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Some Fixed Point Theorems using *C*-Class Functions in S Metric Spaces

Sheeba Khan [1*], **Dr. Abha Tenguriya** [2]
[1] Research Scholar, Department of Mathematics, Barkatullah University Bhopal, Madhya Pradesh, India
[2] Professor and head, Department of Mathematics, M.L.B Girls P.G Autonomous College Bhopal, Madhya Pradesh, India

* Corresponding Author: **Sheeba Khan**

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Abstract
A very important step in the development of the fixed point theory was given by A.H. Ansari by the introduction of a C−class function. Using C−class functions, we generalize some known fixed point results. In this paper, we prove some common fixed point theorems on S metric spaces via C-class functions using some nonnegative reals and give some consequences of the main result and some examples in support of the main result. The obtained result, in this paper generalizes, extend and improve several results from the existing literature regarding S-metric spaces.

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1. Introduction

Fixed point theory is one of the most important research tools in nonlinear analysis. In the last decades, many authors have published papers in fixed point theory and battened continuously. The application potential is the main cause for this involvement. Fixed point theory has an application in many areas such as, physics, chemistry, biology, computer science and many branches of mathematics. The Banach contraction mapping principle ([3]) or the Banach fixed point theorem is the most famous and pioneer result in a complete metric space. The famous Banach contraction mapping principle states that every self-mapping Q defined on a complete metric space (X, d) satisfying the condition:

$$d(Q(x), Q(y)) \leq k\, d(x, y) \quad (1.1)$$

For all $x, y \in X$, where $k \in (0, 1)$ is a constant, has a unique fixed point and for every $x0 \in X$ a sequence $\{Qn\, x0\}n\geq 1$ is convergent to the fixed point.

After this result Most of the works were basically generalizations of the work of Banach. These generalizations include more general metric spaces like b-metric space, M-metric space and s-metric space etc, or more general contractions etc. in our work we determined fixed point using s-metric space

In 2012, Sedghi et al. [28] introduced the concept of a S-metric space which is different from other spaces and proved fixed point theorems in such spaces. They also give some examples of a S-metric space which shows that the S-metric space is different from other spaces. They built up some topological properties in such spaces and proved some fixed point theorems in the framework of S-metric spaces. After this grateful beginning work of Sedghi et al. [28] many authors attracted to study the problems of the fixed point, common fixed point, coupled fixed point and common coupled fixed point by using various contractive conditions for mappings (see, for examples, [5, 6, 8, 13, 18, 29, 30, 31]).

Recently, a large number of authors have published many papers on S-metric spaces in different directions (see, e.g.,g., [9, 10, 11, 12, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27,32, 33] and many others).

In 2014, Ansari [1] introduced the notion of C-class function that is pivotal result in fixed point theory.

In this article, we prove some common fixed point theorems on S-metric spaces via C- class functions and give some consequences of the main result. We also give some examples to demonstrate the validity of the result. Our results generalize, extend and improve several results from the existing literature.

2. Preliminaries

In this section, we recall some basic definitions, lemmas and auxiliary results to prove our main result.

Definition 2.1. ([28]) Let X be a nonempty set and let $S: X^3 \to [0, \infty)$ be a function satisfying the following conditions for all u, v, w, t ∈ X hold with
(S1) $S(u, v, w) = 0$ if and only if $u = v = w$;
(S2) $S(u, v, w) \leq S(u, u, t) + S(v, v, t) + S(w, w, t)$.

Then, the function S is called an S-metric on X and the pair (X, S) is called an S-metric space or simply SMS.

Example 2.2([28]) Let $X = R_n$ and $\|.\|$ a norm on X, then

$$S(u, v, w) = \| v + w - 2u \| + \| v - w \| \text{ is an S-metric on X.}$$

Example 2.3([28]) Let X be a nonempty set and d be an ordinary metric on X. Then $S(u, v, w) = d(u, w) + d(v, w)$ for all u, v, w ∈ X is an S-metric on X.

Example 2.4([28]) Let X = R be the real line. Then $S(u, v, w) = |u - w| + |v - w|$ for all u, v, w ∈ R is an S-metric on X. This S-metric on X is called the usual S-metric on X.

