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Modeling of the flow in a coastal aquifer: Comparison between confined and unconfined groundwater

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Abstract
In this work, we try to model the flow of water in a coastal aquifer in two cases: the case of a confined water table and the case of a groundwater table.

Keywords: hydrodynamics, aquifer, groundwater, confined water table, Darcy's law, flow equation, dif- fusivity equation

1. Introduction

An aquifer is a body of permeable rocks comprising a saturated zone sufficiently conductive of groundwater to allow the significant flow of an underground water table and the capture of an appreciable amount of wa- ter. A coastal aquifer is an aquifer located near a marine or oceanic coast, or even in direct contact with them. In this work, we will study this model in two cases. The first case consists of a coastal aquifer containing a confined water table. The second case is that of a coas- tal aquifer containing an unconfined groundwater table. They are both illustrated in the figures 1 and 2. In the first case, it is a question of studying a saturated po- rous medium. In the second case, it is a porous medium containing three zones, one unsaturated, a second fluc- tuation zone and a saturated deep zone. In addition, in both cases, we do not require any external power sup- ply of pumping or recharging type. We also consider Dupuit's hypothesis, namely that the flow is horizon- tal. Thus, the pressure is constant on the vertical axis of the x.

One of the problems of coastal aquifers is also known as marine intrusion. In fact, it is a natural pollution phenomenon carried out during the infiltration of salt water from the sea under the continent. This can generate soil salinization and vegetation alteration. The coastal water table model can generate an inverse problem where fresh water seeps into the sea and flows over the seawater. This phenomenon is generally illustrated in nature by the presence of two colors indicating the difference in the density of fresh and salty waters.

In the figures 1 and 2, three zones are present. The first one contains only fresh water. A second one contains fresh water and salt water. Finally, the last one contains only salted water. Also, two theoretical problems arise at the level of the second zone.

1. The first problem consists in identifying the interface or the transition zone that separates fresh water from salt water.
2. The second problem consists in determining the fresh water flow vector at the interface, which is assumed to be known this time.

These two subjects make it possible to study the evolution of a possible contamination of the soil by sea salt through the movement of water. As mentioned earlier, the effects of such a situation are very serious on the environment. In addition, it can lead to soil degradation and salinization as a result of irrigation with these waters. Nowadays, several coastal water tables

Are threatened around the world. We mention the El Jadida-Sidi Moussa tablecloth in Morocco, the Annaba-Oran ta- blecloth in Algeria and the El Haouaria tablecloth in Tunisia.

We have selected in the literature two approaches modeling this phenomenon. The first approach is the density approach [6]. We will assume the existence of two liquid phases: fresh water and salt water. In addition, we seek to identify the interface that sefparates these two phases. A classic analytical solution, namely the Ghyben-Herzberg [7] solution, in a uncon- fined groundwater table, is present in Appendix A of the book [5]. The second approach is the transport in solution [8] detailed in Appendix B of the book [5]. We consider only one liquid phase: water. We are trying to determine the concentration rate of the contaminant: salt. The equation obtained at the end of Appendix B has been the subject of a mathematical and numerical study in the book [4].

To solve the physical problems related to porous me- dia, we introduce the following five parameters: the quantity of the fluid is expressed by the mass M [kg]or the volume V [m3], the hydraulic load h(x, y, x, t) [m], the vector flow of the fluid q(x, y, x, t) expressed in [m/s] or [kg/s.m2] and the hydraulic conductivity K [m/s]. The fifth parameter depends on the nature of the water table. If it is confined then we will use the specific storage Ss(x, y, x, t) [m−1]. On the other hand, for a unconfined groundwater table,, the storage coefficient S(x, y, x, t) [m−1] is equal to the drainage porosity ωd. This porosity generally has values between 0.02 and 0.30. Among these parameters, there are three that are related to the flow media: the specific sto- rage or the drainage porosity (depending on the case), the hydraulic conductivity and the hydraulic load. The other two parameters are related to the nature of the flow: the quantity and the flow. We keep the same no- tation for the two cases studied.

The study of the two models go through the same ap- proach. First, a flow equation is calculated using Dar- cy's law and the continuity equation. In a confined groundwater table,, we speak of specific storage Ss. This parameter is intimately linked to the compressibility of the medium and the water. On the other hand, in an unconfined water table, these parameters do not exist; we then speak of a storage coefficient. In the unconfined water layers, the storage coefficient is equal to the effective porosity (gravity water); it is between 0.2 and

0.01. In captive tablecloths, it is much smaller, 0.001 to 0.0001. It is measured in the field by test pumping ope- rations which fold down the water table. These values are extracted from [9].

