Neutrosophic Soft Regular Semiopen Sets in Neutrosophic Soft Topological Spaces

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Abstract
In this paper, we introduce and study the concept of neutrosophic soft regular semiopen and neutrosophic soft regular semiclosed sets in neutrosophic soft topological spaces. By using neutrosophic soft regular semiopen set, we define a new neutrosophic soft interior operator namely neutrosophic soft regular semi interior operator and neutrosophic soft regular semi neighborhood. Also the relation between these sets and counter examples for the reverse relation are given.

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1. Introduction

Molodtsov [22], in the year 1999, introduced the soft set theory as a new mathematical tool. He has established the fundamental results of this new theory and successfully applied the soft set theory in to several directions, such as smoothness of functions, Operation research, Riemann integration, Game theory, Theory of probability and so on. Soft set theory has a wider application and its progress in very rapid in different fields. There is no need of membership function in soft set theory and hence very convenient and easy to apply practice. Shabir and Naz [26] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. Later, Aygunoglu *et al.* [3], Zorlutuna *et al.* [32] and Hussain *et al.* [19] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces.

Smarandache [27, 28, 29] introduced the notion of Neutrosophic set, it is classified into three independent functions namely, membership, indeterminacy and non-membership function that are independently related. Later, Maji [20] has introduced a combined concept Neutrosophic soft set (NSS). Using this concept, several mathematicians have produced their research works in different mathematical structures for instance Arockiarani *et al.* [1, 2], Bera and Mahapatra [4], Deli [14, 15], Deli and Broumi [16], Maji [21], Broumi and Smarandache [11], Salama and Alblowi [24], Saroja and Kalaichelvi [25], Broumi [12], Sahin *et al.* [23]. Later, this concept has been modified by Deli and Broumi [17]. Accordingly, Bera and Mahapatra [4-10] have developed some algebraic structures over the neutrosophic soft set.

In general topology, the notion of regular semiopen set was introduced by Cameron [13] in 1978. Recently, Vadivel and Elavarasan [30, 31] introduced the concept of soft regular semiopen sets and soft regular semi continuous in soft topological spaces. In this concept has been generalized to neutrosophic soft setting. This paper introduce and study the concept of neutrosophic soft regular semiopen (semiclosed) sets in neutrosophic soft topological spaces. By using neutrosophic soft regular semiopen (semiclosed) sets, we define a new neutrosophic soft closure operator namely neutrosophic soft regular semi interior (closure) operator and investigate their properties which are important for further research on neutrosophic soft topology.

2. Preliminaries

Definition 2.1 [27] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of]−0, 1+[. That is $T_A;\ I_A;\ F_A : X \rightarrow]-0, 1+[$. There is no restriction on the sum of $T_A(x);\ I_A(x), F_A(x)$ and so, $-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$.

Definition 2.2 [22] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \rightarrow P(U)$ is a mapping.

Definition 2.3 [20] Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F : A \rightarrow NS(U)$ is a mapping.

This concept has been modified by Deli and Broumi [17] as given below.

Definition 2.4 [17] Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function fN representing a mapping $f_N : E \rightarrow NS(U)$ where fN is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(U)$ and therefore it can be written as a set of ordered pairs, $N = \{(e; \{< x;\ T_{f_{N(e)}}(x);\ I_{f_{N(e)}}(x);\ F_{f_{N(e)}}(x) > : x \in U\}): e \in E\}$ where $Tf_{N(e)}(x);\ If_{N(e)}(x);\ Ff_{N(e)}(x)$ [0; 1], respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_{N(e)}$. Since supremum of each $T;\ I;\ F$ is 1 so the inequality $0 \leq T_{f_{N(e)}}(x) + I_{f_{N(e)}}(x) + F_{f_{N(e)}}(x) \leq 3$ is obvious.