Definition 2.5 Let (X, S) be an S-metric space. For ε > 0 and u ∈ X we define respectively the open ball $B_s(u, \varepsilon)$ and closed ball $B_s[u, \varepsilon]$ with center u and radius ε as follows:

$$B_s(u, \varepsilon) = \{v \in X: S(v, v, u) < \varepsilon\},$$
$$B_s[u, \varepsilon] = \{v \in X: S(v, v, u) \leq \varepsilon\}.$$

Example 2.6([29]) Let X = R. Denote $S(u, v, w) = |v + w - 2u| + |v - w|$ for all u, v, w ∈ R.

Then
$B_s(1, 2) = \{v \in R: S(v, v, 1) < 2\} = \{v \in R: |v - 1| < 1\}$
$= \{v \in R: 0 < v < 2\} = (0, 2)$,

and
$B_s[2, 4] = \{v \in R: S(v, v, 2) \leq 4\} = \{v \in R: |v - 2| \leq 2\}$
$= \{v \in R: 0 \leq v \leq 4\} = [0, 4]$.

Definition 2.7. ([28], [29]) Let (X, S) be an S-metric space and A ⊂ X.
(γ_1) The subset A is said to be an open subset of X, if for every x ∈ A there exists c > 0 such that $B_s(x, c) \subset A$.
(γ_2) A sequence $\{r_n\}$ in X converges to r ∈X if $S(r_n, r_n, r) \to 0$ as $n \to \infty$, that is, for each ε > 0, there exists $n_0 \in N$ such that for all $n \geq n_0$ we have $S(r_n, r_n, r) < \varepsilon$. We denote this by $\lim_{n\to\infty} r_n$ or $r_n \to r$ as $n \to \infty$.
(γ_3) A sequence $\{r_n\}$ in X is called a Cauchy sequence if $S(r_n, r_n, r_m) \to 0$ as $n, m \to \infty$, that is, for each ε > 0, there exists $n_0 \in N$ such that for all $n, m \geq n_0$ we have $S(r_n, r_n, r_m) < \varepsilon$.
(γ_4) The S-metric space (X, S) is called complete if every Cauchy sequence in X is convergent.
(γ_5) Let τ be the set of all A ⊂ X having the property that for every x ∈ A, A contains an open ball centered in x. Then τ is a topology on X (induced by the S-metric space).
(γ_6) A nonempty subset A of X is S-closed if closure of A is equal to A.

Definition 2.8. Let X be a non-empty set and let A, B: X → X be two self mappings of X. Then a point u ∈ X is called a (Ω_1) fixed point of operator A if A (u) = u and a (Ω_2) common fixed point of A and B if A (u) = B (u) = u.

Definition 2.9. ([28]) Let (X, S) be an S-metric space. A mapping A: X → X is said to be a contraction if there exists a constant $0 \leq k < 1$ such that

$$S(Au, Av, Aw) \leq k\, S(u, v, w) \tag{2.1}$$

for all u, v, w ∈ X.

Remark 2.10. ([28]) If the S-metric space (X, S) is complete and A: X → X is a contraction mapping, then A has a unique fixed point in X.

Definition 2.11([28]) Let (X, S) and (X′, S′) be two S-metric spaces. A function R: X → X′ is said to be continuous at a point $x_0 \in X$ if for every sequence $\{r_n\}$ in X with S $(r_n, r_n, x_0) \to 0$, S′(R(r_n), R(r_n), R(x_0)) → 0 as n → ∞. We say that R is continuous on X if R is continuous at every point $x_0 \in X$.

Definition 2.12([1]) A mapping F: [0, ∞) × [0, ∞) → R is called a C-class function if it is continuous and satisfies the following axioms:
(i) F (s, t) ≤ s;
(ii) F (s, t) = s implies that either s = 0 or t = 0, for all s, t ∈ [0, ∞).

Note that for some F, we have that F (0, 0) = 0. The letter C denotes the set of all C-class functions. The following example shows that C is nonempty.

