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Sliding mode path following control of a 4WS mobile robot

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Abstract
This paper presents a method of applying a combination of the sliding mode control (SMC) and target guidance algorithm to control a four-wheel-steering mobile robot (4WS) to follow the trajectory with small curvature, based on visual information captured by a single camera. A mathematic model of four-wheel-steering mobile robot were also used, the proposed controller ensure stability and robustness when operating under uncertainties and disturbances. The process of synthesizing control laws is strictly mathematically guaranteed. Research results are calculated, simulated, and tested on Matlab.

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Keywords: sliding mode control, SMC, 4WS, 4WS, mobile robot, path following

Introduction
The path control problem of mobile robots is one of the most concerned problems in robotics technology. The current trend of using more electric vehicles also creates more opportunities for the application of mobile robots in real life. Four-wheel-steering mobile robots, with very high maneuverability and controllability, are chosen for extensive research, including the path control problem. Many researchers have used sliding mode control to control 4WS mobile robots [1, 2]. Those controllers are usually based on kinematic models and with the assumption of high-precision measurements when returning the exact absolute position of the vehicle, this is even a more difficult problem than creating the controller based on that model.
There are also many studies [4,5] that often use vision-based algorithms to guide the vehicle along a reference or target path, including virtual target navigation algorithms. This paper presents a method that combines sliding mode control (SMC) and target guidance algorithm to control a four-wheel-steering mobile robot to follow a trajectory with small curvature, based on the image information acquired by a single camera. This may be a more efficient and feasible solution.

Methodology
A. Mathematic model of a 4WS mobile robot
1. Dynamic model of a 4WS mobile robot.
There are different methods to create the dynamic model of a 4WS mobile robot. But the method that is used in [10] is more practical and easy to apply (Ackermann and bicycle model). Then the method is adopted for this article.
In which the steering angles of each real wheels as well as the virtual wheels F and R (as shown in Figure 1) are determined through the Ackermann steering method with the instantaneous center of rotation I_{CR}.

$$\delta_c = \tan^{-1}\frac{\tan(\delta_f)+\tan(\delta_r)}{2} \quad (1)$$

$$R = \frac{l}{\cos(\delta_c)*(\tan(\delta_f)-\tan(\delta_r))} \quad (2)$$

Where: $\delta_f, \delta_r, \delta_c$ are the virtual steering angles at points F, R, C; wheel base $l = 2 * l_f = 2 * l_r$, d is the vehicle width. R is the trajectory radius, as in fig. 1: $R = I_{CR}C$; $\delta_i, i = 1,2,3,4$ are the real steering angles at real wheels – angles between the longitudinal car axis and wheel planes.

The real steering angles of each wheel can also be calculated from δ_c, R

$$\delta_1 = \tan^{-1}\frac{\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)-\frac{d}{2}} \tag{3}$$

$$\delta_2 = \tan^{-1}\frac{-\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)-\frac{d}{2}} \tag{4}$$

$$\delta_3 = \tan^{-1}\frac{-\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)+\frac{d}{2}} \tag{5}$$

$$\delta_4 = \tan^{-1}\frac{\frac{l}{2}+R*\sin(\delta_c)}{R*\cos(\delta_c)+\frac{d}{2}} \tag{6}$$