Definition 2.5 [17] The complement of a neutrosophic soft set N is denoted by N^c and is defined by

$$N^c = \{(e; \{< x;\ F_{f_{N(e)}}(x);\ 1 - I_{f_{N(e)}}(x);\ T_{f_{N(e)}}(x) > : x \in U\}): e \in E\}$$

Definition 2.6 [17] Let N_1 and N_2 be two NSSs over the common universe $(U; E)$. Then N_1 is said to be the neutrosophic soft subset of N_2 if $\forall\ e \in E$ and $\forall$ x∈U, $T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x)$_$I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x), F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x)$. We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

Definition 2.7 [17] Let N_1 and N_2 be two NSSs over the common universe $(U; E)$. Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as:

$$N_3 = \left\{\left(e; \left\{< x;\ T_{f_{N_3}(e)}(x);\ I_{f_{N_3}(e)}(x);\ F_{f_{N_3}(e)}(x) > : x \in U\right\}\right): e \in E\right\}$$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \circ T_{f_{N_2}(e)}(x), I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x), F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x)$.

Their intersection is denoted by $N_1 \cap N_2 = N_4$ and is defined as:

$$N_4 = \left\{\left(e; \left\{< x;\ T_{f_{N_4}(e)}(x);\ I_{f_{N_4}(e)}(x);\ F_{f_{N_4}(e)}(x) > : x \in U\right\}\right): e \in E\right\}$$

where $T_{f_{N_4}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x), I_{f_{N_4}(e)}(x) = I_{f_{N_1}(e)}(x) \circ I_{f_{N_2}(e)}(x), F_{f_{N_4}(e)}(x) = F_{f_{N_1}(e)}(x) \circ F_{f_{N_2}(e)}(x)$.

Definition 2.8 [6] Let M, N be two NSSs over $(U; E)$. Then $M - N$ may be defined as, ∀ x∈U; e ∈ E,
$M - N = \{< x;\ T_{f_{M(e)}}(x) * F_{f_{N(e)}}(x);\ I_{f_{M(e)}}(x) \circ (1 - I_{f_{N(e)}}(x));\ F_{f_{M(e)}}(x) \circ T_{f_{N(e)}}(x) >\}$
A neutrosophic soft set N over $(U; E)$ is said to be null neutrosophic soft set if $T_{f_{N_4}(e)}(x) = 0;\ I_{f_{N_4}(e)}(x) = 1;\ F_{f_{N_4}(e)}(x) = 1, x \in U. e \in E$. It is denoted by Φ_u.
A neutrosophic soft set N over $(U; E)$ is said to be absolute neutrosophic soft set if $T_{f_{N_4}(e)}(x) = 1;\ I_{f_{N_4}(e)}(x) = 0;\ F_{f_{N_4}(e)}(x) = 0, x \in U. e \in E$. It is denoted by 1_u..

Clearly, ${\Phi_u}^c = 1_u$ and ${1_u}^c = \phi_u$.

Definition 2.9 [6] Let $NSS(U; E)$ be the family of all neutrosophic soft sets over U via parameters in E and $\tau_u \subset NSS(U; E)$. Then τ_u is called neutrosophic soft topology on (U, E) if the following conditions are satisfied.

1. $\phi_u, 1_u \in \tau_u$
2. the intersection of any finite number of members of τ_u u also belongs to τ_u.
3. the union of any collection of members of τ_u belongs to τ_u.

Then the triplet (U, E, τ_u) is called a neutrosophic soft topological space (for short, nsts). Every member of τ_u is called τ_u-open neutrosophic soft set. An NSS is called τ_u-closed iff it's complement is τ_u-open. There may be a number of topologies on (U, E). If ${\tau_u}'$ and ${\tau_u}''$ are two topologies on (U, E) such that _${\tau_u}' \subset {\tau_u}''$, then ${\tau_u}'$ is called neutrosophic soft strictly weaker (coarser) than τ_u" and in that case τ_u" is neutrosophic soft strict finer than τ_u'. Moreover $NSS(U,E)$ is a neutrosophic soft topology on (U,E).

Definition 2.10 [6] Let (U,E,τ_u) be a neutrosophic soft topological space over (U,E) and $M \in NSS(U,E)$ be arbitrary. Then the interior of M is denoted by M^0 or NSInt(M, E) and is defined as :
M^0=∪{N : N is neutrosophic soft open and N⊂M}.
i.e., it is the union of all open neutrosophic soft subsets of M.