Example 2.13. ([1]) Each of the functions F: [0, ∞) × [0, ∞) → R defined below are elements of C.
1. F (s, t) = s − t;
2. F (s, t) = ms, 0 < m < 1;
3. F (s, t) = $\frac{s}{(1+t)^t}, r \in (0, \infty)$
4. F (s, t) = $\frac{log\,(t+a^S)}{1+t}, a > 1;$
5. F (s, t) = = $\frac{in\,(1+a^S)}{2}, a > e$
6. F (s, t) = $(s + l)^{\frac{1}{(1+t)^r}} - l, l > 1, r \in (0, \infty);$
7. F (s, t) = $slog_{t+a}\, a, a > 1;$
8. F (s, t) = $s - \left(\frac{1+s}{2+s}\right)\left(\frac{t}{1+t}\right)$;
9. F (s, t)= $s\beta(s), where \beta: [0, \infty) \to [0, \infty) and\ is\ continuous$;
10. F (s, t)= $s - \left(\frac{t}{k+t}\right)$;
11. F (s, t)=$\frac{s}{(1+s)^r}, r \in (0, \infty);$

Definition 2.14([1]) A function ψ: [0, ∞) → [0, ∞) is called an altering distance function if the following properties are satisfied:
(ψ1) ψ is non-decreasing and continuous function;
(ψ2) ψ (t) = 0 if and only if t = 0.

Remark 2.15 We denote Ψ the class of all altering distance functions.

Definition 2.16([1]) A function ϕ: [0, ∞) → [0, ∞) is said to be an ultra altering distance function, if it is continuous, non-decreasing such that ϕ (t) > 0 for t > 0.

We denote by Φu the class of all ultra altering distance functions.

Lemma 2.17([28], Lemma 2.5) Let (X, S) be an S-metric space. Then, S (u, u, v) = S (v, v, u) for all u, v ∈ X.
Lemma 2.18([28], Lemma 2.12) Let (X, S) be an S-metric space. If $r_n \to r$ and $p_n \to p$ as
n → ∞ then $S(r_n, r_n, p_n) \to S(r, r, p)$ as n → ∞.
Lemma 2.19([6], Lemma 8) Let (X, S) be an S-metric space and A be a nonempty subset of X. Then A is S-closed if and only if for any sequence $\{r_n\}$ in A such that $r_n \to r$ as n → ∞, then r ∈ A.
Lemma 2.20([28]) Let (X, S) be an S-metric space. If c > 0 and x ∈ X, then the ball $B_s(x, c)$
is a subset of X.

Lemma 2.21([29]) The limit of a convergent sequence in a S-metric space (X, S) is unique.

Lemma 2.22([28]) In a S-metric space (X, S), any convergent sequence is Cauchy.

3. Main Results

In this section, we shall prove common fixed point theorem on S-metric spaces via C-class functions and rational type contraction.

Theorem 3.1 Suppose (X, S) be a complete S-metric space and t_1, t_2 : X → X be two self-mappings satisfying the inequality:

$$\psi(S(t_1x, t_1y, t_2z,) \leq F\big(\psi\big(\eta(x, y, z)\big), \varphi(\eta(x, y, z))\big) \tag{3.1}$$

Where

$$\eta(x,y,z) = \alpha_1 S(x,y,z) + \alpha_2 S(x,x,t_1x) + \alpha_3 S\left(\frac{S(z,z,t_2z)}{1+S(x,y,z)}\right)$$

For all $x, y, z \in X$, where $\alpha_1, \alpha_2, \alpha_3 > 0$ are non negative reals with $\alpha_1+\alpha_2+\alpha_3 < 1, \psi \in \Psi, \varphi \in \Phi_u$ and $F \in C$.then t_1, t_2 have a unique common fixed point in X.