In the two cases studied, Darcy's law is the same: Darcy's law is strictly applicable only for homogeneous media where the flow of water is laminar. It cannot be used in particular for karst networks because they are considered as a heterogeneous medium. More generally, Darcy's law is written:

$$q = -K \text{ grad } h \tag{1}$$

Where K is the hydraulic conductivity.

Hypothesis 1 In the following, we consider the follo- wing hypotheses:

— neglect of the compressibility of the water and that of the porous medium.

Dupuit's hypothesis: all speeds are horizontal and parallel to each other on the same vertical.

the permeability tensor admits the vertical as one of its main directions. This causes the absence of the vertical load from where h(x, y) is independent of x.

The last hypothesis is realized when the coast of the free surface of the unconfined groundwater table, moves away from the outlets or the ridge of the water table.

2. Coastal aquifer with a confined water layer

In this work, the model is presented in the figure 1. As seen in the book [5], the study of the flow of water

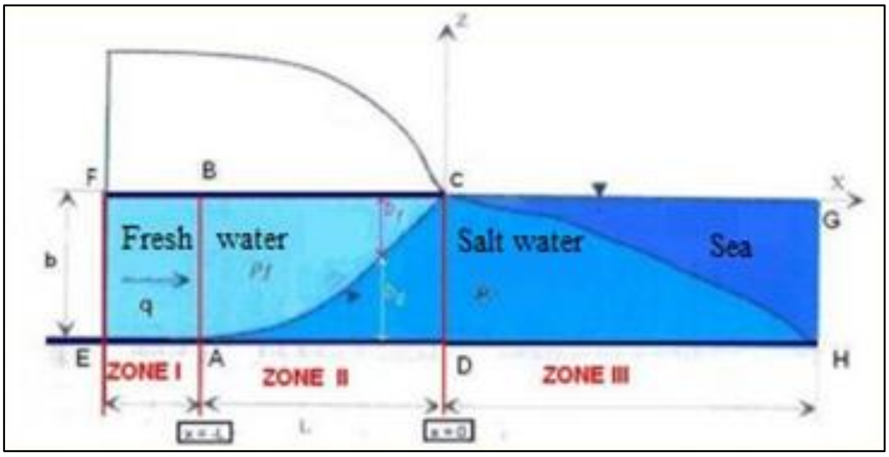

Fig 1: Coastal aquifer with a confined water layer

Assumes that the porous media are saturated and isothermal. First, the simplified special case of the figure 1 is modeled by the flow equation. The demonstration involves two basic equations: the continuity equation and Darcy's law. Then, a more general equation is presented describing a diffusive original flow. His demonstration exists in [8].

2.1. Flow equation

Theorem 2 In an isothermal saturated porous medium, the flow equation is:

$$S_s \frac{\partial h}{\partial t} = div(K\ grad\, h) \quad \forall x\ \forall t \tag{2}$$

Demonstration 3 (Theorem 2:) Let's start by recalling the law of conservation of mass, in our case. It corresponds to the continuity equation expressed by the following PDE:

$$\frac{\partial \rho n}{\partial t} + div(\rho n \vec{u}(x,t)) = 0 \forall x, \forall t \tag{3}$$

Now the interstitial velocity is related to the flow in a saturated porous medium via the relation: $q = n\vec{u}$. Therefore, we find:

$$\frac{\partial \rho n}{\partial t} + div(\rho q) = 0 \forall x, \forall t \tag{4}$$

By combining Darcy's law 1 and equality 4, we find:

$$\frac{\partial \rho n}{\partial t} = div(\rho\, K\, grad\, h) \quad \forall x,\ \forall t \tag{5}$$

We will now refine the first member of the equation. On the one hand, we have:

$$\begin{aligned} \frac{\partial}{\partial t}\rho n &= \rho \frac{\partial n}{\partial t} + n \frac{\partial \rho}{\partial t} && \forall x,\ \forall t \\ &= \rho\left(\frac{\partial n}{\partial p} + \frac{n}{\rho}\frac{\partial \rho}{\partial p}\right)\frac{\partial p}{\partial t} && \forall x,\ \forall t \end{aligned} \tag{6}$$

