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Modulation of elements in porous media and in pipelines

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Abstract
A fluid can be in the form of a liquid or a gas. Its movement, generally described by the term "flow" is characterized by the flow rate. This is the amount that passes through a given surface. According to the experiments, the flow rate depends as well on the fluid as on the support of its flow, whether it is a pipe or a porous medium. In this work, a porous medium can be cracked or granulated. In the second part of the article, the author presents some forces that can be exerted both on the medium and on the fluid. They are classified into inhibitory and catalyzing. Finally, an example of an energy balance is described in the form of Bernoulli's law.

Keywords: hydrodynamics, fluide, flow, Forces, constraints, Darcy Law, Higen-Poiseuille law, Bernoulli law

1. Introduction

The flow rate is the quantity of a quantity that passes through a given surface per unit of time. It makes it possible to quantify a displacement of matter or energy. In this work, the aim is to model the flow rate of a fluid in a pipe and then in a porous medium. In pipelines, the flow rate of a laminar flow under certain fixed conditions is formulated by the Hagen Poiseuille law. On the other hand, for a turbulent flow, there is no fixed mathematical formula describing the flow rate. However, thanks to the Reynolds number, the movement of the fluid can be predicted according to a predefined classification. In porous media, the flow rate of an incompressible fluid appears in Darcy's law. According to experience, this formula relates the flow rate to the height of the fluid in laminar flow. But mathematically, this can be translated by a gradient and therefore by an average speed. In a turbulent flow, the flow rate fluctuates continuously. We then use the Darcy Weisbach formula.

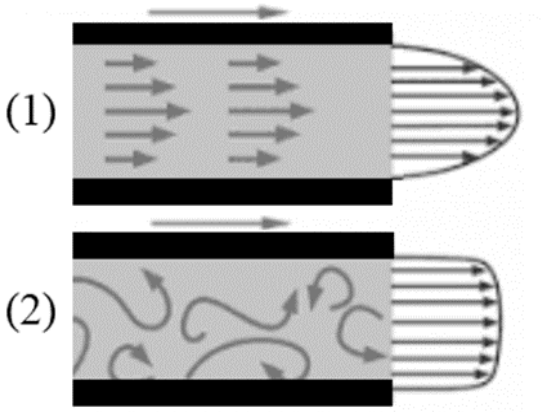

Fig 1: (1) laminar flow (2) turbulent flow

1.1 In the pipeline

The flow rate of a laminar flow: the Poiseuille-Hagen law

Poiseuille's law, also called Hagen-Poiseuille's law, describes the laminar flow of a viscous liquid. In general, Poiseuille's law theoretically states the relationship between the flow rate of a flow, the viscosity of the fluid, the pressure difference at the ends of the pipe, the length and the radius of this pipe. This relationship is verified experimentally in pipes with small radii and is often used in viscometers because it states in particular that the flow rate is inversely proportional to the viscosity. Experimentally, in a cylindrical tube or pipe, the speed distribution is a paraboloid of revolution, as indicated in the figure??. Discovered independently in 1840 by the French doctor and physicist Jean-Lard-Marie Poiseuille and by the Prussian engineer Gotthilf Hagen, it constitutes the first attempt to overcome the notion of average velocity of a flow, until then in use like the formulas of Ch and Prony. A Poiseuille flow is a flow that follows a Poiseuille law.

Poiseuille's law establishes that

$$Q = \frac{\pi r^4}{8\mu} \times \frac{\Delta P}{L} \quad (1)$$

Here, Q designates the flow rate, sometimes confused with the liquid flow velocity, r the internal radius of the pipe, ΔP the pressure difference between the two ends of the pipe, L the length of the pipe and μ the viscosity of the liquid. The law presumes that the liquid has a laminar flow, that is to say regular and without turbulence.

Example 1: This law has practical applications in medicine and particularly for the study of the flow in the blood vessels.

Remark 1: The term r4 attests to the impact of the radius of a tube in the calculation of the flow Q. If all the other parameters are identical, a doubling of the width of the tube leads to a multiplication by 16 of the value of Q. This means that we would need 16 tubes to drain as much water as through a single tube of double diameter.

- -Poiseuille's law is used to show the dangers of atherosclerosis: if the radius of a coronary artery is divided by two, the speed of the blood will be divided by 16.
- -This law also explains why, when someone drinks from a straw, the wider it is, the higher the amount of drink sucked in.

Under certain conditions, the flow is symmetrical in the cylindrical pipe and the flow can be studied in two dimensions and not in 3 dimensions, as indicated in the figure??.