Proof: for each $x_0 \in X$.Let $x_{2n+1} = t_1x_{2n}$ and $x_{2n+2} = t_2x_{2n+1}$ for n=0, 1, 2…..we prove that $\{x_n\}$ is a Cauchy sequence in (X, S). It follows from (3.1) x=y= x_{2n}, z = x_{2n-1} and using $S_1$and S_2 and lemma (2.18).we have

$$\psi(S(x_{2n+1},x_{2n+1},x_{2n})) = \psi(S(t_1x_{2n},t_1x_{2n},t_2x_{2n-1},))$$

$$\leq F\left(\psi(\eta(x_{2n},x_{2n},x_{2n-1})),\varphi(\eta(x_{2n},x_{2n},x_{2n-1},))\right), \quad (3.2)$$

Where $\eta(x_{2n},x_{2n},x_{2n-1}) = \alpha_1 S(x_{2n},x_{2n},x_{2n-1}) + \alpha_2 S(x_{2n},x_{2n},t_1x_{2n}) + \alpha_3 S\left(\frac{S(x_{2n-1},x_{2n-1},t_2x_{2n-1})}{[1+S(x_{2n},x_{2n},x_{2n-1})]}\right)$

$$= \alpha_1 S(x_{2n},x_{2n},x_{2n-1}) + \alpha_2 S(x_{2n},x_{2n},x_{2n+1}) + \alpha_3 S\left(\frac{S(x_{2n-1},x_{2n-1},x_{2n})}{[1+S(x_{2n},x_{2n},x_{2n-1})]}\right)$$
$$\leq \alpha_1 S(x_{2n},x_{2n},x_{2n-1}) + \alpha_2 S(x_{2n+1},x_{2n+1},x_{2n}) + \alpha_3 S\left(\frac{S(x_{2n},x_{2n},x_{2n-1})}{[1+S(x_{2n},x_{2n},x_{2n-1})]}\right)$$
$$\leq \alpha_1 S(x_{2n},x_{2n},x_{2n-1}) + \alpha_2 S(x_{2n+1},x_{2n+1},x_{2n}) + \alpha_3 S\,(x_{2n},x_{2n},x_{2n-1}) \quad (3.3)$$

Using equation (3.3)in equation (3.2) and using the property of F,we get $\psi(S(x_{2n+1},x_{2n+1},x_{2n})) \leq F\left(\psi(\,(\alpha_1 + \alpha_3)S(x_{2n},x_{2n},x_{2n-1})) + \alpha_2(S\,(x_{2n+1},x_{2n+1},x_{2n})\right)$,

$$\leq \psi\left((\,\alpha_1 + \alpha_3)S(x_{2n},x_{2n},x_{2n-1})) + \alpha_2(S\,(x_{2n+1},x_{2n+1},x_{2n})\right), \quad (3.4)$$

Since $\psi \in \Psi$, so using the property of ψ, we deduce that

$$S(x_{2n+1},x_{2n+1},x_{2n}) \leq \psi\left((\,\alpha_1 + \alpha_3)S(x_{2n},x_{2n},x_{2n-1})) + \alpha_2(S\,(x_{2n+1},x_{2n+1},x_{2n})\right),$$

Or

$$S(x_{2n+1},x_{2n+1},x_{2n}) \leq \left(\frac{\alpha_1+\alpha_3}{1-\alpha_2}\right) S(x_{2n},x_{2n},x_{2n-1}) \quad (3.5)$$
$$= pS(x_{2n},x_{2n},x_{2n-1})$$

Where $p = \left(\frac{\alpha_1+\alpha_3}{1-\alpha_2}\right) < 1$

$\because\ \alpha_1 + \alpha_3 < 1$. This implies that:

$$S(x_{n+1},x_{n+1},x_n) \leq pS(x_n,x_n,x_{n-1}) \quad (3.6)$$

For n =0, 1, 2, 3…..

let $D_n = S(x_{n+1},x_{n+1},x_n)$ *and* $D_{n-1} = S(x_n,x_n,x_{n-1})$. Then from equation (3.6), we conclude that

$$D_n \leq pD_{n-1} \leq p^2D_{n-2} \leq \cdots \leq p^nD_0 \quad (3.7)$$

There for since $0 \leq p < 1$, taking the limit as $n \to \infty$, we have

$$\lim_{n\to\infty,} S(x_{n+1},x_{n+1},x_n) = 0 \quad (3.8)$$

Now we shall show that $\{x_n\}$ is a Cauchy sequence in(X, S)

