International Journal of Multidisciplinary Research and Growth Evaluation.

Fourier transform, fast Fourier transform, wavelet transform application on emotions

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Article Info

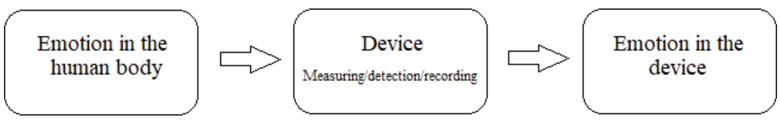
ISSN (online): 2582-7138
Volume: 05
Issue: 04
July-August 2024
Received: 28-06-2024
Accepted: 05-08-2024
Page No: 1194-1197

Abstract
Neurons are the seat of movements of charged particles, which create microcurrents. Thanks to devices such as the EEG and the MEG, these brain electrical waves can be recorded on the surface of the skull. The frequency of these waves reflects the state of alertness and consciousness. The mathematical tools used to study emotions as an electromagnetic signal are diverse. Indeed, electromag- netic waves have been studied from different angles. The mathematical modeling of such a phenomenon leads back to the resolution of the system formed by the equation of Maxwell, Faraday, Gauss and Am- père. For this, we need the theory of the Fourier transformation. However, a study of an EEG graph, will lead a numeritian to be interested in the analysis of a signal and therefore in the theory of the discrete Fourier transformation and its algorithm the fast Fourier transformation. As the brain waves are repre- sented by functions corresponding to small oscillations, it is possible to use the analsye of the wavelet transformation.

Keywords: Fourier transformation, Discret Fourier transformation, Fast Fourier tranformation, Wavelet transformation, Emotion

1. Introduction

There are several mathematical models for studying emotions and emotional activity. The theoretical study of these models uses several mathematical and computer tools. In this article, the author is interested in Fourier analysis. Indeed, this branch evolved a lot during the time from 1810 with Joseph Fourier until 1980 with Yves Meyer. Indeed, in this article, the author describes the different forms of transformations of approximation of functions: that of Fourier, the DFT / FFT and finally the wavelet transformation. As well as their application in emotional signal processing, whether visual, graphic, electromagnetic or phonetic. In this context, it is necessary to differentiate between the study of emotional activity in the human body, its detection by a device and the processing of the recorded signal, as indicated in the figure 1:



Thus, in this article, we model the emotional activity in the human body by Maxwell's equations, as in the section 2. Indeed, emotions are generated by electro-chemical phenomena originating in the nervous system and hormonal glands. On the other hand, once recorded by devices such as cameras, microphones, EEG and MEG; emotions are considered as signals to be processed differently depending on the data obtained and the database used.This is no longer an absolute part of the mathematical field but rather of the field of computer algorithms, database and signal processing.

2. The Fourier transformation (FT)

This section is carried out thanks to the documents [6], [7], [8], [9] and [10].

2.1 Mathematical tools

Around 1810, Joseph Fourier states that any function can be decomposed as an infinite sum of the two sinusoidal functions sine and cosine by means of simple calculations.

Definition 1 *Fourier series*

Given a periodic function f: R C of period 2π and bounded. We call Fourier series of the formal series of the form:

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n cos(nwt) + b_n sin(nwt)) \tag{1}$$

The coefficients a0, an and bn are independent of time and are given by the following integrals :

$$a_0 = \frac{1}{T} \int_0^T f(t)\, dt \tag{2}$$

$$a_n = \frac{2}{T} \int_0^T f(t) cos(nwt)\, dt \tag{3}$$

$$b_n = \frac{2}{T} \int_0^T f(t) sin(nwt)\, dt \tag{4}$$

Remark 1 in complex notation, any physical periodic signal can be written with ν = 1 and

$$f_T(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{in2\pi\nu t} \tag{5}$$

with $\nu = \frac{1}{T}$ *and*

with $n \in \mathbb{Z}$.

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)\, e^{-in2\pi\nu t}\, dt \tag{6}$$

Theorem 1 Any physical signal can be decomposed into a Fourier integral, of the form

$$f(t) = \int_{-\infty}^{+\infty} \hat{f}(\nu)\, e^{-in2\pi\nu t}\, d\nu \tag{7}$$

where f̂(ν) denotes the Fourier transform of the signal. f̂(ν) is a continuous complex-valued function, defined by

$$\hat{f}(\nu) = \int_{-\infty}^{+\infty} f(t)\, e^{-in2\pi\nu t}\, dt \tag{8}$$

Remark 2 The theorem 1 concerns the summable square signals for which ∫R | f (t)| dt is finite, that is to say signals which transport a finite energy as is the case in physics.

Remark 3 The Fourier transform has limits :

1. it gives the quantity of each frequency present in the signal for the entire observation period. The Fourier theory therefore becomes ineffective for a signal whose frequency spectrum varies consid- erably over time.
2. It does not work well when it has to describe locally a function having discontinuities.

2.2 Application on emotions

The tissues of the head work like passive conductors, so the researchers consider that electric currents and magnetic fields behave in a stationary way at all times. Consequently, the brain electromagnetic fields (E, B) verify the following Maxwell's equations:

$$\begin{cases} \nabla \times E = 0 \\ \nabla . E = \frac{\rho}{\varepsilon_0} \\ \nabla \times B = \mu_0 J \\ \nabla \end{cases} \tag{9}$$

with

1. J the density of the current.
2. ρ the density of the load.
3. ε0 the magnetic permeability of the vacuum.

$\varepsilon_0 = 8.8510^{-12}\ F.m^{-1}$

with F/m is the farad per meter.