Now, the compressibility of the medium α and the compressibility of the fluid β are expressed by

$$\begin{aligned} \alpha &= \frac{-1}{1-n}\frac{\partial n}{\partial p} \\ \beta &= \frac{1}{\rho}\frac{\partial \rho}{\partial p} \end{aligned} \tag{7}$$

Then:

$$\frac{\partial \rho n}{\partial t} = \rho((1-n)\alpha + n\beta)\frac{\partial p}{\partial t} \quad \forall x,\ \forall t \tag{8}$$

On the other hand, the hydraulic load is expressed by:

$$h = z + \frac{p}{\rho g} \tag{9}$$

By drifting with respect to time:

$$\frac{\partial h}{\partial t} = \frac{\partial z}{\partial t} + \frac{1}{\rho g}\frac{\partial p}{\partial t} \tag{10}$$

Since z is constant over time, we find:

$$\rho g \frac{\partial h}{\partial t} = \frac{\partial p}{\partial t} \tag{11}$$

Finally,

$$\frac{\partial \rho n}{\partial t} = \rho S_s \frac{\partial h}{\partial t} \quad \forall x,\ \forall t \tag{12}$$

Where

$$S_s = \rho g((1-n)\alpha + n\beta) \tag{13}$$

Is the specific storage coefficient. By combining the equation (12) and the equation (5), we obtain the equation of the flow:

$$S_s \frac{\partial h}{\partial t} = div(K\ grad\, h) \quad \forall x\ \forall t \tag{14}$$

2.2 diffusivity equation: general case

In chapter 2 of the book [5], the study of the flow of water in saturated or unsaturated porous media involves three basic equations: the continuity equation, Darcy's law and the isothermal equation of state of the fluid.

Theorem 4 The confined layer diffusivity equation is

$$div\left[\bar{\bar{K}}\,(grad\,p + \rho\, g\, grad\, z)\right] = \rho\omega g\left[\beta_l - \beta_S + \frac{\alpha}{\omega}\right]\frac{dp}{dt}\,. \tag{15}$$

The coefficient Ss = ρω g hβl − βS + α ω i is called the specific storage coefficient of the groundwater table,. βS is of the order of 1 25βl. It is often overlooked in the calculations. On the other hand, α and βl are of the same order of magnitude, which is about 5 × 10−10MKS. More simplified expressions exist in the book [8].

Demonstration 5 The demonstration contains 6 steps:

1. Equation of continuity of the fluid in an elementary volume.
2. Equation of continuity of the flow of solid grainsin the elementary volume.
3. Darcy's law.
4. Combination of the three equalities.
5. The equations of state of the liquid and the solid.
6. Synthesis: diffusivity equation. Simplifications.

The demonstration is extracted from the book [8].

3 Coastal aquifer with a unconfined ground water layer

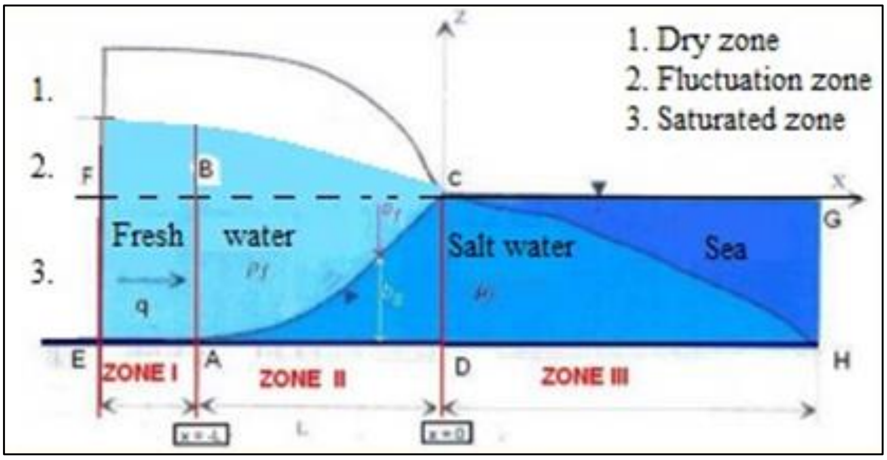

Fig 2: Coastal aquifer with unconfined ground water table

An unconfined water layer in a porous medium that is saturated only in one part of the aquifer, the other part is unsaturated, or even dry. The groundwater table, is generally limited by an impermeable substrate from below. The height of the layer coincides with the piezometric height. It traces the free surface of the water table. A unconfined table water can be fed by springs or rivers or the sea or even by rain. Likewise, the height of the unconfined groundwater table, may decrease due to evaporation or by well drilling.