Definition 2.11 [6] Let (U,E,τ_u) be a neutrosophic soft topological space over (U,E) and $M \in NSS(U,E)$ be arbitrary. Then the closure of M is denoted by $\overline{M}$ or NSCl(M, E) and is defined as :
$\overline{M}$=∩{N : N is neutrosophic soft closed and M⊂ N}.
i.e., it is the intersection of all closed neutrosophic soft supersets of M.

Definition 2.12 [6] 1. A neutrosophic soft point in an NSSN is defined as an element
$(e, f_N(e))$ of N, for $e \in E$ and is denoted by e_N, if $f_N(e) \notin \phi_u$, and $f_N(e') \epsilon \phi_u$; $e' \in E - \{e\}$.
2. The complement of a neutrosophic soft point e_N is another neutrosophic soft point ${e_N}^c$ such that ${f^c}_N(e) = (f_N(e))^c$.
3. A neutrosophic soft point $e_N \in M, M$ being an NSS if for the element $e \in E$, $f_N(e) \leq f_M(e)$.

Definition 2.13 [18] A nsts (U,E,τ_u), and a Ns $\tilde{R} = \{(e; \{< x; T_{f_N(e)}(x); I_{f_N(e)}(x); F_{f_N(e)}(x) > : x \in U\}) : e \in E\}$ over (U,E). Then ($\tilde{R}$, A) is called:
1. neutrosophic soft regular open (for short, nsro) iff (R, A)=NSInt(NSCl($\tilde{R}$, A)).
2. neutrosophic soft semi-open (for short, nsso) iff ($\tilde{R}$, A)⊆ NSCl(NSInt($\tilde{R}$, A)).

The complement of nsro, nsso sets are called nsrc, nssc sets.

3. Neutrosophic soft regular semiopen and neutrosophic regular semiclosed sets

Definition 3.1. A nsts (X, E, τ), then ($\tilde{R}$, E) is called:
(i) neutrosophic soft regular semiopen (for short, nsrso) set if ∃ an nsro set ($\tilde{S}$, E) ∋ ($\tilde{S}$, E)⊆($\tilde{R}$, E) ⊆NSCl($\tilde{S}$, E).
(ii) neutrosophic soft regular semiclosed (for short, nsrsc) set if ∃ an nsrc set ($\tilde{S}$, E) ∋ NSInt($\tilde{S}$, E)⊆($\tilde{R}$, E)⊆($\tilde{S}$, E).
We shall denote the family of all nsrso (nsrsc) sets of a nsts (X, E, τ) by NSRSOS(X), NSRSCS(X).

Example 3.1 Let X={a, b}, E={e}. Let us consider the neutrosophic soft sets ($\tilde{R}$, E), ($\tilde{S}$, E) and ($\tilde{T}$, E) over the universal set X as follows:
($\tilde{R}$, E)={e, ⟨(a, 0.40, 0.50, 0.60)⟩, ⟨(b, 0.50, 0.50, 0.50)⟩}
($\tilde{S}$, E)={e, ⟨(a, 0.40, 0.50, 0.40)⟩, ⟨(b, 0.50, 0.50, 0.50)⟩}
($\tilde{T}$, E)={e, ⟨(a, 0.50, 0.50, 0.60)⟩, ⟨(b, 0.50, 0.50, 0.50)⟩}
Clearly τ ={$0_{(X, E)}$, $1_{(X, E)}$, ($\tilde{R}$, E), ($\tilde{S}$, E)} is a nsts over X. The Nss ($\tilde{T}$, E) is nsrso set of X, since ∃ a nsro set ($\tilde{S}$, E) ∋ ($\tilde{S}$, E)⊆($\tilde{T}$, E)⊆NSCl($\tilde{S}$, E).

Example 3.2 In Example 3.1, the Nss ($\tilde{T}$, E) is nsrso set, but ($\tilde{T}$, E) not nsro set, because ($\tilde{T}$, E) ≠ NSInt(NSCl($\tilde{T}$, E)).

