2019) [60].

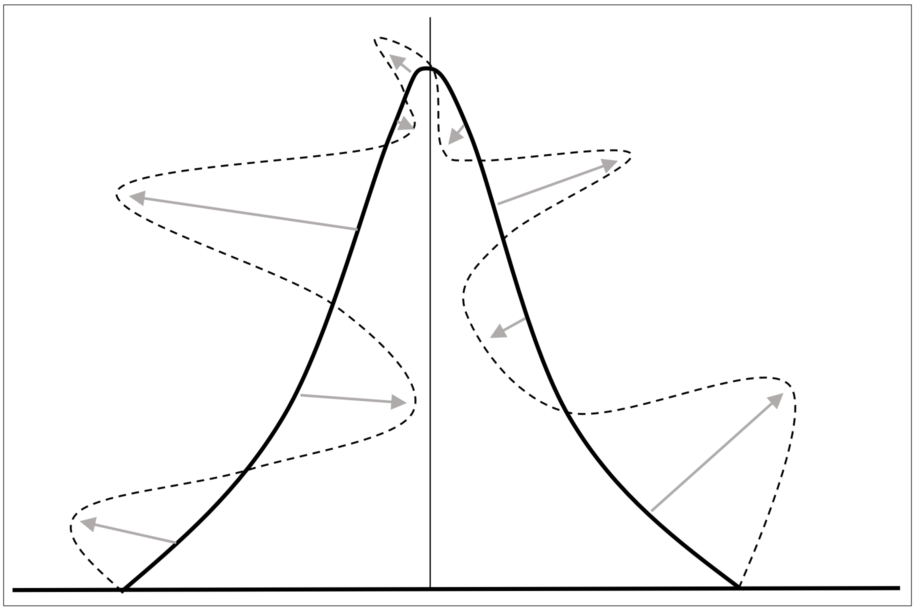
According to Jensen and Maheu (2018) [41], conventional methods found in existing literature that typically use the GARCH approach and regression analysis may be misestimating risk, contributing to the problem of inconclusive results. The novel nonparametric Bayesian approach overcomes the problems presented by parametric methods.

From the review of international empirical literature, it can be seen that the risk-return relationship is investigated widely by a number of studies (Savva and Theodossiou, 2018) [74]. However, the results vary a great deal since many studies show a positive or negative, nonlinear or linear, significant or insignificant relationship. This confirms the inconclusive empirical backing to the theoretical risk-return relationship (Maneemaroj *et al*., 2019) [60]. It can further be seen that despite the twenty-year gap, from 1988 to 2018, the same line of conventional quantitative finance methods and econometric models have been typically used in the investigation of the risk-return relationship (Savva and Theodossiou, 2018) [74]. However, recently there has been an inclination toward more nonlinear and nonparametric approaches, particularly an inclination to more mathematical and statistical based models, according to the studies by Demirer *et al*. (2019) [21] and Jensen and Maheu (2018) [41].

That is, complex mathematical and statistical theories are transformed into relatively simple and practical computational methods where one can easily obtain results (Bartlett and Keogh, 2016) [7]. This is made possible as a result of technological advancements and relevant up to date software (Karabatsos, 2017) [47]. Take the Bayesian method which originated by Bayes (1763) [8] for example; this method consists of intense mathematical integration which refers to the process of averaging out the uncertainty surrounding a variable. In nonparametric Bayesian modelling, this is considered as a golden standard method and is increasingly being used in different fields, mainly because of computational ease (Karabatsos, 2017) [47]. During the investigation of the risk-return relationship, it is noted that there is always some source of variability to take into account as this affects risk estimation (Cenesizoglu and Reeves, 2018) [14].

The theories by Maneemaroj *et al*. (2019) [60] and Jensen and Maheu (2018) [41], are highlighted to the extent of relevance on the area of concern in the empirical review. The limitations of the models in conjunction with the two theories allow for a single problem to be highlighted which is "return exposure" or "return risk."