Thus for any $n, m \in \mathbb{N}$ with $m > n$ and using lemma (2.18), then we have

$$S(x_n,x_n,x_m) \leq 2S(x_n,x_n,x_{n+1}) + S(x_m,x_m,x_{n+1})$$
$$= 2S(x_n,x_n,x_{n+1}) + S(x_{n+1},x_{n+1},x_m)$$
$$\leq 2S(x_n,x_n,x_{n+1}) + 2S(x_{n+1},x_{n+1},x_2) + S(x_{n+2},x_{n+2},x_m)$$
$$\leq 2S(x_n,x_n,x_{n+1}) + 2S(x_{n+1},x_{n+1},x_2) + 2S(x_{n+2},x_{n+2},x_{n+3}) + \cdots + S(x_{m-1},x_{m-1},x_m)$$
$$\leq 2(p^n + p^{n+1} + p^{n+2} + \cdots + p^{m-1})S(x_0,x_0,x_1)$$
$$= 2(p^n + p^{n+1} + p^{n+2} + \cdots + p^{m-1})D_0$$

$$\leq \left(\frac{2p^n}{1-p}\right) D_0 \to 0 \; as \; n, m \; \to \infty$$

since $0 \leq t < 1$. Thus, the sequence $\{x_n\}$ is a Cauchy sequence in the space (X, S). By the completeness of the space, there exists $u \in X$ such that $\lim_{n\to\infty} x_n = u$

Now, we shall show that u is a fixed point of t_2 For this, using the given inequality (3.1) for $x = y = x_{2n}$ and $z = u$, we have

$$\psi(S(x_{2n+1}, x_{2n+1}, t_2u)) = \psi(S(t_1x_{2n}, t_1x_{2n}, t_2u,))$$
$$\leq F\left(\psi(\eta(x_{2n}, x_{2n}, u)), \varphi(\eta(x_{2n}, x_{2n}, u,))\right) \tag{3.9}$$

Where

$$\eta(x_{2n}, x_{2n}, u) = \alpha_1 S(x_{2n}, x_{2n}, u) + \alpha_2 S(x_{2n}, x_{2n}, t_1x_{2n}) + \alpha_3 S\left(\frac{S(u,u,t_2u)}{[1+S(x_{2n},x_{2n},u)]}\right)$$
$$= \alpha_1 S(x_{2n}, x_{2n}, u) + \alpha_2 S(x_{2n}, x_{2n}, x_{2n+1}) + \alpha_3 S\left(\frac{S(u,u,t_2u)}{[1+S(x_{2n},x_{2n},u)]}\right)$$

letting $n \to \infty$in the above inequality and using (S1), we get

$$\eta(x_{2n}, x_{2n}, u) = (\alpha_3)S(u, u, t_2u) \tag{3.10}$$

using equation (3.10) in (3.9) and using the property F, We have

$$\psi(S(x_{2n+1}, x_{2n+1}, t_2u)) \leq F(\psi((\alpha_3)S(u, u, t_2u)), \varphi(\alpha_3)S(u, u, t_2u)))$$
$$\leq \psi((\alpha_3)S(u, u, t_2u))) \tag{3.11}$$

Letting $n \to \infty \; in \; equation$ (3.11) we obtain

$$\psi(S(u, u, t_2u)) \geq \psi((\alpha_3)S(u, u, t_2u)) \tag{3.12}$$

Since $\in \Psi$, so using the property of ψ in equation (3.12),we deduce that

$$S(u, u, t_2u) \leq (a_3)S(u, u, t_2u)$$
$$\leq (a_1 + a_2 + a_3)S(u, u, t_2u)$$
$$< S(u, u, t_2u) \text{ Since } a_1 + a_2 + a_3 < 1$$

Which is a contradiction.Hence $S(u, u, t_2u) = 0$, that is $t_2u = u$. This shows that u is a fixed point of g.By similar fashion, we can shows that $t_1u = u$. Consequently, u is a common fixed point of t_1 and t_2.