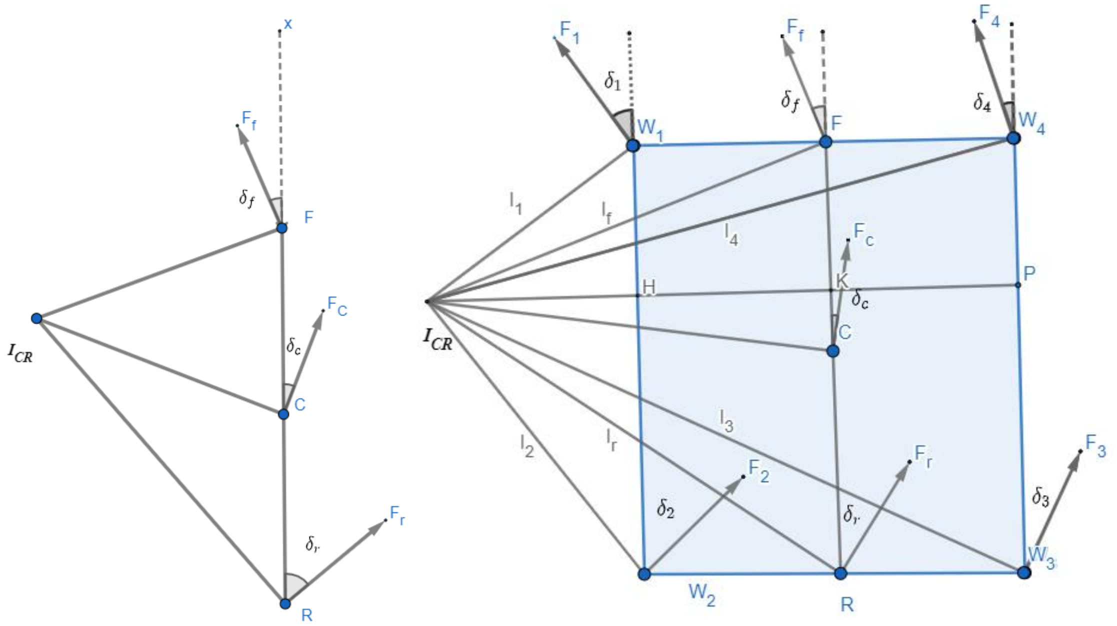

Fig 1: The simplified two-wheeled bicycle kinematic model

Because the steering angle at the vehicle's center of gravity C, $\delta_c = 0$, the direction of the resultant force at C coincides with the direction of the vehicle's longitudinal axis. Then the longitudinal acceleration of the vehicle will be calculated by

$$a_x = F_c/m \tag{7}$$

The centripetal acceleration is the acceleration along the horizontal direction of the vehicle body calculated by:

$$a_y = v^2/R_0 \tag{8}$$

With the steering wheel, it is assumed that there is a steering angle control system available so that the steering angle is a first order inertial of the steering angle control voltage:

$$\delta = \frac{K}{Ts+1}u \tag{9}$$

Then the derivative of the steering angle can be calculated by:

$$\dot{\delta} = \frac{K}{T}u - \frac{1}{T}\delta \tag{10}$$

Then the dynamic equations of the robot vehicle on the vehicle body coordinate system at the center of gravity C will be:

$$\begin{cases} \dot{x} = v\cos\psi \\ \dot{y} = v\sin\psi \\ \dot{v} = \frac{F_{th}}{m} = \frac{F_c}{m} - k_v v^2 \\ \dot{\psi} = \frac{v}{R_0} = 2 * \frac{v}{l} * \tan\delta \\ \dot{\delta} = \frac{K}{T}u - \frac{1}{T}\delta \end{cases} \quad (11)$$

in which, it is assumed that the resistance force moving along the longitudinal axis of the vehicle body is proportional to the velocity v: $F_{ms} = -mk_v v^2$, with k_v being a proportional coefficient that is always positive.
Then system (11) is the system of dynamic equations for a 4WS mobile robot.

2. Mathematic model of path following problem of a 4WS mobile robot.

Assume that the vehicle needs to follow a given trajectory with required coordinates: (x_d, y_d, ψ_d) in a fixed ground coordinate system. The control objective is to ensure that the error between the actual values and the required values is zero:

$$\begin{cases} e_x = x - x_d \\ e_y = y - y_d \\ e_\psi = \psi - \psi_d \end{cases} \quad (12)$$

To simplify the system, we choose a coordinate system on the required trajectory with the x_d axis along the trajectory and the y_d axis perpendicular to it. Accordingly, the Dx_dy_d coordinate system will deviate at an angle ψ_d from the fixed ground coordinate system.

Then the equations in system (11) when placed on the Dx_dy_d coordinate system (see figure 2) become:

$$\begin{cases} \dot{x} = v\cos(-\psi + \psi_d) \\ \dot{y} = -v\sin(-\psi + \psi_d) \\ \dot{v} = \frac{F_c}{m} - k_v v^2 \\ \dot{\psi} = \frac{v}{l} * 2\tan\delta \\ \dot{\delta} = \frac{K}{T}u - \frac{1}{T}\delta \end{cases} \quad (13)$$