Volatility arises from changes in price data as a result of the reaction and response of investors to news (Hussain *et al*., 2019) [40]. Price movements usually occur due to various factors such as volatility feedback, the leverage effect, inefficient information, behavioral biases and different investor sentiment. Essentially, resulting in a nonlinear and asymmetric return distribution since returns are derived from the price data which deviate from their fundamental value, as supported by Gyldberg and Bark (2019) [33]. Since the return distribution is strongly linked to risk, the variability found in price data can lead to misestimating risk. Therefore, although various studies investigate various sources of risk arising from macroeconomic and financial factors, the "return exposure" may be overlooked. That is, the fundamental risk that arises from the asymmetric nature of returns. This can be a major contributor to misestimating risk, contributing to the inconclusive results of the empirical risk-return relationship. Figure 4 illustrates the asymmetric nature of a return's distribution.



Source: Authors own

Fig 4: Asymmetric nature of returns

From Figure 4, the symmetric and bell-shaped curve represents the fundamental values of price data. The arrows represent the respective increase and decrease movements in price data as a result of various factors. This includes volatility, volatility feedback, the leverage effect, inefficient information, behavioral biases, different investor sentiment and so on. Consequently, this results in an asymmetric distribution of price data as shown by the broken grey line due to the random price movements. Hence, an asymmetric return distribution since returns is derived from price data. By accounting for this risk, due to the asymmetric nature of returns, this provides a more efficient measure of risk and fundamentally addresses the omitted variable bias by Kim and Kim (2018) [50].

The relationship between a return distribution and risk distribution may be linear or nonlinear as well as positive or negative, in relation to one another (Aboura and Chevallier, 2018) [1]. However, the very nature of a return distribution on its own is nonlinear, more so when taking on a higher level of risk which yields greater price movements (Hussain *et al*., 2019) [40]. Therefore, a major contrast in this study is the investigation of the conditional mean of excess returns, instead of the traditional conditional variance typically used by the GARCH approach (Jensen and Maheu, 2018) [41]. Although excess returns are a source of risk, emphasis is placed on the asymmetric nature of returns. That is, the "return exposure" independent or dependent, due to taking on a higher level of risk. A greater risk, by the excess returns measure, simply lends greater exposure due to greater price movements (Hussain *et al*., 2019) [40].

Most importantly, this measure is latent and stochastic or random in nature and cannot be observed directly (Harris, 2017) [34]. Thus, a certain level of uncertainty is attached to this measure (Jensen and Maheu, 2018) [41]. Consequently, certain models that do not have the appropriate model specification/s heavily contributes to misleading results due to not accounting for this measure of risk (Jin, 2017) [42]. Specifically, regression analysis, the VAR framework, causality tests and GARCH approach are regarded as irrelevant in estimating the risk-return relationship (Jensen and Maheu, 2018) [41]. Due to these methods shortcomings, limitations and model misspecifications, they are prone to misestimating risk, heavily contributing to misleading results (Savva and Theodossiou, 2018; Jensen and Maheu, 2018; Jin, 2017; Karabatsos, 2017) [42, 41, 47, 74]. The ongoing debate regarding the risk-return relationship can benefit by recognizing that these methods are limited and that advancements have been made in literature to deal with such issues. At the same time, encourage unconventional Bayesian and nonparametric methods that are robust, efficient and effective in risk estimation (Demirer *et al*., 2019; Jensen and Maheu, 2018; Jin, 2017; Karabatsos, 2017) [21, 47, 42, 41].

The Bayesian approach has the ability to average out uncertainty affecting parameters and the nonparametric approach has the ability to account for every possible risk-return relationship (Waldmann, 2018) [84]. This provides improved risk estimation which ensures a credible estimation of risk and thus the risk-return relationship (Demirer *et al*., 2019) [21]. Further, in a nonparametric framework, the robustness of any model is enhanced as model misspecifications are corrected (Apergis *et al*., 2017) [5]. As a result, there is no need for model extensions, specifications and accounting for different sources of variability to address the omitted variable bias (Kim and Kim, 2018) [50]. The actual nature of the data is modelled, thus, allowing "the data to speak for itself" and state the relationship held (Jensen and Maheu, 2018) [41]. This is in line with Bekiros *et al*. (2017) [9], who finds that the "actual returns are the most important factors" in the context of investigating the risk-return relationship. Thus, to account for these risks, this study the highlights Bayesian approach by Jensen and Maheu (2018) [41], in the context of the nonparametric approach.