The flow rate of a turbulent flow

A turbulent flow in a pipeline is difficult to model mathematically. This fluid state is determined using the Reynolds number Re > 3500. If the flow rate is not modeled mathematically, the causes of the turbulence can be described and modeled. These are pressure losses caused by friction or viscosity or even the change in the diameter of the pipe. There are two types of pressure drops in pipelines: Linear pressure drops and Singular pressure drops. The first is due to the friction of the walls, the second is related to the existence of obstacles in the pipes.

In a turbulent flow, the flow rate is disturbed. The flow lines are chaotic instead of linear, see figure??. There are also other variables allowing the study of a turbulent flow such as the turbulence intensity and the turbulence viscosity.

In other words, in a turbulent flow, the different layers of fluid become entangled by swirling and exchange energy with each other. The form of flow that is formed is characterized by three-dimensional, unpredictable and non-stationary movements. A laminar boundary layer remains in part, but only in the peripheral zone of the pipe. The speed distribution is practically constant over a large part of the pipe section unlike laminar flow, the pressure drop is proportional to the square of the average fluid velocity.

1.2 In porous media

The study of the displacement of water in a porous medium was conducted experimentally by Darcy in 1856. For the same hydraulic load, Darcy defines a permeability coefficient K, measured in [m/s], depending on the type of porous medium. The quantity of water passing through this medium is proportional to the total cross-section crossed A [m2], to the permeability coefficient K [m/s] of the medium and to the hydraulic load h [m] and inversely proportional to the length l of the medium crossed:

$$Q = K.A.\frac{h}{l} \quad [m^3/s] \quad (2)$$

Or

$$Q = K.A.i \quad [m^3/s] \quad (3)$$

We note i = hl the pressure drop per unit length, called hydraulic gradient. On the ground, the latter is calculated by placing two piezometers L meters apart. The gradient is the ratio between the level difference h of the piezometers and the distance L. Darcy's law is strictly applicable only for homogeneous media where the water flow is laminar (non-turbulent). It cannot be used in particular for karst networks (heterogeneous medium).

More generally, Darcy's law is written:

$$q = -K \text{ grad } h \quad (4)$$

Where K is the hydraulic conductivity.

Example 2: Au chapitre 2 du livre [?], l'de de l'ulement de l'eau dans des milieux poreux saturu insaturait intervenir trois ations de base: l'ation de continuita loi de Darcy et l'ation d't isotherme du fluide.

Example 3: The blood circulates in macroscopic blood vessels such as arteries, arterioles, veins and venules. However, on a microscopic scale, capillaries are considered as tissues and the modeling of flows can be compared to a flow in a porous medium. The table?? Summarizes the flow properties in each case:
Conclusion 4 Darcy's law is strictly applicable only for homogeneous media where the water flow is laminar. It cannot be used in particular for karst networks because they are considered a heterogeneous medium.

More generally, Darcy's law is written:

$$Q = -K \text{ grad } h \quad (5)$$

Where K [m/s] is the hydraulic conductivity and h(x,y,z,t) [m] is the hydraulic load.

2. Les forces

A force can be exerted either on a medium, on a fluid or on both.
Example 5: An aquifer with an unconfined groundwater table undergoes both types of force simultaneously. Indeed, several forces can be exerted. On the one hand, the hydraulic conductivity K is a force related to the texture of the medium. On the other hand, the internal dispersive force linked to moving water molecules is a force linked to molecular friction.
In addition, a force can be inhibiting or catalyzing a fluid flow in a studied medium.

Example 6: Catalytic forces: gravitational force, potential force, mechanical force.
- Inhibitory forces: pressure, viscosity or even elastic stresses.

2.1. Inhibitory forces

Stresses are forces that dissipate energy in another form For example:
1. Shear strength: A shear stress τ is a stress applied in a parallel or tangential way to a face of a material, as opposed to normal stresses that are applied perpendicularly, such as pressure. It is the ratio of a force to a surface:

$$\tau = F/S \quad (6)$$

Where τ is the intensity of the shear; also called the shear stress, or cission ; F is the tangential force applied and S is the area of the surface on which the shear is exerted.
The shear stress therefore has the dimension of a pressure, expressed in pascals or for large values in megapascals : MPa. Shear stress is not a notion practiced or measured like blood pressure. However, this scalar intervenes in the mathematical modeling of fluids at small scales.
The deformation speed also called the speed gradient