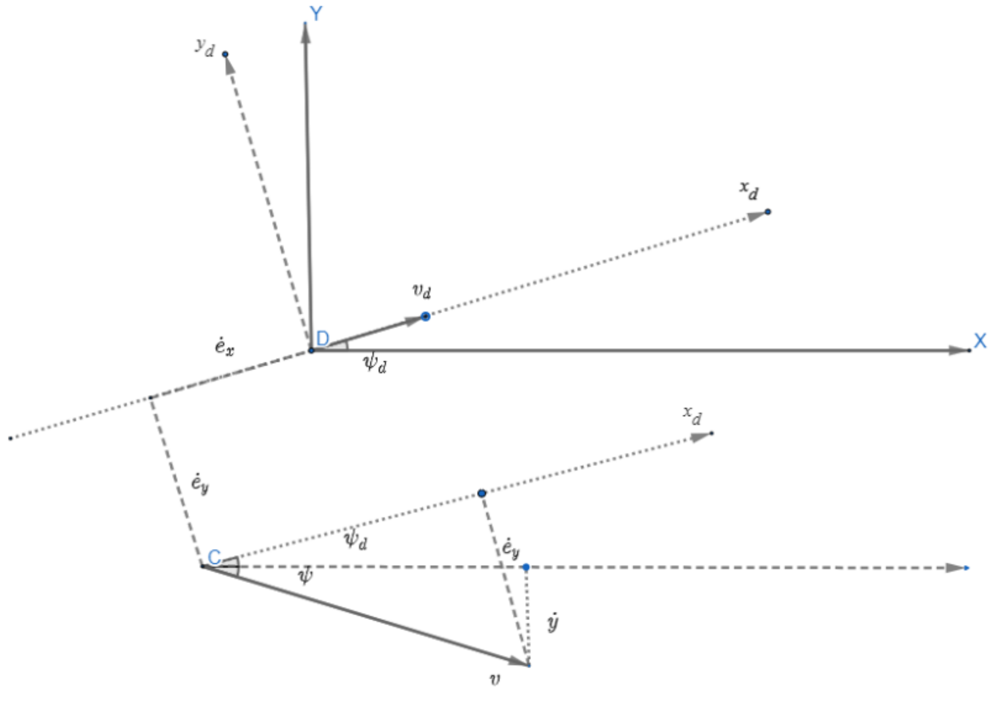

Fig 2: Path following dynamic model on required trajectory coordinate system

Where $\dot{v}$ of equation (13) is actually $\dot{v}_{xd}$, i.e. the acceleration along the x_d axis of the required trajectory coordinate system. In fact, we can choose $y_d = 0$ because it is considered as a reference point for tracking the trajectory, then we have:

$$e_y = y \quad (14)$$

Then the derivative of the tracking errors will be:

$$\begin{cases} \dot{e}_x = \dot{x} - \dot{x}_d = v\cos(\psi - \psi_d) - v_d \\ \dot{e}_y = \dot{y} = v\sin(\psi - \psi_d) \\ \dot{e}_\psi = \dot{\psi} - \dot{\psi}_d \end{cases} \quad (15)$$

In which, for the path following problem without speed requirements, that is, only requiring accuracy when tracking y_d, ψ_d, the dynamic equation system will have the following form:

$$\begin{cases} \dot{e}_y = v\sin(e_\psi) \\ \dot{v} = \frac{F_c}{m} - k_v v^2 \\ \dot{e}_\psi = -\frac{v}{l} * 2\tan\delta + \dot{\psi}_d \\ \dot{\delta} = \frac{K}{T}u - \frac{1}{T}\delta \end{cases} \quad (16)$$

Most of the path following control problems for wheeled mobile robots are lane tracking control problems, that is, ensuring that the error $|e_y| = |y - y_d|$ is minimized. However, there are also path following control problems that require speed, in which case the system of dynamic equations will become:

$$\begin{cases} \dot{e}_x = v\cos(e_\psi) - \dot{x}_d = v\cos(e_\psi) - v_d \\ \dot{e}_y = v\sin(e_\psi) \\ \dot{v} = \frac{F_c}{m} - k_v v^2 \\ \dot{e}_\psi = -\frac{V}{l} * 2\tan\delta + \dot{\psi}_d \\ \dot{\delta} = \frac{K}{T}u - \frac{1}{T}\delta \end{cases} \quad (17)$$

B. Path following control system for 4WS mobile robot

1. PID control for path following problem.

The state feedback controllers installed on current mobile vehicles are mostly based on information provided by high-resolution cameras located above, in the center of the front of the vehicle body, about the vehicle's position relative to the lane markings on both sides or the center line in the middle of the lane. In this topic, the researcher chooses the observation conditions for the camera as two lane markings on both sides of the vehicle body. For the control system to operate, these lane markings must always be within the camera's field of view (as shown in Figure 2.1), so there are certain initial condition constraints when using this method.