3. Bayesian approach

3.1. Background

According to Herath (2019) [37], classical or frequentist statistics is often used in the conventional approaches of quantitative finance where theory is either accepted or rejected based on the empirical results. On the other end of the spectrum, is Bayesian statistics which is used for estimation, inference and modelling of data where the theory and empirical model are closely related. This is made possible by accounting for prior information. Bayesian statistics is an extensive field of study built on Bayes (1763) theorem which is the probability estimation of a relationship given prior information. Although most researchers may not use Bayes theorem directly, the underlying idea of the concept is fundamental to aid one's understanding. That is, in terms of conditioning variables, how the probability of one variable, representing a relationship, theory or event, affects the probability of another (Hatjispyros *et al*., 2019) [35]. In addition, the updating of existing theories as additional data becomes available (Cai, 2018) [12].

The inability of the classical or frequentist approach to consider prior information suggests an inflexible approach (Herath, 2019) [37]. The empirical results of a classical approach are often presented in the form of *p*-values or confidence intervals, whereas the Bayesian approach presents a posterior parameter estimate (Wagenmakers *et al*., 2018) [83]. The posterior refers to an updated probability estimation as will be defined in the next section of 3.2. According to Brooks (2014) [11], a conventional 95% confidence interval defines a range of values that one can be 95% certain contains a parameter estimate. Wagenmakers *et al*. (2018) [83] notes that the confidence interval procedure is limited as one cannot specify the interval bounds and then find out the probability or confidence that the parameter estimate lies within that specified interval.

In contrast, to a regular confidence interval, is a Bayesian interval which is also known as a credible or density interval (Karabatsos, 2017; Jensen and Maheu, 2018) [47, 41]. A credible interval has two advantages by Wagenmakers *et al*. (2018) [83]. First, a credible interval accounts for conditional prior information. This leads to the second advantage, which means that the parameter estimate is a posterior parameter estimate where the data has updated to given information. To aid understanding this critical difference in data estimation, a classical approach can be thought of as "pre-data," whereas a Bayesian approach is more of a "post-data" estimate due to taking into account prior information including uncertainty.

One of the main advantages of a Bayesian approach over a frequentist approach is the ability to average out uncertainty surrounding a parameter (Waldmann, 2018) [84]. In the context of risk estimation, according to Aliu *et al*. (2017) [4], the probabilities of possible future outcomes can be estimated given prior information. Meaning, risk allows an individual to have some probability of knowledge, whereas in contrast, uncertainty does not. Therefore, a method that has the ability to account for uncertainty immediately suggests a more robust and informative measure of risk (Herath, 2019) [37].

This is made possible by the fact that a Bayesian approach introduces parameters as random variables instead of a number or fixed value, such as returns as a linear function of volatility, in a parametric model (Kim and Kim, 2018) [50]. Structural breaks are treated the same way, allowing for the parameters to change in relation to these breaks in a quantitative rather than qualitative manner (Wang and Tsay, 2018) [85]. In contrast, the traditional approach by the GARCH school of modelling is to use dummy variables to account for extreme events (Mandimika and Chinzara, 2012) [59]. Thus, not only does the Bayesian approach has a greater ability to capture extreme events but accounts for the uncertainty associated with the random stochastic nature of the variables (Cai, 2018; Gong *et al*., 2019) [12, 29].

Literature highlights the Bayesian approach as a novel approach used in a number of fields and real-life practical situations such as the medical field, psychology and economics (Karabatsos, 2017; Wagenmakers *et al*., 2018; Herath, 2019) [47, 83, 37]. The fundamentals of a Bayesian approach remain the same whether the model is simple or complex because the common important feature is the posterior estimate (Wagenmakers *et al*., 2018) [83]. When the posterior estimate cannot be determined analytically, it can be drawn from computational sampling techniques such as Markov Chain Monte Carlo (MCMC) methods (Herath, 2019) [37]. MCMC is often used to derive a probability estimation of a density given limited information about the distribution (Martino *et al*., 2018; Gu *et al*., 2019; Griffin *et al*., 2018) [62, 31].