Table 1: Comparison between the flow in porious medium and pipelines

	Transport in the Canals	**Transport in Tissues**
Example	Arteritis, arteriole vein, venules	capillary vessels
Type of flow	Hydrodynamic regime depends on the Reynolds number	laminar flow Reynolds number<< 1
Flow	Poiseuille's Law $Q = \Delta P/R$	Darcy's Law $Q = \Delta P/R$
	$R = 8\mu L/r^2$: flow resistance μ : viscosity of the fluid, P $a.s$ L : vessel length, m r : radius of the vessel, m	$R = \mu e/\alpha$: flow resistance μ : viscosity of the fluid, P $a.s$ e :thickset thickness, m α : permeability coefficient, m^2

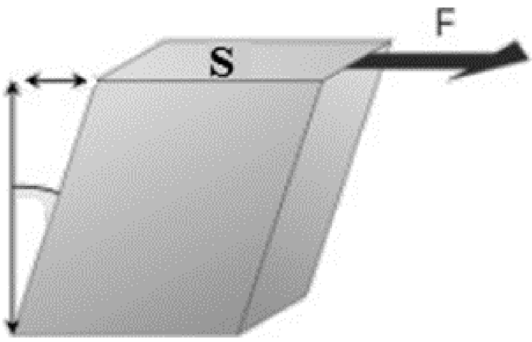

Fig 2: The shear stress: F is the tangential force at the surface S

Or the shear speed is denoted γ Its unit is m.s^{-1} and its expression is

$$\dot{\gamma} = \frac{1}{h}\frac{\partial u}{\partial t} \quad (7)$$

With u is the displacement of the fluid, in meters, h the height of the sheared area, in meters. We note γ the relative deformation of the fluid after a time dt, we have

$$\gamma = \frac{u}{h} \quad (8)$$

The deformation is the ratio of two lengths, so it is a dimensionless quantity. Thus, the rate of deformation is in inverse time units, s^{-1}.
2. Pressure force: The pressure load on a fluid is calculated by the formula

$$P = F/S \quad (9)$$

Where P is the pressure, F is the force, S is the surface on which the pressure is exerted. This formula determines the pressure necessary to move a load between two points. During acceleration, the potential energy of the pressure is transformed into the kinetic energy of the fluid movement. During deceleration, this kinetic energy is transformed into potential energy. This is represented mathematically by Bernoulli’s law. When friction is present, the load is no longer constant. We then speak of pressure drop.

Example 7: Certain concepts such as hydraulic loading exist not only in porous media but also in pipelines. These are notions related to the energies provoking the flow. For example, the hydraulic load defined by the height of the water column is none other than the load of the gravitational pressure on a water column.

3. Viscosity strength μ = F/S.

2.2 Catalytic forces
These are forces that transmit positive energies such as the driving force or even cause non-inhibitory reactions or that it increases the speed at which the reactions occur. We will present as examples: thermo gravitation, thermo diffusion, thermal convection, thermo solution convection, thermo vibrational convection, or vibrational thermo solution convection.

3. Energy balance
The unit of energy is the joule that is to say in newton meters. Although this article is dedicated to a study of the differences between flows in porous media and pipeline flows, several variables and parameters persist in both media. These are parameters that model the properties of the fluid or the energy necessary for the flow. Generally, liquids are in motion thanks to the potential energies of gravitational origin and kinetic energy. This is the case, of hot water flows in plumbing pipes in a bathroom or even groundwater flows in aquifers. However, several other energies exist such as the elastic energy of the blood vessels allowing the continuity of hemodynamics. In the first section, we present the theory of the law of conservation of mechanical energy. It makes it possible to define Bernoulli’s law. A very practical formula in fluid mechanics.

3.1. The potential energy
Mechanical potential energy is an energy that is ex-changed by a body when it moves while being subjected to a conservative force. It is expressed in joules. This potential energy, defined to an arbitrary constant, depends only on the position of the body in space. It is called potential because it can be stored by a body and can then be transformed, for example, into kinetic energy when the body is set in motion. Each conservative force gives rise to a potential energy. We can thus distinguish: gravity potential energy, gravitational potential energy, elastic potential energy, electrostatic potential energy, magnetic potential energy, inertia

potential energy, drive potential energy (in certain simple situations), pressure potential energy.
The potential energy is defined to within an additive constant. This has no influence on the results since the potential energy is used in derivation operations (calculation of a conservative force) or variation (calculation of a job). These two operations make the constant disappear, the choice of the latter is therefore purely arbitrary and its determination is generally done in such a way as to simplify the calculations.