Example 3.3 Let X={a, b}, E={e}. Let us consider the neutrosophic soft sets ($\tilde{R}$, E), ($\tilde{S}$, E) and ($\tilde{T}$, E) over the universal set X as follows:
($\tilde{R}$, E)={e, ⟨(a, 0.30, 0.50, 0.60)⟩, ⟨(b, 0.50, 0.50, 0.50)⟩}
($\tilde{S}$, E)={e, ⟨(a, 0.60, 0.50, 0.50)⟩, ⟨(b, 0.50, 0.50, 0.50)⟩}
($\tilde{T}$, E)={e, ⟨(a, 0.40, 0.50, 0.60)⟩, ⟨(b, 0.50, 0.50, 0.50)⟩}

Clearly τ ={$0_{(X, E)}$, $1_{(X, E)}$, ($\tilde{R}$, E), ($\tilde{S}$, E)} is a nsts over X. The Nss ($\tilde{T}$, E) is nsso set of X, since ∃ a nso set ($\tilde{R}$, E) ∋ ($\tilde{R}$, E)⊆($\tilde{T}$, E)⊆NSCl($\tilde{R}$, E). But the Nss ($\tilde{T}$, E) is not nsrso set of X.

Remark 3.1. From definition of nsrso (nsrsc) sets and Examples 3.1, 3.2 and 3.3 it is clear that
1. Every nsros (nsrcs) is a nsrsos (nsrscs) but not conversely.
2. Every nsrsos is a nsso set but not conversely.

Remark 3.2. $0_{(X, E)}$, and $1_{(X, E)}$ are always nsrsc and nsrso sets.
Theorem 3.1. Arbitrary union of nsrso sets is a nsrso set.
Proof. Let {($\tilde{R}$i, E): i ϵ I} be a family of nsrso sets over the universc set X. Then ∃ a nsro sets ($\tilde{S}$i, E) such that ($\tilde{S}$i, E)⊆($\tilde{R}$i, E) ⊆NSCl($\tilde{S}$i, E) for each i. Thus ∪($\tilde{S}$i,E)⊆∪($\tilde{R}$i, E)⊆∪NSCl($\tilde{S}$i, E) and ($\tilde{S}$i, E) is nsros. So ($\tilde{R}$i, E) is a nsrso set.
Remark 3.3. Arbitrary intersection of nsrsc sets is a nsrsc set.
Theorem 3.2. If a nrsos ($\tilde{R}$, E) is such that ($\tilde{R}$, E)⊆($\tilde{T}$, E)⊆NSCl($\tilde{R}$, E), then ($\tilde{T}$, E) is also nsrsos.
Proof. As ($\tilde{R}$, E) is nsrso set, ∃ an nsro set ($\tilde{S}$, E) ∋ ($\tilde{S}$, E)⊆($\tilde{R}$, E)⊆NSCl($\tilde{S}$, E). Then, by hypothesis, ($\tilde{S}$, E)⊆($\tilde{T}$, E) and NSCl($\tilde{R}$, E)⊆NSCl($\tilde{S}$, E). So ($\tilde{T}$, E)⊆NSCl($\tilde{R}$, E)⊆NSCl($\tilde{S}$, E) i.e., ($\tilde{S}$, E)⊆($\tilde{T}$, E)⊆NSCl($\tilde{S}$, E). Hence ($\tilde{T}$, E) is nsrso set.

Theorem 3.3. If a nsrscs $(\tilde{R}, E)$ is such that NSInt$(\tilde{R}, E)\subseteq(\tilde{S}, E)\subseteq(\tilde{R}, E)$ then $(\tilde{S}, E)$ is also nsrsc set.
Definition 3.2. Let (X, E, τ) be a nsts. Then,
(i) the neutrosophic soft regular semiclosure of $(\tilde{R}, E)$ defined by nsrscl$(\tilde{R}, E)$={$(\tilde{S}, E)$|$(\tilde{R}, E)\subseteq(\tilde{S}, E)$ and $(\tilde{S}, E)\in$ NSRSCS(X, E, τ)} is a Nss.
(ii) the neutrosophic soft regular semi interior of $(\tilde{R}, E)$ defined by nsrsint$(\tilde{R}, E)$ ={$(\tilde{S}, E)$|$(\tilde{S}, E)\subseteq(\tilde{R}, E)$ and $(\tilde{S}, E)\in$ NSRSOS(X, E, τ)} is a Nss.
Clearly, nsrscl$(\tilde{R})$ is the smallest nsrscs containing $(\tilde{R}, E)$ and nsrsint$(\tilde{R}, E)$ is the largest nsrsos contained in $(\tilde{R}, E)$.