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In the modeling of a unconfined groundwater table the compressibility of the water can be neglected, as well as that of the porous medium. Indeed, as descry bed above, a variation in the hydraulic load causes a movement of the free surface; by saturating or desaturating the porous media. In other words, the variation of the load causes a storage or destocking of the water in the aquifer. From a mathematical point of view, a piece of elementary volume of the variable free surface must be taken into account in the continuity equation, according to the author of the book [8].

I n this study, we consider the unconfined coastal groundwater table, of the figure 2. In addition, we neglect any external power supply of pumping or recharging type.

3.1 The flow equations in the unconfined water table

As for captive groundwater, the study of the flow of water in unconfined groundwater is modeled by two equations: first, the flow equation; then, the diffusivity equation. These two cases involve two basic equations: the continuity equation and Darcy's law. The medium is considered isothermal. The demonstration of the second model exists in [8].

Theorem 6 The flow equation in the saturated zones of an unconfined water layer, is:

$$\omega_d \frac{\partial h}{\partial t} = div(K\ grad\, h) \quad \forall x\ \forall t \tag{16}$$

With ωd the drainage porosity.

Demonstration 7 (Theorem 6) Law of conservation of mass:

The mass equation is:

$$M(t) = \int_{x\in\Omega} \rho n d\omega(x) \tag{17}$$

Where ρ is the density of the water, n the porosity, dω(x) an element of volume, x ∈ R3. The global mass conservation equation without mass source terms is

$$\frac{\partial M(t)}{\partial t} = 0 \tag{18}$$

By combining the two equations above, we find:

$$\frac{\partial M(t)}{\partial t} = \int_{x\in\Omega} \frac{\partial \rho n}{\partial t} d\omega(x) - \int_{x\in\Sigma} \rho n (V\cdot n(x)) d\sigma(x) = 0 \tag{19}$$

The negative sign in front of the second integral is explained by the outgoing normal. Applying Green's formula to the second integral, we finally find:

$$\int_{x\in\Omega} \left(\frac{\partial \rho n}{\partial t} + div(\rho n V(x,t))\right) d\omega(x) = 0 \tag{20}$$

This is valid for any fixed t and for any volume Ω(t) obtained from a given initial volume Ω(t0). By making Ω(t) tend towards an infinitesimal volume, we obtain the following EDP:

$$\frac{\partial \rho n}{\partial t} + div(\rho n V(x,t)) = 0 \forall x, \forall t \tag{21}$$

Darcy's Law

A demonstration similar to that of the case of a confined groundwater table, is repeated. Indeed, by combining Darcy's law in a unconfined water table and the equality 21, we find:

$$\frac{\partial \rho n}{\partial t} = div(\rho\, K\, grad\, h) \quad \forall x,\ \forall t \tag{22}$$

On the other hand, the hydraulic load is expressed by:

$$h = z + \frac{p}{\rho g} \tag{23}$$

By drifting with respect to time:

$$\frac{\partial h}{\partial t} = \frac{\partial z}{\partial t} + \frac{1}{\rho g}\frac{\partial p}{\partial t} \tag{24}$$

Since z is constant over time, we find:

$$\rho g \frac{\partial h}{\partial t} = \frac{\partial p}{\partial t} \tag{25}$$

Finally,

$$\frac{\partial \rho n}{\partial t} = \rho n \frac{\partial h}{\partial t} \quad \forall x,\ \forall t \tag{26}$$

By combining the equation (26) and the equation (22), we obtain

$$n\rho \frac{\partial h}{\partial t} = \rho div(K\ grad\, h) \quad \forall x\ \forall t \tag{27}$$

However, for a unconfined groundwater table,, the storage coefficient S is equal to the drainage porosity ωd. This porosity generally has values between 0.02 and 0.30. Thus, the porosity coincides with the drainage porosity n = ωd. In addition, it is assumed that the porous medium is incompressible and therefore ρ is constant.