Example 8: rgie potentielle de pesanteur A simple example is that of a terrestrial body held aloft (and the refore possessing a potential energy of gravity due to its height) which, once released, transforms this potential energy into kinetic energy when its speed increases during its fall. On Earth, gravity is often considered to be a vector field, directed downwards, approximately towards the center of the Earth. Its intensity, denoted g, is approximately independent of the altitude, with

$9 \leq g \leq 10N/kg$ usuellement, $g = 9{,}81N/kg$

The energy received by a body of mass m moving from the altitude point z0 to the altitude point z1 is expressed by the potential energy difference. It is worth:

$$\Delta Epp = mg\,(z_1 - z_0) \tag{10}$$

Depending on the problem, an arbitrary origin of the potentials can be fixed. For example, if we study the fall of an object to the floor of a laboratory, we fix the origin of the potentials at the level of this floor, regardless of the real altitude of the laboratory. By choosing the sea level as the origin of the potentials, then this energy is expressed in joules by:

$$\Delta\ Epp = mgh$$

Where m (kg) is the mass and h (m) is the altitude.

3.2 Kinetic energy
Kinetic energy is a fundamental notion in physics, in particular in the study of the dynamics of fluid and solid particles. Like any energy, kinetic energy is expressed in joules (J). There are two forms of kinetic energy:
1. The kinetic energy of translation
2. The kinetic energy of rotation

In the case of a body in translation, of mass m and speed v, the kinetic energy Ec is proportional to the mass of the body and the square of its speed, i.e. the relation:

$$E_c = \frac{1}{2} m v^2 \tag{11}$$

In the case of a rotating body, the kinetic energy is proportional to the square of the angular velocity ω, according to the relation:

$$E_c = \frac{1}{2} I \omega^2 \tag{12}$$

Where I represents the moment of inertia of the system. The kinetic energy theorem states that the variation of the kinetic energy between two instants t1 and t2 is equal to the algebraic sum of the work of the external forces applied to this system between these two instants.

3.3 Law of conservation of mechanical energy: Bernoulli's law
For an isolated system, without interaction with other systems or friction, the sum of kinetic energy and potential energy is constant over time:

$$\begin{aligned} E_c + E_p &= E_m \\ &= constante \end{aligned} \tag{13}$$

With Ec the kinetic energy, Ep the potential energy of gravity and Em the mechanical energy. In fluid mechanics, Bernoulli's equation (??) is a direct application of the law of conservation of energy. It describes an energy balance that oscillates between potential energy and kinetic energy.

Kinetic energy
The kinetic energy is expressed by

$$\begin{aligned} E_c &= \frac{1}{2V}.m.v^2 \\ &= \frac{1}{2}.\rho.v^2 \end{aligned} \tag{14}$$

With V is the volume of a homogeneous body, m is the weight, v is the velocity and ρ is the density of a homogeneous body.

Potential energy

The potential energy admits an elevation component (potential gravity energy) and a pressure component also called pressure charge:

$$E_p = \rho.g.z + p \tag{15}$$

With p is the pressure, z is the altitude, g is the intensity of gravitation and ρ is the density of a homogeneous body.

Energy balance

The law of conservation of mechanical energy states that the sum of kinetic and potential energies are conserved and equal to a constant.

$$\frac{1}{2}.\rho.v^2 + \rho.g.z + p = constante \tag{16}$$

With p(x) is the pressure [P a or N/m2], ρ(x) is the density [kg/m3], v(x) is the fluid velocity [m/s], g is the acceleration of gravity [N/kg or m/s2], z is the altitude [m], the reference level for the elevation is arbitrary.
Generally, it is the average sea level that is fixed.

Bernoulli's law

This equation comes from the study of a non-swirling flow, of a non-viscous incompressible fluid, in steady state, along a current line, where heat transfers are neglected. Under these conditions, the studied system preserves the energy balance and the law of conservation of energy?? is applicable. By dividing the two terms of the equality?? by ρ.g; we get Bernoulli's law:

$$\frac{v^2}{2g} + z + \frac{p}{\rho g} = constante \tag{17}$$

With p(x) is the pressure [P a or N/m2], ρ(x) is the density [kg/m3], v(x) is the fluid velocity [m/s], g is the acceleration of gravity [N/kg or m/s2], z is the altitude [m], the reference level for the elevation is arbitrary. The constant intervening in the second member of the equation is not universal but specific to the flow, it is a constant along a current line, called load.