Theorem 3.4. Let (X, E, τ) be a nsts, $(\tilde{R}, E)$, $(\tilde{S}, E)$ are Nss's of X. Then
(i) $(\tilde{R}, E)\in$ NSRSCS(X)⇔$(\tilde{R}, E)$=nsrscl$(\tilde{R}, E)$,
(ii) $(\tilde{R}, E)\in$NSRSOS(X)⇔$(\tilde{R}, E)$=nsrsint$(\tilde{R}, E)$,
(iii) (nsrscl$(\tilde{R}, E)$)c=nsrsint$(\tilde{R}$c E),
(iv) (nsrsint$(\tilde{R}, E)$)c=nsrscl$(\tilde{R}$c, E),
(v) $(\tilde{R}, E)\subseteq(\tilde{S}, E)$⇒nsrsint$(\tilde{R}, E)\subseteq$nsrsint$(\tilde{S}, E)$,
(vi) $(\tilde{R}, E)\subseteq(\tilde{S}, E)$⇒nsrscl$(\tilde{R}, E)\subseteq$nsrscl$(\tilde{S}, E)$,
(vii) nsrscl(0(X, E))= 0(X, E) and nsrscl(1(X, E)) = 1(X, E),
(viii) nsrsint(0(X, E))= 0(X, E) and nsrsint(1(X, E)) = 1(X, E),
(ix) nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$)=nsrscl$(\tilde{R}, E)\cup$ nsrscl$(\tilde{S}, E)$,
(x) nsrsint($(\tilde{R}, E)\cap(\tilde{S}, E)$) = nsrsint$(\tilde{R}, E)\cap$ nsrsint$(\tilde{S}, E)$,
(xi) nsrscl($(\tilde{R}, E)\cap(\tilde{S}, E)$)⊂ nsrscl$(\tilde{R}, E)\cap$nsrscl$(\tilde{S}, E)$,
(xii) nsrsint($(\tilde{R}, E)\cup(\tilde{S}, E)$)⊂nsrsint$(\tilde{R}, E)\cup$nsrsint$(\tilde{S}, E)$,
(xiii) nsrscl(nsrscl$(\tilde{R}, E)$)=nsrscl$(\tilde{R}, E)$,
(xiv) nsrsint(nsrsint$(\tilde{R}, E)$)=nsrsint$(\tilde{R}, E)$.