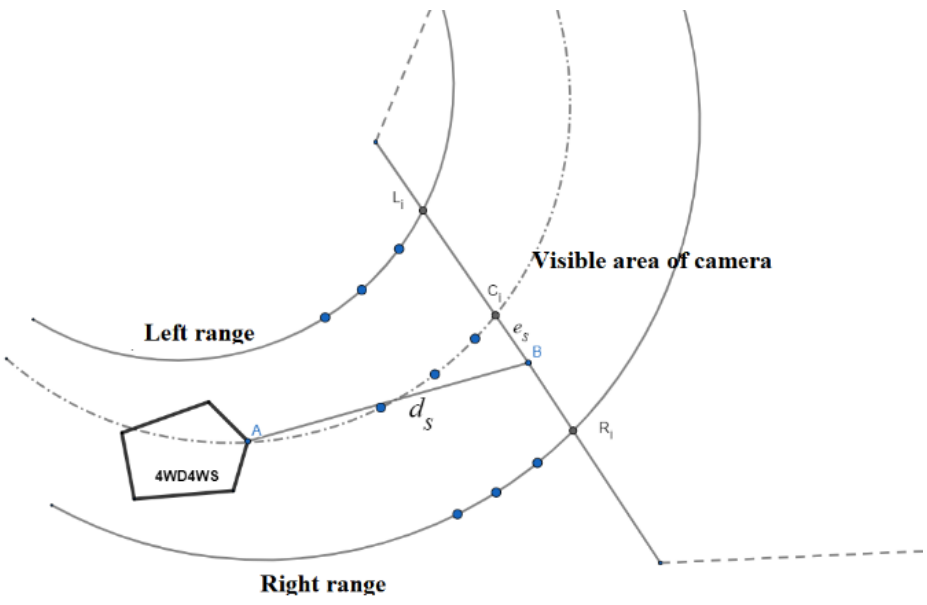

Fig 3: Camera determines trajectory tracking error

Then the camera points scanned and recognized as lying on the left and right ranges of the lane (L_i and R_i) will be continuously updated and from there use the function approximation method to create continuous trajectories in the form of a time-dependent function [1]. From there, the required lane centerline trajectory can be determined:

$$C_i = \frac{1}{2}(L_i + R_i) \tag{18}$$

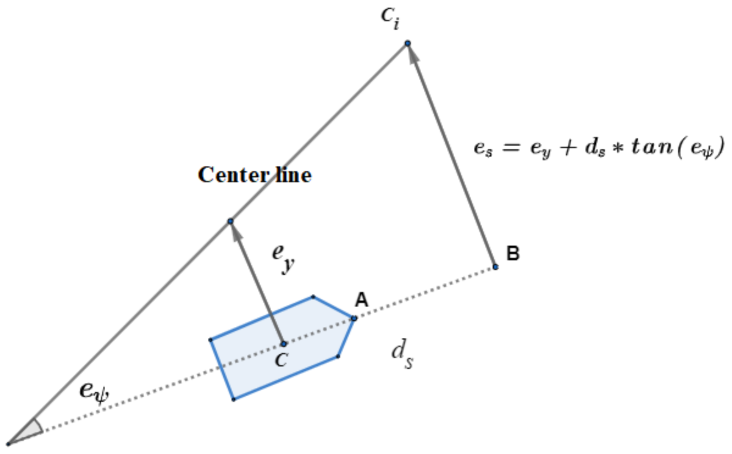

Fig 4: Camera determines trajectory tracking error

Then, according to Figure 4, the error that the camera can determine is based on the calculation of the deviation of the road centerline with the midpoint of the lower edge of the camera's observed area. And this error is calculated according to the following formula:

$$e_s = e_y + d_s * \tan(e_\psi) \tag{19}$$

In which, d_s is the distance from the lower edge of the camera's visible area to the center of gravity of vehicle C. With a small deviation angle e_ψ, the error e_s can be calculated by:

$$e_s = e_y + d_s * e_\psi \tag{20}$$

$$\text{Set: } \begin{cases} x_1 = e_y \\ x_2 = v \\ x_3 = e_\psi \\ x_4 = \delta \end{cases} \tag{21}$$

Then (17) becomes:

$$\begin{cases} \dot{x}_1 = x_2 \sin(x_3) \\ \dot{x}_2 = \frac{F_c}{m} - k_v x_2^2 \\ \dot{x}_3 = -\frac{x_2}{l} * 2\tan x_4 + \dot{\psi}_d \\ \dot{x}_4 = \frac{K}{T} u - \frac{1}{T} x_4 \end{cases} \tag{22}$$

Then the PID controller from the error feedback e_s can be used to control the system (22) without measuring the values of e_y, e_ψ.

2. SMC for path following problem

To build a SMC path following controller for the 4WS robot, it is necessary to change the dynamic equation because it is impossible to get the values e_y, e_ψ into the equivalent control expression, these values are completely unmeasurable. At that time, to be able to use only the measurement error from the camera placed on the car body to get the feedback value, it is necessary to use the virtual target guidance algorithm. The control process is divided into two stages, the first stage is the guiding loop to find the desired direction angle, the second stage is to follow the desired direction angle.

Thus, there will be 3 direction angles: the actual direction angle of the car body ψ, the desired direction angle ψ_d and the direction angle of the required trajectory ψ_r. Then, the value of the deviation of the direction angle from the desired direction angle can be calculated by:

$$\tan(\psi_d - \psi) = \tan(\Delta\psi) = \frac{e_s}{d_s} \quad (23)$$

That is, if the camera error is known, the direction angle error $\Delta\psi$ can always be calculated, and vice versa:

$$e_s = d_s \tan(\psi_d - \psi) = d_s \tan(\Delta\psi) \quad (24)$$

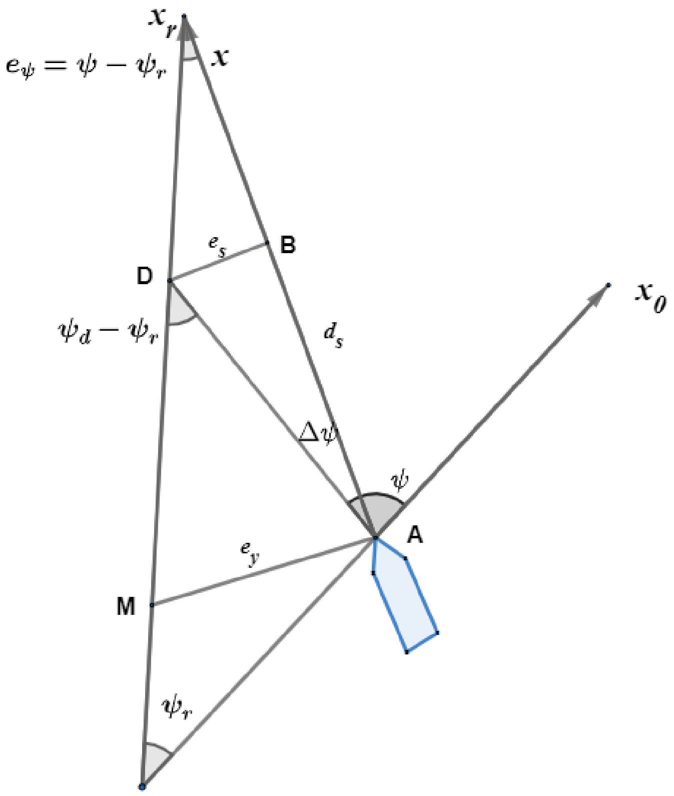

Fig 5: Trajectory tracking error determined by the camera

According to Figure 5, the controller's task after determining the angular deviation $\Delta\psi$ is to drive the vehicle so that $\Delta\psi$ approaches 0. Thus, in the dynamic equation system for the sliding mode controller, Δψ must be a variable, in addition, its derivative must also approach 0 at the same time, so $\Delta\dot{\psi}$ is also a variable.
Combine with (2.2), one can get:

$$\begin{cases} \dot{v} = \frac{F_c}{m} - k_v v^2 \\ \Delta\dot{\psi} = -\frac{v}{l} * 2\tan\delta + \dot{\psi}_d \\ \dot{\delta} = \frac{K}{T}u - \frac{1}{T}\delta \end{cases} \quad (25)$$

Then:

$$\Delta\ddot{\psi} = \frac{2}{l}\dot{v}\tan(\delta) + \frac{2v}{l}\frac{1}{\cos^2(\delta)}\dot{\delta} + \ddot{\psi}_d$$

$$= \frac{2}{l}\left(\frac{F_c}{m} - k_v v^2\right)\tan(\delta) + \frac{2v}{l}\frac{1}{\cos^2(\delta)}\left(\frac{K}{T}u - \frac{1}{T}\delta\right) + \ddot{\psi}_d \quad (26)$$

$$\text{Set: } \begin{cases} x_1 = v \\ x_2 = \Delta\psi \\ x_3 = \Delta\dot{\psi} \\ x_4 = \delta \end{cases} \quad (27)$$

Then (25) becomes:

$$\begin{cases} \dot{x_1} = \frac{F_c}{m} - k_v x_1^2 \\ \dot{x_2} = x_3 \\ \dot{x_3} = \frac{2}{l}\left(\frac{F_c}{m} - k_v x_1^2\right)\tan(x_4) + \frac{2x_1}{l}\frac{1}{\cos^2(x_4)}\left(\frac{K}{T}u - \frac{1}{T}x_4\right) + \ddot{\psi}_d \\ \dot{x_4} = \frac{K}{T}u - \frac{1}{T}x_4 \end{cases} \quad (28)$$

Choose sliding variable

$$s = \lambda x_2 + x_3 \quad (29)$$

Where λ is a positive parameter. Then its derivative:

$$\dot{s} = \lambda \dot{x}_2 + \dot{x}_3 \quad (30)$$

Subtitute (28) to (30), then:

$$\dot{s} = G_1 + G_2 + G_3 + G_u \quad (31)$$

Where:

$$G_1 = \frac{2}{l}\left(\frac{F_c}{m} - k_v x_1^2\right)\tan(x_4) \quad (32)$$

$$G_2 = -\frac{2x_1}{l}\frac{1}{\cos^2(x_4)}\frac{1}{T}x_4 \quad (33)$$

$$G_3 = \lambda x_3 + \ddot{\psi}_d \quad (34)$$

$$G_u = \frac{2x_1}{l}\frac{1}{\cos^2(x_4)}\frac{K}{T}u \quad (35)$$

The control input in the sliding mode controller consists of two components, continuous control and discontinuous control.