This development of MCMC methods has been made possible as a result of technological advancements and relevant up to date software (Herath, 2019) [37]. According to Karabatsos (2017) [47], MCMC has been specifically designed to fit Bayesian models which are uniquely beneficial from conventional quantitative finance methods which address a number of shortcomings. This includes parametric models such as regression analysis, the VAR model, causality tests and the GARCH approach, as highlighted in the empirical review. The application of Bayesian and MCMC methods in the fields of psychology and medicine demonstrates its level of usefulness in the real-world due to its practicality and effectiveness (Nishiura *et al*., 2020; van Doremalen *et al*., 2020; Wagenmakers *et al*., 2018; Karabatsos, 2017) [43, 47, 83].

In conclusion, the Bayesian approach is suitable for models that understand the complexity of financial data, especially the nature of returns which has a nonlinear, asymmetric, volatile, stochastic and latent nature (Wagenmakers *et al*., 2018) [83]. Thus, it is only fair to apply this method to the field of finance to improve conclusive findings.

3.2. Research design

There are three specific reasons by Ferson (2005) [27], where one can use the Bayesian method for a scientific analysis which is applicable in the context of the risk-return relationship. Firstly, it provides a framework to structure a problem where there could exist the following three sub-challenges:

- First, there is a lack of existing literature regarding the subject of interest (Ferson, 2005) [27]. For example, the risk-return relationship topic in South Africa relative to other countries, respectively (Savva and Theodossiou, 2018) [74]. Although South Africa is the largest market in Africa, the investigation of the risk-return relationship is relatively limited. This is in terms of volume over the years as highlighted by Mandimika and Chinzara (2012) [59], and the methods employed by Steyn and Theart (2019) [78]. In contrast, international empirical literature, particularly for the developed countries, have more literature as documented in the study by Savva and Theodossiou (2018) [74].
- Second, there may be a need to incorporate a probabilistic approach rather than a deterministic one (Ferson, 2005) [27]. Return exposure has a stochastic and latent nature which means that it can be statistically analysed but not necessarily forecasted with certain precision (Harris, 2017; Jin, 2017) [34, 42].
- Third, there exists a substantial amount of uncertainty surrounding the parameters and model (Ferson, 2005) [27]. With respect to the parameters, due to the nature of returns exposure, there is a certain level of uncertainty attached to the variable returns. In terms of the model, this can be shown by previous risk-return empirical studies that use methods that do not effectively account for return exposure. Specifically, the conventional parametric models that are not designed to handle the asymmetric nature of returns and the uncertainty associated to the variable (Jin, 2017) [42]. Thus, affecting the estimation of risk and contributing to the inconclusive results of the risk-return relationship (Jensen and Maheu, 2018) [41].

Secondly, the ability to estimate probability distributions which are made up of two parts, namely, priors and posteriors (Hatjispyros *et al*., 2019) [35]. A prior is an initial probability estimation based on existing information (Goudarzi *et al*., 2019) [30]. A prior has the ability to update given the availability of more data by means of a likelihood function which consists of new observed data (Karabatsos, 2017) [47]. The combination of a prior and likelihood by means of model estimation results in the posterior which is an updated probability estimation (Cai, 2018) [12].

With respect to the nonparametric Bayesian approach in this study, the prior is estimated by the Bayesian Dirichlet Process by Ferguson (1973) [26], derived by the stick-breaking process by Sethuram (1994) [76]. The posterior is estimated by a slice sampler by Kalli *et al*. (2011) [45] and a Gibbs sampling technique. This follows Jensen and Maheu (2018) [41], which is the first and only study to apply the nonparametric Bayesian approach to the risk-return relationship and volatility feedback topic, to the best of the authors knowledge. These are golden standard nonparametric Bayesian methods (Dirichlet Process, slice and Gibbs sampler) which involve random sampling methods whereby every distribution has an equal chance of being drawn. These methods suggest low levels of bias and systematic error, and a high level of reliability, validity and viability (Etikan and Bala, 2017) [24]. Thus, ensuring accurate estimates and reliable results (Karabatsos, 2017) [47].