Proof. Let $(\tilde{R}, E)$ and $(\tilde{S}, E)$ be two Nss's over X.
(i) Let $(\tilde{R}, E)$ be a nsrsc set. Then it is the smallest nsrsc set containing itself. Thus $(\tilde{R}, E)$ =nrscl$(\tilde{R}, E)$. On the other hand, let $(\tilde{R}, E)$ =nsrscl$(\tilde{R}, E)$ and nsrscl$(\tilde{R}, E)\in$NSRSCS(X). Then $(\tilde{R}, E)\in$NSRSCS(X).
(ii) Similar to (1).
(iii) (nsrscl$(\tilde{R}, E)$)c = (∩{$(\tilde{S}, E)$|$(\tilde{R}, E)\subseteq(\tilde{S}, E)$ and $(\tilde{S}, E)\in$NSRSCS(X)})c =∪{$(\tilde{S}, E)$c|$(\tilde{R}, E)\subseteq(\tilde{S}, E)$ and $(\tilde{S}, E)\in$NSRSCS(X)} =∪{$(\tilde{S}, E)$c|$(\tilde{S}, E)$c$\subseteq(\tilde{R}, E)$c and $(\tilde{S}, E)$c$\in$ NSRSOS(X)} = nsrsint($(\tilde{R}, E)$c).
(iv) Similar to (iii).
(v) Follows from definitions.
(vi) Follows from definitions.
(vii) Since 0(X, E) and 1(X, E) are nsrsc sets so nsrscl(0(X, E)) = 0(X, E) and nsrscl(1(X, E)) =1(X, E).
(viii) Since 0(X, E) and 1(X, E) are nsrso sets so nsrsint(0(X, E))=0(X, E) and nsrsint(1(X, E))=1(X, E).
(ix) We have $(\tilde{R}, E)\subseteq(\tilde{R}, E)\cup(\tilde{S}, E)$ and $(\tilde{S}, E)\subseteq(\tilde{R}, E)\cup(\tilde{S}, E)$. Then by (vi), nsrscl$(\tilde{R}, E)\subseteq$ nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$) and nsrscl$(\tilde{S}, E)\subseteq$ nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$). Thus nsrscl$(\tilde{S}, E)$ ∪ nsrscl$(\tilde{R}, E)$ ⊆ nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$). Since, nsrscl$(\tilde{R}, E)$, nsrscl$(\tilde{S}, E)\in$NSRSCS(X), nsrscl$(\tilde{S}, E)$ ∪ nsrscl$(\tilde{R}, E)$ ∈ NSRSCS(X). Then $(\tilde{R}, E)\subseteq$ nsrscl$(\tilde{R}, E)$ and $(\tilde{S}, E)\subseteq$ nsrscl$(\tilde{S}, E)$ imply that $(\tilde{R}, E)\cup(\tilde{S}, E)\subseteq$ nsrscl$(\tilde{R}, E)$ ∪ nsrscl$(\tilde{S}, E)$. Thus, nsrscl$(\tilde{R}, E)$ ∪ nsrscl$(\tilde{S}, E)$ is nsrsc set containing $(\tilde{R}, E)\cup(\tilde{S}, E)$. But nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$) is the smallest nsrsc set containing $(\tilde{R}, E)\cup(\tilde{S}, E)$. So nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$) ⊆ nsrscl$(\tilde{R}, E)\cup$ nsrscl$(\tilde{S}, E)$. Hence nsrscl($(\tilde{R}, E)\cup(\tilde{S}, E)$)=nsrscl$(\tilde{R}, E)\cup$ nsrscl$(\tilde{S}, E)$.
(x) Similar to (ix).
(xi) We have $(\tilde{R}, E)\cap(\tilde{S}, E)\subseteq(\tilde{R}, E)$ and $(\tilde{R}, E)\cap(\tilde{S}, E)\subseteq(\tilde{S}, E)$. Then nsrscl($(\tilde{R}, E)\cap(\tilde{S}, E)$) ⊆nsrscl$(\tilde{R}, E)$ and nsrscl($(\tilde{R}, E)\cap(\tilde{S}, E)$)⊆nsrscl$(\tilde{S}, E)$. Thus nsrscl($(\tilde{R}, E)\cap(\tilde{S}, E)$) ⊆ nsrscl$(\tilde{R}, E)\cap$nsrscl$(\tilde{S}, E)$.
(xii) Similar to (xi).
(xiii) Since nsrscl$(\tilde{R}, E)\in$NSRSCS(X) so by (i), nsrscl(nsrscl$(\tilde{R}, E)$)=nsrscl$(\tilde{R}, E)$.
(xiv) Since nsrsint$(\tilde{R}, E)\in$NSRSOS(X) so by (ii), nsrsint(nsrsint$(\tilde{R}, E)$)=nsrsint$(\tilde{R}, E)$.

Definition 3.3. A nsts (X, E, τ), and a Nss $(\tilde{R}, E)$ is said to be neutrosophic soft semi-regular if it is both nsso set and nssc set. Equivalently, a Nss $(\tilde{R}, E)$ is said to be neutrosophic soft semi regular open if $(\tilde{R}, E)$ =nssint(nsscl$(\tilde{R}, E)$).
The family of all neutrosophic soft semi-regular set of (X, E, τ) is denoted by NSSRS(X).