$$u = u_{eq} + u_d \quad (36)$$

The continuous control component, u_{eq} is defined as the solution of the equation $\dot{s} = 0$, then

$$G_u = -G_1 - G_2 - G_3 \quad (37)$$

Or:

$$\frac{2x_1}{l}\frac{1}{\cos^2(x_4)}\frac{K}{T}u_{eq} = -G_1 - G_2 - G_3 \quad (38)$$

Then u_{eq} can be calculated:

$$u_{eq} = -(G_1 + G_2 + G_3)\frac{lT}{2K}\frac{\cos^2 x_4}{x_1} \quad (39)$$

When applying $u = u_{eq} + u_d$ to (31), u_{eq} will exclude G_1, G_2, G_3, there will be only u_d in G_u expressions:

$$\dot{s} = \frac{2x_1}{l}\frac{1}{\cos^2(x_4)}\frac{K}{T}u_d \quad (40)$$

Choose Lyapunov function:

$$V = \frac{1}{2}s^2 \quad (41)$$

Then:

$$\dot{V} = s\dot{s} \quad (42)$$

To make $\dot{V} \leq 0$. Assume $\dot{V}$ has the form of:

$$\dot{V} = -\eta|s| \leq 0, \text{ where } \eta > 0 \quad (43)$$

Combine (42) and (43):

$$\dot{s} = -\eta \text{ sign}(s) \quad (44)$$

Substitute to (40):

$$\frac{2x_1}{l}\frac{1}{\cos^2(x_4)}\frac{K}{T}u_d = -\eta \text{ sign}(s) \quad (45)$$

With $\cos^2 x_4 \geq 0$, then choose:

$$u_d = -K_d \text{ sign}(s)\text{sign}(x_1), \text{ where } K_d > 0 \quad (46)$$

Where the value of K_d affects the convergence time of the system or the time required for the robot to return to its trajectory.

C. Simulations

The research results were verified and simulated on Matlab with the following parameters. The robot car has a mass of m= 400 kg, a body length of l= 2 m, and a wind resistance parameter of K_v=0.025 Ns^2/m^2. The coefficients in the steering angle controller are T=10; K=1.

The initial condition is that the car deviates from the lane centerline by -1 m, the initial velocity of the car body is 0.5 m/s, the initial angular deviation is 17 degrees, and the initial steering angle is also 17 degrees.

Assume that the car is pushed by a total force of 100 N, and the required lane trajectory has a constant angular velocity of 18 degrees/s, and the distance from the midpoint of the bottom edge of the camera screen to the center of gravity of the car is 1.5 m.

The above PID and sliding mode controllers are simulated on Matlab Simulink software for the object with system parameters and initial state as in section B, demonstrating good trajectory tracking ability with relatively small errors, under the condition of changes in the required lane trajectory.

Results and Discussion

The above PID and sliding mode controllers are simulated on Matlab Simulink software for the object with system parameters and initial state as in section B, demonstrating good trajectory tracking ability with relatively small errors, under the condition of changes in the required lane trajectory.