4. μ0 the magnetic permeability of the vacuum.

$\mu_0 = 4\pi\ 10^{-7}\ H.m^{-1}$

with H/m is the henry per meter.

The 9 system can be linked to a couple of wave equations ,see article [10], solvable thanks to Fourier analysis, see article [6] and [9].

3. The Discrete/Fast Fourier transformation (DFT and FFT)

This section is carried out thanks to the documents [1], [2], [3] and [4].

3.1 Mathematical tools

The discrete Fourier transformation (DFT) is the discrete spectral representation in the frequency domain of a sampled signal. It is used to process a digital signal; unlike the continuous Fourier transform used to process an analog signal. In other words, it constitutes a discrete equivalent of the continuous Fourier transformation.

Definition 2 The discrete Fourier transformation of a signal s of N samples (s(0), s(1),..., s(N − 1))

Discrete spectral representation of the sampled signal s(n) is thus obtained.

Definition 2 *The discrete Fourier transformation of a signal s of N samples* $(s(0), s(1), \ldots, s(N-1))$ *is the vector* $(S(0), S(1), \ldots, S(N-1))$ *defined by :* $S(k) = \sum_{n=0}^{N-1} s(n)\, e^{-2i\pi k \frac{n}{N}}$ *pour* $0 \leqslant k < N$ *A discrete spectral representation of the sampled signal s(n) is thus obtained.*

Remark 4 *The inverse discrete Fourier transform is given by :* $s(n) = \frac{1}{N} \sum_{k=0}^{N-1} S_k\, e^{2i\pi n \frac{k}{N}}$

The fast Fourier transformation (FFT) is a particular algorithm for calculating the discrete Fourier trans- form (DFT). It is published in 1965 by James Cooley and John Tukey and has been adopted by pro- fessionals in signal processing and telecommunications. Several other researchers have worked on and improved this algorithm before this date.

Algorithm 1 To calculate the N samples of the DFT: {S0, . . . , SN−1}, we use the basic expression :

$$S_m = \sum_{k=0}^{N-1} s_k W_N \ , form \in \{0, \ldots, N-1\}$$

$$avec : W_N^{km} = e^{-2i\pi \frac{km}{N}}$$

- $Let's\ pose\ W_N^{km} = e^{-2i\pi \frac{km}{N}}$
- $For\ 0 \le t \le N/2 - 1\ do$ $\quad S_t = S_{1,t} + W_N^t S_{2,t}$
- $For\ N/2 \le i \le N - 1\ do$
  - $Let's\ pose\ t = i + N/2$
  - $Calculate$ $\quad S_i = S_{1,t} + W_N^t S_{2,t}$

4. The Wavelet transformation (WT)
This section is carried out thanks to the documents [1], [2], [3], [4] and [5].

4.1 Mathematical tools
In the 1980, based on reflections on Fourier decomposition and thanks to the advent of digital technology; Yves Meyer, one of the founders of wavelets, receives the prestigious Abel Prize in 2017. Wavelet analysis is used in signal processing.
The Wavelet transform (WT) can be expressed with the following equation:

$$F(a, b) = \int_{-\infty}^{+\infty} f(x)\, \psi^*(x)\, dx \qquad (10)$$

Where the symbol denotes the complex conjugate and ψ is a given function. This function can be chosen arbitrarily provided and obeys certain rules.
The Discrete Wavelet transform (DWT) is an implementation using a discrete set of scales and wavelet translations obeying certain rules. In other words, this transform decomposes the signal into a set of mu- tually orthogonal wavelets.
The Continuous Wavelet Transform (CWT) is an implementation of the wavelet transform using arbi- trary scales as well as practically arbitrary wavelets. The wavelets used are not orthogonal and the data obtained by this transform are highly correlated.
The Wavelet packet transform (WPT) looks a lot like the (DWT). However, the wavelet packet trans- form scaling coefficients and the the wavelet coefficients, are passed through the low-pass and high-pass filters.

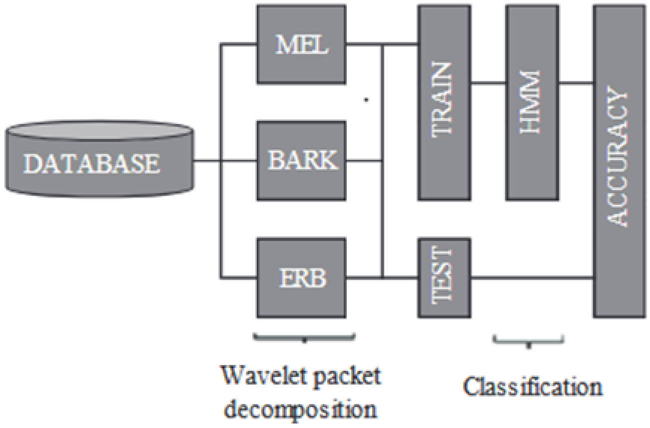


Fig 1: Speech emotion recognition system description [1].

4.2 Application on emotions
In the article "speech emotion recognition in acted and spontaneous context" [1], the authors make a study from two databases: SAVEE and IEMOCAP in order to classify the emotions according to wavelet packet decomposition as in the figure 1.

5. Conclusion
In this article, the author shows the importance of Fourier analysis in the study of emotions from a mathematical point of view. This field has evolved a lot and is approached by several scientific disciplines such as modeling and signal processing. Therefore, they each use mathematical tools that seem different, given the vocabulary used, but all come together in Fourier analysis in its improved forms such as DFT and wavelet transform.

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