Theorem 3.5. If $(\tilde{R}, E)$ is any Nss in a nsts (X, E, τ) then following are equivalent:
(i) $(\tilde{R}, E)\in$NSSRS(X),
(ii) $(\tilde{R}, E)$=nssint(nsscl$(\tilde{R}, E)$),
(iii) ∃ a nsro set $(\tilde{S}, E)$ ∋ $(\tilde{S}, E)\subseteq(\tilde{R}, E)\subseteq$NSCl($(\tilde{S}, E)$).

Proof. (i)⇒(ii) If $(\tilde{R}, E)\in$NSSRS(X), then nssint(nsscl$(\tilde{R}, E)$)=nssint$(\tilde{R}, E)$=$(\tilde{R}, E)$.
(ii)⇒(iii) Suppose $(\tilde{R}, E)$=nssint(nsscl$(\tilde{R}, E)$). Since NSInt(NSCl$(\tilde{R}, E)$)⊆nsscl$(\tilde{R}, E)$ for any Nss $(\tilde{R}, E)$ of X, NSInt(NSCl$(\tilde{R}, E)$)⊆nssint(nsscl$(\tilde{R}, E)$)=$(\tilde{R}, E)$. Since $(\tilde{R}, E)\in$NSSOS(X) we have $(\tilde{R}, E)\subseteq$NSCl(nssint$(\tilde{R}, E)$). Then, we obtain NSInt(NSCl$(\tilde{R}, E)$) ⊆ $(\tilde{R}, E)$ ⊆ NSCl(NSInt$(\tilde{R}, E)$)⊆NSCl(NSInt(NSCl$(\tilde{R}, E)$)). Since NSInt(NSCl(NSInt(NSCl$(\tilde{R}, E)$))) = NSInt (NSCl$(\tilde{R}, E)$),

NSInt(NSCl($\tilde{R}$, E)) is nsro set.
(iii)⇒(i) It is obvious that ($\tilde{R}$, E) ∈NSSOS(X). We have NSInt(NSCl($\tilde{R}$, E))= NSInt(NSCl($\tilde{S}$, E))= ($\tilde{S}$, E) ⊆($\tilde{R}$, E). Then ($\tilde{R}$, E) is nssc set. Thus, we obtain ($\tilde{R}$, E) ∈NSSOS(X).

Proposition 3.1. If ($\tilde{R}$, E) ∈NSSOS(X), then nsscl($\tilde{R}$, E)∈NSSRS(X).

Proof. Since nsscl($\tilde{R}$, E) is nssc set, we show that nsscl($\tilde{R}$, E) ∈ NSSOS(X). Since ($\tilde{R}$, E) ∈ NSSOS(X), ($\tilde{S}$, E)⊆($\tilde{R}$, E)⊆NSCl($\tilde{S}$, E) for nsos ($\tilde{S}$, E) of X. Then, we have ($\tilde{S}$, E) ⊆ nsscl($\tilde{S}$, E) ⊆ nsscl($\tilde{R}$, E)⊆NSCl($\tilde{S}$, E). Thus nsscl($\tilde{R}$, E)∈NSSRS(X).

Proposition 3.2. If ($\tilde{R}$, E) is nsrso set in (X, E, τ), then ($\tilde{R}$, E)c is also nsrso set.

Proof. Follows from the Definition 3.1.

Proposition 3.3. In a nsts (X, E, τ), the nsrc, nsro and nsrclopen sets are nsrso sets.

Definition 3.4. A Nss ($\tilde{R}$, E) in a nsts (X, E, τ) is called neutrosophic soft regular semi neighborhood (briefly, nsrsnbd) of the neutrosophic soft point e_R of X if ∃ a nsrso set ($\tilde{S}$, E) such that e_R∈($\tilde{S}$, E)⊆($\tilde{R}$, E). The neutrosophic soft regular semi neighborhood system of a neutrosophic soft point e_R denoted by NSRSN(e_R), is the family of all its neutrosophic soft regular semi neighborhoods.