Thirdly, choosing and estimating the parametric or nonparametric approach to accompany the Bayesian model (Demirer *et al*., 2019) [21]. By definition, a parametric model refers to a set number of parameters with respect to the sample size (Jin, 2017) [42]. This is in contrast to a nonparametric model where the number of parameters increases as the sample size increases (Apergis *et al*., 2018) [6]. In other words, this means that as more data becomes available, the number of parameters increases, allowing for a greater number of possibilities (Demirer *et al*., 2019) [21]. Further, the likelihood function of new observations can be captured due to the access or availability of additional data (Cai, 2018) [12]. Essentially, the nonparametric approach implies a model free approach, where this study highlights the normality assumption being relaxed, allowing for an array of asymmetric properties (Jensen and Maheu, 2018) [41].

In the context of this study, according to Karabatsos (2017) [47], a nonparametric Bayesian model is often referred to as being an infinite-mixture model. An infinite-mixture model describes a model that takes into account an infinite number of clusters. The cluster is a component of a mixture of, in this case, weights and parameters. The nonparametric Bayesian model assumes an infinite number of clusters, whereas the parametric Bayesian model assumes a finite number of clusters. As a result, the nonparametric Bayesian model is the more robust model, due to having greater flexibility in effectively accounting for higher moment asymmetric forms of the risk-return relationship, in an infinite sample space. A sample space refers to the number of possible outcomes of a random variable. That is, the nonparametric approach is designed to effectively account for an infinite sample space, whereas a parametric approach is limited to a finite sample space by definition, as drawn from Jensen and Maheu (2018) [41] and Karabatsos (2017) [47].

3.3. Quantification of return exposure

The variables of interest (specifically returns) are defined, motivated and then quantified. The choice of the risk and return variables are excess returns (r_t) and realized variance (R), in line with the studies by Jensen and Maheu (2018) [41] and Kim and Kim (2018) [50].

Excess returns are defined as the returns obtained due to taking on a higher level of risk by definition. Excess returns are synonymous with abnormal returns and the risk premium which refers to the risk-return relationship (He *et al.*, 2018). The choice of excess returns over standard returns is motivated by the application of Bayes (1763) [8] theorem to the risk-return relationship. Equation 1 is statistically defined as the conditional mean probability of r given R, which is equal to the joint probability of r and R, divided by the unconditional probability of R:

$$P(r|R) = \frac{P(r \cap R)}{P(R)} \quad (1)$$

where: P(r|R) is the probability of r conditional on R
P(r ∩ R) is the joint probability of r and R
P(R) is the unconditional probability of R
Equation 2 is derived from the cross multiplication of Equation 1:

$$P(r \cap R) = P(r|R) * P(R) \quad (2)$$

Given that r is defined as return and R is defined as risk, Equation 2 is defined as the probability estimation of the relationship of risk and return which is equal to the risk premium (return given risk) and risk. In the context of this study, the probability estimation of the risk-return relationship is equal to the relationship between excess returns and realized variance. The use of a risk-based measure of returns, emphasizes capturing return exposure.

Excess returns refer to the returns earned due to taking on a higher level of risk (He *et al.*, 2018). However, return exposure is a return inherent risk that arises from the fundamental nature of returns, independent or dependent, due to taking on a higher level of risk. A greater risk simply lends greater exposure due to greater price movements (Hussain *et al.*, 2019) [40]. Essentially, in comparison to standard returns, excess returns lends greater return risk exposure to be captured, ultimately improving risk estimation, in line with Jensen and Maheu (2018) [41].

4. Conclusions

The risk-return relationship holds fundamental importance to the fields of finance and economics as well as useful information to various market participants. Due to conflicting results over the years, this has caused an ongoing debate to arise. There are a number of factors and theories that attempt to explain the magnitude of varying results which motivated the pursuit of this research. From a broad analysis, results can easily vary, from study to study, as a result of different choices such as data frequency, sample period and model specification, as noted by Savva and Theodossiou (2018) [74]. However, this study identified a trend in the use of conventional methods over a twenty-year gap, despite the drawbacks of the models being highlighted in literature. The foremost being the parametric GARCH approach which is subject to a number of nonnegativity constraints, limited in its ability to capture asymmetric properties and fully capture risk. This heavily contributes to the problem of inconclusive results regarding the risk-return relationship, thus, offering no conclusive solution to the ongoing debate.