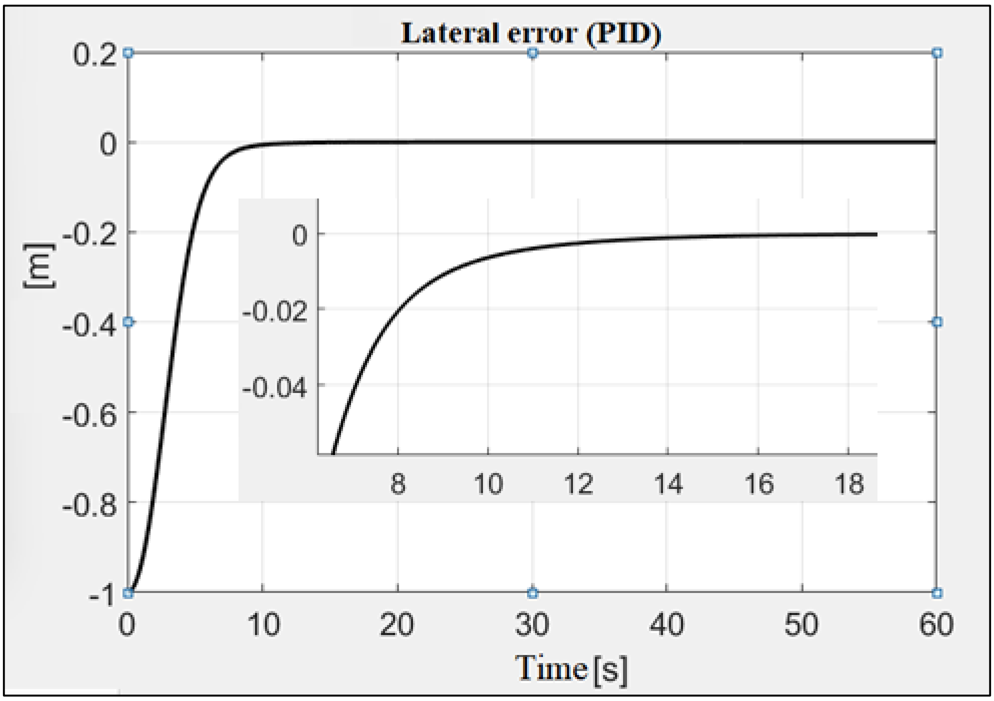

Fig 6: Lateral error by using PID control

Through the simulation results in Figures 6 to 10, both controllers work well with the ability to track the trajectory with high precision and stabilize the vehicle speed and steering angle after tracking at similar values. However, the sliding mode controller shows better trajectory tracking quality with faster convergence time, the sliding variable S, the camera measurement error and the angle deviation converge to zero at 6.8s, so the lateral tracking errors of the vehicle body and the angle error will converge at the same time. While with the PID controller, the lateral error and angle error at the same period are still at 0.015 m and 1.5 degrees. The biggest difference lies in the steering angle of the robot car. In the case of using a sliding mode controller, the steering angle is a smooth line, ensuring that the car does not shake during operation, while the steering angle of the car when using a PID controller has a large shaking in reality, which may be difficult to operate (Figure 9).