Definition 3.5. A Nss ($\tilde{R}$, E) in a nsts (X, E, τ) is called a neutrosophic soft regular semi neighborhood (briefly, nsrsnbd) of the neutrosophic soft set ($\widetilde{S}$, E) if ∃ a nsrso set ($\tilde{T}$, E) ∋ ($\tilde{S}$, E) ⊆($\tilde{T}$, E) ⊆($\tilde{R}$, E).

Theorem 3.6. The neutrosophic soft regular semi neighborhood system NSRSN(e_R) at e_R of a nsts (X, E, τ) has the following:
(i) If ($\tilde{R}$, E)∈NSRSN(e_R), then e_R ∈($\tilde{R}$, E).
(ii) ($\tilde{R}$, E) ∈ NSRSN(e_R) and ($\tilde{R}$, E)⊆($\tilde{S}$, E), then ($\tilde{S}$, E)∈NSRSN(e_R).
(iii) ($\tilde{R}$, E), ($\tilde{S}$, E) ∈ NSRSN(e_R), then ($\tilde{R}$, E)∩($\tilde{S}$, E)∈NSRSN(e_R).
(iv) ($\tilde{R}$, E) ∈ NSRSN(e_R), then ∃ a ($\tilde{S}$, E)∈NSRSN(e_R) such that ($\tilde{R}$, E)∈NSRSN(e_S) for each e_S ∈ ($\tilde{S}$, E).

Proof. (i) If ($\tilde{R}$, E)∈NSRSN(e_R), then ∃ a ($\tilde{S}$, E)∈NSRSOS(X) such that e_R ∈ ($\tilde{S}$, E)⊆($\tilde{R}$, E). Then, we have e_R ∈ ($\tilde{R}$, E).
(ii) Let ($\tilde{R}$, E)∈NSRSN(e_R) and ($\tilde{R}$, E)⊆($\tilde{S}$, E). Since ($\tilde{R}$, E)∈NSRSN(e_R), then ∃ a ($\tilde{T}$, E) ∈ NSRSOS(X) such that e_R ∈($\tilde{T}$, E) ⊆($\tilde{R}$, E). Then, we have e_R ∈($\tilde{T}$, E)⊆($\tilde{R}$, E)⊆($\tilde{S}$, E). Thus ($\tilde{S}$, E) ∈ NSRSN(e_R).
(iii) If ($\tilde{R}$, E), ($\tilde{S}$, E) ∈ NSRSN(e_R), then ∃ ($\tilde{T}$, E), ($\tilde{U}$, E) ∈ NSRSOS(X) such that e_R ∈ ($\tilde{T}$, E) ⊆($\tilde{R}$, E) and e_R ∈($\tilde{U}$, E)⊆($\tilde{S}$, E). Thus e_R∈ ($\tilde{T}$, E)∩($\tilde{U}$, E)⊆($\tilde{R}$, E)∩($\tilde{S}$, E). Since ($\tilde{T}$, E)∩($\tilde{U}$, E) ∈ τ, we have ($\tilde{R}$, E)∩($\tilde{S}$, E)∈NSRSN(e_R).
(iv) If ($\tilde{R}$, E)∈NSRSN(e_R), then ∃ a ($\tilde{U}$, E)∈NSRSOS(X) such that e_R∈($\tilde{U}$, E)⊆($\tilde{R}$, E). Put ($\tilde{T}$, E)=($\tilde{U}$, E). Then for every e_S∈($\tilde{T}$, E), e_S∈($\tilde{T}$, E)⊆($\tilde{U}$, E)⊆($\tilde{R}$, E). This implies ($\tilde{R}$, E) ∈ NSRSN(e_S).

4. Conclusion

In this paper, we have studied the concept of nsrso and nsrsc sets in nsts's. By using nsrso (nsrsc) sets, we define a new neutrosophic soft interior, closure operators namely neutrosophic soft regular semi interior, neutrosophic soft regular semi closure operator. Also the relation between these sets and counter examples for the reverse relation are given. In future, we can be extended to neutrosophic soft regular semi continuous, neutrosophic soft regular semi irresolute and neutrosophic soft regular semi contra continuous in nsts.

5. References

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