Now, we shall show the uniqueness. Let u_1 be another common fixed point of t_1 and t_2

Such that $t_1u_1 = u_1 = t_2u_1$ with $u_1 \neq u$. Using given contractive condition (3.1) for x=y=u=z=u_1 and using (S1) and Lemma 2.18, we obtain

$$\psi(S(u, u, u_1)) = \psi(S(t_1u, t_1u, t_2u_1,))$$
$$\leq F\left(\psi(\eta(u, u, u_1)), \varphi(\eta(u, u, u_1))\right) \tag{3.13}$$

Where

$$\eta(u, u, u_1) = \alpha_1 S(u, u, u_1) + \alpha_2 S(u, u, t_1u) + \alpha_3 S\left(\frac{S(u_1,u_1,t_2u_1)}{[1+S(u,u,u_1)]}\right)$$
$$= \alpha_1 S(u, u, u_1) + \alpha_2 S(u, u, u) + \alpha_3 S\left(\frac{S(u_1,u_1,u_1)}{[1+S(u,u,u_1)]}\right)$$
$$= \alpha_1 S(u, u, u_1)$$

Substituting in equation (3.13) and using the property of F,we have

$$\psi(S(u, u, u_1)) \leq F(\psi((\alpha_1)S(u, u, u_1)), \varphi((\alpha_1)S(u, u, u_1)))$$
$$\leq \psi((\alpha_1)S(u, u, u_1)) \tag{3.14}$$

Since$\psi \in \Psi$, so using the property of ψ in equation (3.14),we deduce that

$$S(u, u, u_1) \leq ((\alpha_1)S(u, u, u_1))$$
$$\leq (a_1 + a_2 + a_3)S(u, u, u_1)$$
$$< S(u, u, u_1), \text{ Since } a_1 + a_2 + a_3 < 1 \tag{3.15}$$

Which is a contradiction.Hence $S(u,u,u_1) = 0$, that is $u = u_1$.This shows the uniqueness of the common fixed point of t_1 and t_2.this completes the proof.

Putting $t_1 = t_2$ in Theorem 3.1, then we obtain the following result.

Corollary 3.2 Let (X, S) be a complete S-metric space and f: X → X be a self-mapping satisfying the following inequality:

$$\psi(S(t_1x, t_1y, t_1z,)) \leq F(\psi(\Theta(x,y,z)), \varphi(\Theta(x,y,z))) \quad (3.16)$$

Where

$$\Theta(x,y,z) = \alpha_1 S(x,y,z) + \alpha_2 S(x,x,t_1x) + \alpha_3 S\left(\frac{S(z,z,t_1z)}{1+S(x,y,z)}\right)$$

For all $x, y, z \epsilon X$, where $\alpha_1, \alpha_2, \alpha_3 > 0$ are non negative reals with $\alpha_1 + \alpha_2 + \alpha_3 < 1, \psi \in \Psi, \varphi \in \Phi_u$ and $F \in C$.then t_1 has a unique common fixed point in X.
proof. This result immediately follows from Theorem 3.1 by taking $t_1 = t_2$.

If we take F(s,t)=ms for some $m \in [0,1)$ *and* $\psi(t) = t$ *for all* $t \geq 0$ in theorem 3.1 then we have the following result (with $ma_1 \to a_1, ma_2 \to a_2, ma_3 \to a_3$)

Corollary 3.3 Let (X, S) be a complete S-metric space and $t_1, t_2: X \to X$ be two self mappings satisfying the inequality

$$S(t_1x, t_1y, t_2z) \leq \alpha_1 S(x,y,z) + \alpha_2 S(x,x,t_1x) + \alpha_3 S\left(\frac{S(z,z,t_2z)}{1+S(x,y,z)}\right) \quad (3.17)$$

for all x, y, z ∈ X, where $a_1, a_2, a_3 > 0$ are nonnegative reals with $a_1 + a_2 + a_3 < 1$ Then t_1 and t_2 have a unique common fixed point in X.

Proof Follows from Theorem 3.1 by taking F (s, t) = ms for some m ∈ [0, 1) and ψ(t) = t for all t ≥ 0 with $ma_1 \to a_1, ma_2 \to a_2, ma_3 \to a_3$

Putting g = f in Theorem 3.1, then we obtain the following result.