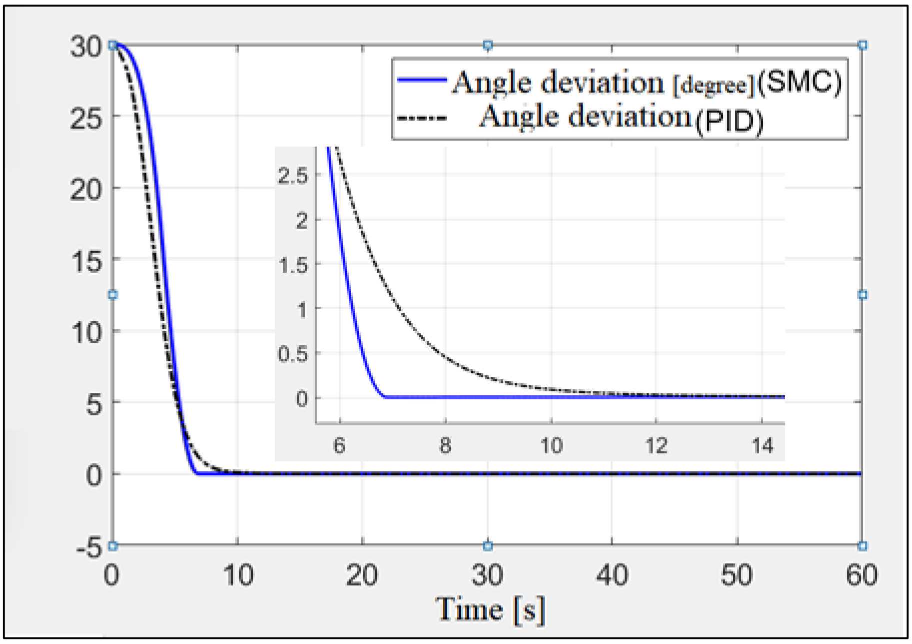

Fig 7: Angle deviation when using PID control and SMC

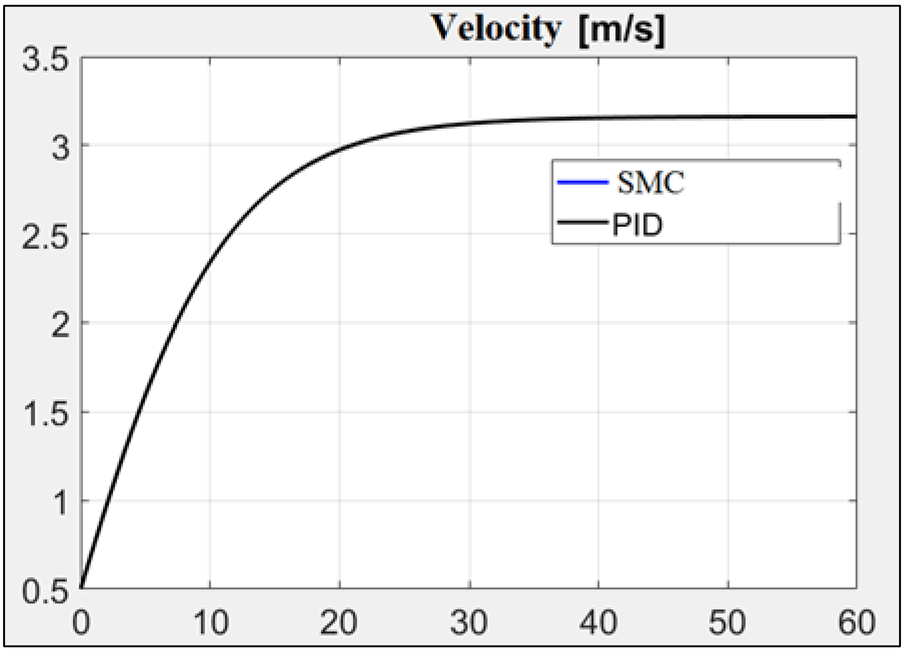

Fig 8: Velocity of robot when using PID control and SMC

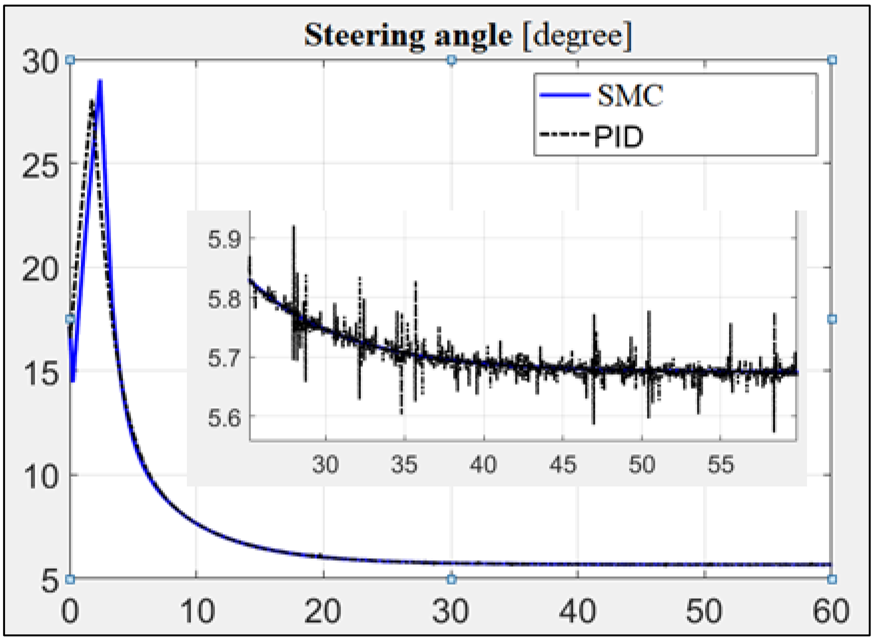

Fig 9: Steering angle when using PID control and SMC

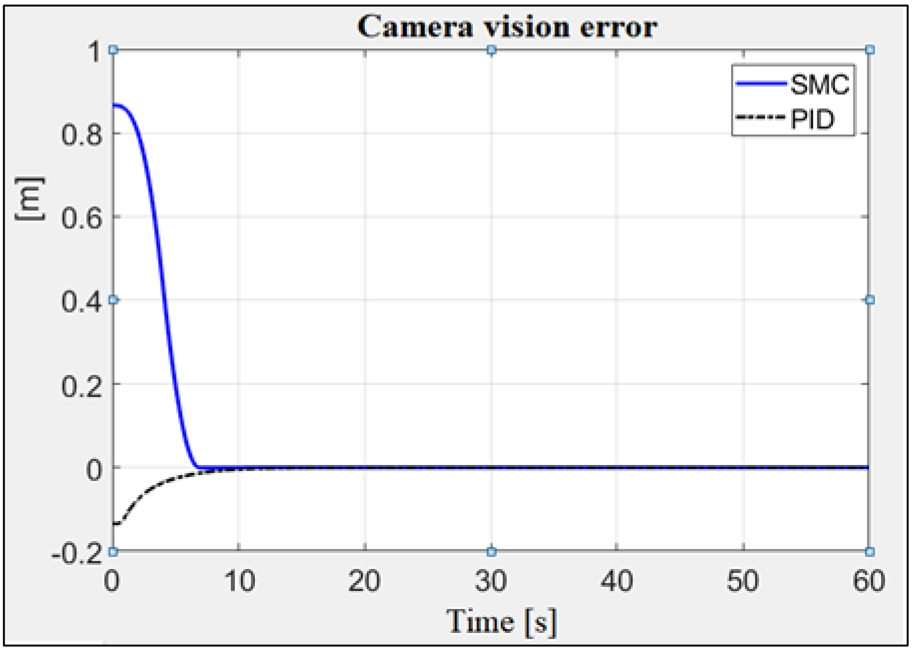

Fig 10: Lateral error that is detected by camera when using PID control and SMC

Conclusions

This article has presented the specific application of sliding mode control and PID control to the problem of lane tracking control for the 4WS mobile robot by experimental simulation on Matlab Simulink software. The simulation results show that by applying the sliding mode control method, the system's tracking quality can be improved, significantly reducing the vibration of the steering wheel during operation. Those results prove that the sliding mode controller can be used in the control system of the 4WS lane tracking robot when performing the required task.

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