The flow equation:

$$\omega_d \frac{\partial h}{\partial t} = div(K\ grad\, h) \quad \forall x\ \forall t \tag{28}$$

Theorem 8 The flow equation for the unsaturated zones in a unconfined groundwater, is:

$$-\rho g \frac{d\Theta}{d\psi}\frac{\partial h}{\partial t} = div(K(\Theta)\ grad\, h) \quad \forall x,\ \forall t \tag{29}$$

With Θ the water content.

Demonstration 9 (Theorem 8) In the partially saturated zone, the volume of the water in the pores has different properties from that of the empty pores. Thus, it is assumed that the air phase is immobile in unsaturated soils and the movement is calculated only in the water phase. In the second chapter of the book [5], the notion of water content was introduced via the variable Θ, to solve this problem.

Darcy's equation

$$\vec{u} = -K(\Theta)\ grad\, h \tag{30}$$

Where K is the hydraulic conductivity. The load has the same expression:

$$h = \frac{p}{\rho g} + z \quad (31)$$

The water pressure is negative.

The law of conservation of mass
The mass equation is:

$$M(t) = \int_{x\in\Omega} \rho\Theta d\omega(x) \quad (32)$$

Where ρ is the density of water, Θ the water content, dω(x) an element of volume, x ∈ R3. The global mass conservation equation without mass source terms is

$$\frac{\partial M(t)}{\partial t} = 0 \quad (33)$$

By combining the two equations above, we find:

$$\frac{\partial M(t)}{\partial t} = \int_{x\in\Omega} \frac{\partial \rho\Theta}{\partial t} d\omega(x) - \int_{x\in\Sigma} \rho(V\cdot n(x))d\sigma(x) = 0 \quad (34)$$

The negative sign in front of the second integral is explained by the outgoing normal. Applying Green's formula to the second integral, we finally find:

$$\int_{x\in\Omega} (\frac{\partial \rho\Theta}{\partial t} + div(\rho V(x,t)))d\omega(x) = 0 \quad (35)$$

It is assumed that the medium is incompressible and that ρ is constant. Therefore,

$$\int_{x\in\Omega} (\frac{\partial \Theta}{\partial t} + div(V(x,t)))d\omega(x) = 0 \quad (36)$$

This is valid for any fixed t and for any volume **Ω**(t) obtained from a given initial volume **Ω**(t0). By making **Ω**(t)tend towards an infinitesimal volume, we obtain the following PDE:

$$\frac{\partial \Theta}{\partial t} + div(V(x,t)) = 0 \forall x, \forall t \quad (37)$$

Flow equation
By combining Darcy's law in a unconfined water table and the equality 37, we find:

$$\frac{\partial \Theta}{\partial t} = div(K(\Theta)\, grad\, h) \quad \forall x, \ \forall t \quad (38)$$

Thanks to an experimental relationship, called suction-water content, we can assume the existence of a function ψ (Θ) such that

$$\begin{aligned} d\Theta &= \frac{d\Theta}{d\psi} d\psi \\ &= -\frac{d\Theta}{d\psi} dp \end{aligned} \quad (39)$$

As the pressure variation is related to the load variation at a given point by,

$$dp = \rho g dh \tag{40}$$

We finally get,

$$-\rho g \frac{d\Theta}{d\psi}\frac{\partial h}{\partial t} = div(K(\Theta)\ grad\, h) \quad \forall x,\ \forall t \tag{41}$$

This flow equation is nonlinear. It is solved only numerically. For more details, the reader can refer to [8].

3.2 diffusivity equation: general case

Theorem 10 The unconfined groundwater diffusivity equation is

$$\frac{\partial}{\partial x}\left[\int_\sigma^h K_{xx} dz.\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[\int_\sigma^h K_{yy} dz.\frac{\partial h}{\partial y}\right] = \omega_d \frac{\partial h}{\partial t} + Q \tag{42}$$

Demonstration 11 The demonstration contains 4 steps

1. Expression of the incoming mass flow.
2. Variation of the mass of the element.
3. The volume flow rate of fluid sampled.
4. The assessment and the conclusion.

The demonstration is extracted from the book [8].

4 Conclusion

In this article, the author studies a particular case of an aquifer along a bed of salt water, it can be a sea, an ocean or a lake of water. The objective is to model the flow of fresh water in the aquifer in order to predict the direction of salt infiltration and therefore, to prevent environmental pollution. Mathematically, this is expressed by the flow equation. In this work, we have presented two cases: That of a unconfined groundwater table and that of a confined groundwater table.

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