Example 3.4 Let $X = [0.1]$ and $t_1, t_2: X \to X$ be given by $t_1(x) = \frac{x}{2}$ and $t_2(x) = \frac{x}{4}$ for all $x \in X$. Define the function $S: X^3 \to [0, \infty)$ by $S(x,y,z) = \max\{x,y,z\}$ for all $x, y, z \in X$, then S is an S Metric on X. Let $x, y, z \in X$ such that $x \geq y \geq z$.
We have

$$S(t_1x, t_1y, t_2z) = \max\left\{\frac{x}{2}, \frac{y}{2}, \frac{z}{4}\right\} = \frac{x}{2}$$
$$S(x,y,z) = \max\{x,y,z\} = x$$
$$t_2S(x,x,t_1x) = \max\left\{x, x, \frac{x}{2}\right\} = x$$
$$S(z,z,t_2z) = \max\left\{z, z, \frac{z}{4}\right\} = z$$

Consider the inequality (3.17) of corollary 3.3, we have

$$\frac{x}{2} \leq \alpha_1 x + \alpha_2 x + \alpha_3 \frac{z}{1+z}$$

Putting x=1, $y = \frac{1}{2}, z = \frac{1}{3}$, then we have

$$\frac{1}{2} \leq \alpha_1 + \alpha_2 + \frac{1}{6}\alpha_3$$
$$3 \leq 6\alpha_1 + 6\alpha_2 + \alpha_3$$

The above inequality is satisfied for: (1) $\alpha_1 = \frac{1}{3}, \alpha_2 = \frac{1}{4}, \alpha_3 = 0$; (2) $\alpha_2 = \frac{1}{2}, \alpha_1 = \alpha_3 = 0$, $a_1 + a_2 + a_3 < 1$. Thus all the condition of corollary 3.3 are satisfied.Hence by applying corollary 3.3, t_1 and t_2 have a unique common fixed point in X. Indeed $0 \in X$ is the unique fixed point of t_1 and $t_2$in this case.

4. Conclusion

In this paper, we prove some common fixed point theorems in the setting of complete S-metric spaces via C-class functions and rational type contraction, we give example in support of our result. Also, we give some consequences as corollaries of the established result. The result obtained in this paper extend, generalize and enrich several results from the existing literature regarding complete S-metric spaces.