When it came to modelling the risk-return relationship, there was a progression in the methods applied to the risk-return relationship in international literature. At first, the majority of studies applied the GARCH approach based on its conventional use such as Chou (1988) [18], Park *et al.* (2017) [16] and Savva and Theodossiou (2018) [74]. Then in recent years, a number of studies began using the nonparametric approach in conjunction with conventional methods. This was in order to derive the benefits of a nonparametric approach such as accounting for asymmetry and model misspecifications (Apergis *et al.*, 2018; Demirer *et al.*, 2019) [21, 6]. The nonparametric approach was applied to methods of interest such as the VAR model by Umutlu (2019) [80] and causality tests by Apergis *et al.* (2018) [6]. Additionally, more unconventional methods were introduced such as the unified framework by Kim and Kim (2018) [50] and the nonparametric Bayesian approach by Jensen and Maheu (2018) [41].

In the build up to this research's objective, the limitations of conventional methods such as regression analysis, VAR, causality tests and the GARCH approach were highlighted. It was noted that a number of studies individually recommended the nonparametric approach and Bayesian approach, respectively for more robust data estimation (Karabatsos, 2017; Jin 2017; Chang *et al.*, 2017; Waldmann, 2018; Wagenmakers *et al.*, 2018; Apergis *et al.*, 2018; Jensen and Maheu, 2018; Demirer *et al.*, 2019) [84, 83, 47, 42, 41, 21, 6].

The Bayesian approach has the ability to automatically adjust for sources of uncertainty and measurement errors surrounding parameters; thus, ensuring an efficient estimation of risk. The nonparametric approach has the ability to effectively account for asymmetric properties such as skewness, kurtosis and multiple modes in an infinite sample space. In direct contrast, to the design of the conventional parametric approach where the number of parameters is restricted to the sample size. Hence, the parametric model has an inability to account for every possible risk-return relationship that can hold, particularly higher moment asymmetric forms of the risk-return relationship.

The nonparametric framework is a "model free" approach where there are no assumptions or constraints imposed on the data. Model misspecifications are adjusted for and as a result, there is no need for model extensions, specifications and accounting for various sources of asymmetry. In contrast, to the GARCH family where a number of modifications have been made over the years to the standard GARCH (1, 1) model. However, despite these modifications, the drawbacks of the parametric approach essentially still hold such as the assumptions and nonnegativity constraints imposed on the data as well as the risk that simply remains uncaptured. This is because data analysis of real-world data often requires a method that relaxes parametric assumptions. Thus, allow for flexibility that enables the actual fundamental nature of data to be captured.

A model that satisfies these conditions is the nonparametric Bayesian approach by Jensen and Maheu (2018) [41]. The nonparametric Bayesian approach is a combination of the two most robust methods recommended by literature, respectively in the estimation of data with nonlinear, asymmetric, latent and stochastic properties (Karabatsos, 2017; Wagenmakers *et al*., 2018) [47, 83]. Consequently, this produces a powerful method for the estimation of the risk-return relationship. The methodology of Jensen and Maheu (2018) [41] made use of golden standard nonparametric Bayesian methods, namely, the Dirichlet Process, the slice sampler and Gibbs sampling technique.

To conclude, if a model can effectively estimate risk, there is no need for model extensions, specifications and omitted variables biases. This includes accounting for sources of asymmetry that seem manifold, considering there are so many factors and theories. This includes volatility feedback, the leverage effect, skewness, macroeconomic fundamentals, inefficient information, behavioral biases and different investor sentiment. Moreover, a model designed to capture nonlinear and asymmetric properties is more likely to effectively capture these properties and estimate a nonlinear risk-return relationship. Given the magnitude of international literature, the importance and ongoing debate regarding the risk-return relationship, this study offers a significant perspective and contribution.

In order to make a meaningful contribution, a study should not employ methods that can be considered irrelevant and obsolete, given the existence of more robust methods such as the nonparametric Bayesian approach by Jensen and Maheu (2018) [41]. According to, “Experience with real-world data, however, soon convinces one that both stationarity and Gaussianity are fairy tales invented for the amusement of undergraduates.” Thus, sophisticated and unconventional methods are encouraged as it can inspire a new perspective, a way of thinking and an approach to a problem. Additionally, a robust method is more likely to give a reliable result paving the way for progression in any field and topic.

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