Application: Hydraulic load

The hydraulic load h is a potential scalar that governs the transfer of mass or energy and is measured in meters [m]. Its gradient is of great importance for Darcy's law. Thanks to Bernoulli's equation (??), the hydraulic load can be perceived as a potential energy. This equation comes from the study of a non-swirling flow, of a non-viscous incompressible fluid, in steady state, along a current line, where heat transfers are neglected:

$$\frac{v^2}{2g} + z + \frac{p}{\rho g} = constante \tag{18}$$

With p(x) is the pressure [P a or N/m2], ρ(x) is the density [kg/m3], v(x) is the fluid velocity [m/s], g is the acceleration of gravity [N/kg or m/s2], z is the altitude [m], the reference level for the elevation is arbitrary. Generally, it is the average sea level that is fixed.
The constant intervening in the second member of the equation is not universal but specific to the flow, it is a constant along a current line, called load. By multiplying Bernoulli's equation (??) by ρ.g, we identify the energy balance??. The first term of the sum is the kinetic energy?? Ec = 12 .ρ.v2. The potential energy admits an elevation component (potential gravity energy) and a pressure component also called pressure charge: Ep = ρ.g.z + p. In a saturated porous medium, the fluid velocity being low, the kinetic energy is neglected. Only the potential is responsible for the flow. Hence the equality?? Becomes.

$$z + \frac{p}{\rho g} = constant \tag{19}$$

Thus, the constant of the Bernoulli equation corresponds to the hydraulic load h. Indeed, the potential is the work done during the flow. It therefore represents the energy required to move a fluid between two points.

$$h = z + h_p \tag{20}$$

avec

$$h_p = \frac{p}{\rho g} \tag{21}$$

The quantity hp is applied pore pressure. In a porous soil, this refers to the pressure exerted by the water contained in the pores. This same definition is applicable for a fluid flowing in a porous medium.

Example 9: In the study of a groundwater table, the quantity hp is applied pore pressure of the water with respect to atmospheric pressure. For more details, consult the course manual [?] and its value is defined to a constant:

$$h_p = \frac{p - p_{atm}}{\rho g} \tag{22}$$

So that h coincides with the coast of the free surface of a groundwater table and the piezometric coast of a confined water table.

Example 10: Under the assumption of Dupuit, see the book [?] for more details, we will take a closer look at the quantity hp. Since the flow is plane, the pressure is assumed to be vertically hydrostatic, as in the figure??.
Thus, by noting zinf the height of the floor of a ground water table and zsup the height of the free surface, we find

$$\begin{aligned} h(x,y,t) &= z_{inf} + h_p(x,y,t) \\ &= z_{sup} \end{aligned} \tag{23}$$

Therefore, the pore pressure coincides with the thickness of the groundwater table

$$h_p(x,y,t) = z_{sup}(x,y,t) - z_{inf}(x,y) \tag{24}$$

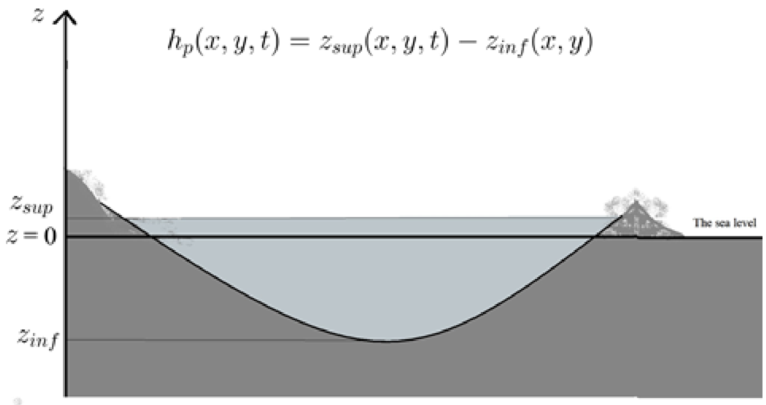

Fig 3: The hydraulic load for a groundwater table

4. Conclusion

As explained in the introduction, certain parameters do not depend on the support of the flow, whether it is a porous medium or a pipe. Fluid dynamics is characterized by the flow rate. The flow rate is a parameter related to the type of the medium of the flow. Thus, in pipe flows, the flow rate of an incompressible fluid is described by Poiseuille's law, on the other hand in porous media, it is modeled by Darcy's law. A second parameter is the Reynolds number. It describes the type of flow, laminar or intermediate or turbulent. The fluids can be viscous. This physical property makes it possible to classify fluids according to their resistance to an external force in Newtonian and non-Newtonian.
These parameters are important for physical and ma thematically modeling. For example, the friction and the viscosity of the fluid cause speed differences depending on the distance from the walls of the pipes or the porous mass. These speed variations generate pressure losses and transform a laminar flow into a turbulent one.

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