References

1. Ansari H. Note on ϕ−ψ\phi - \psiϕ−ψ-contractive type mappings and related fixed points. The 2nd Regional Conference on Mathematics and Applications, Payame Noor University; 2014:377-80.
2. Ansari AH, Chandok S, Ionescu C. Fixed point theorems on bbb-metric spaces for weak contractions with auxiliary functions. Journal of Inequalities and Applications. 2014;2014:429.
3. Banach S. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. Fundamenta Mathematicae. 1922;3:133-81.
4. Branciari A. A fixed point theorem for mappings satisfying a general contractive condition of integral type. International Journal of Mathematics and Mathematical Sciences. 2002;29(9):531-6.
5. Van Dung N, Hieu NT, Radojevic´ S. Fixed point theorems for ggg-monotone maps on partially ordered SSS-metric spaces. Filomat. 2014;28:1885-98.
6. Gupta A. Cyclic contraction on SSS-metric space. International Journal of Analysis and Applications. 2013;3(2):119-30.
7. Hoxha E, Ansari AH, Zoto K. Some common fixed point results through generalized altering distances on dislocated metric spaces. Proceedings of EIIC; 2014:403-9.
8. Kim JK, Sedghi S, Shobkolaei N. Common fixed point theorems for the RRR-weakly commuting mappings in SSS-metric spaces. Journal of Computational Analysis and Applications. 2015;19:751-9.
9. Kim JK, Sedghi S, Gholidahneh A, Rezaee MM. Fixed point theorems in SSS-metric spaces. East Asian Mathematical Journal. 2016;32(5):677-84.
10. Mlaiki N, Celik U, Tas N, Özgür NY, Mukheimer A. Wardowski-type contractions and the fixed-circle problem on SSS-metric spaces. Journal of Mathematics. 2018;Article ID 9127480:9 pp.
11. Mlaiki N, Özgür NY, Tas N. New fixed point theorems on an SSS-metric space via simulation functions. Mathematics. 2019;7:583. doi:10.3390/math7070583.
12. Nashine HK, Saluja GS, Ibrahim RW. Some fixed point theorems for (ψ−ϕ)(\psi - \phi)(ψ−ϕ)-almost weak contractions in SSS-metric spaces solving conformable differential equations. Journal of Inequalities and Applications. 2020;2020:139.
13. Özgür NY, Tas N. Some fixed point theorems on SSS-metric spaces. Matematički Vesnik. 2017;69:39-52.
14. Özgür NY, Tas N. New contractive conditions of integral type on complete SSS-metric spaces. Mathematical Sciences. 2017;11(3):231-40.
15. Özgür NY, Tas N. Some new contractive mappings on SSS-metric spaces and their relationships with the mapping S25S_{25}S25. Mathematical Sciences. 2017;11(1):7-16.
16. Özgür NY, Tas N. The Picard theorem on SSS-metric spaces. Acta Mathematica Scientia. 2018;38(4):1245-58.
17. Özgür NY, Tas N. Some fixed-circle theorems on metric spaces. Bulletin of the Malaysian Mathematical Sciences Society. 2019;42:1433-49.
18. Rahman MU, Sarwar M. Fixed point results of Altman integral type mappings in SSS-metric spaces. International Journal of Analysis and Applications. 2016;10:58-63.
19. Saluja GS. Some fixed point results on SSS-metric spaces satisfying implicit relation. Journal of Advanced Mathematical Studies. 2019;12(3):256-67.
20. Saluja GS. Common fixed point for generalized (ψ−ϕ)(\psi - \phi)(ψ−ϕ)-weak contractions in SSS-metric spaces. The Aligarh Bulletin of Mathematics. 2019;38(1–2):41-62.
21. Saluja GS, Dinu S, Petrescu L. Fixed points for (ψ−ϕ)(\psi - \phi)(ψ−ϕ)-weak contractions in SSS-metric spaces. UPB Scientific Bulletin, Series A. 2020;82(2):119-32.
22. Saluja GS. Fixed point results under implicit relation in SSS-metric spaces. The Mathematics Student. 2020;89(3-4):111-26.
23. Saluja GS. Fixed point theorems on cone SSS-metric spaces using implicit relation. Cubo, A Mathematical Journal. 2020;22(2):273-88.
24. Saluja GS. Some fixed point theorems for weak contraction mappings in SSS-metric spaces. Jnanabha. 2020;50(1):20-6.
25. Saluja GS. Some existence results for contractive-type mappings in cone SbSbSb-metric spaces. Analele Universității din Oradea, Fascicula Matematică. 2020;27(1):63-77.
26. Saluja GS. Some fixed point theorems under implicit relation on SSS-metric spaces. Bulletin of the International Mathematical Virtual Institute. 2021;11(2):327-40.
27. Saluja GS. Fixed point results for F(S,T)F(S, T)F(S,T)-contraction in SSS-metric spaces using implicit relation with applications. International Journal of Mathematics and Combinatorics. 2022;1:17-30.
28. Sedghi S, Shobe N, Aliouche A. A generalization of fixed point theorems in SSS-metric spaces. Matematički Vesnik. 2012;64(3):258-66.
29. Sedghi S, Dung NV. Fixed point theorems on SSS-metric spaces. Matematički Vesnik. 2014;66(1):113-24.
30. Sedghi S, Shobe N, Desˇenovic´ T. Fixed point results in SSS-metric spaces. Nonlinear Functional Analysis and Applications. 2015;20:55-67.
31. Sedghi S, Rezaee MM, Desˇenovic´ T, Radenovic´ S. Common fixed point theorems for contractive mappings satisfying Φ\PhiΦ-maps in SSS-metric spaces. Acta Universitatis Sapientiae Mathematica. 2016;8:298-311.

32. Tas N, Özgür NY. Common fixed points of continuous mappings on SSS-metric spaces. Mathematical Notes. 2018;104(4):587-600.
33. Tas N. Suzuki-Berinde-type fixed-point and fixed-circle results on SSS-metric spaces. Journal of Linear and Topological Algebra. 2018;7:233-44.