The Impact of Thermal Radiation and Viscous Dissipation on Magnetohydrodynamics (MHD) Heat and Mass Transfer Mixed Convective Nanofluid Flow over a Stretching Wedge-Shaped Surface

Areo AO [1*], Oladosu YA [2], Odebiyi O A [3], Olajide OA [4], Adetunde IA[5]

[1-4] Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Oyo State, Nigeria

[5] Adetunde I.A. W.V.S. Tubman University College of Engineering and Technology, Department of Mechanical Engineering, Harper, Maryland County, Republic of Liberia

* Corresponding Author: **Areo AO**

Article Info

ISSN (online): 2582-7138
Volume: 05
Issue: 06
November-December 2024
Received: 23-09-2024
Accepted: 25-10-2024
Page No: 800-810

Abstract

Heat and mass transfer in nanofluid flow has gained significant attention due to its applications in various engineering and industrial processes. The incorporation of magnetic fields (Magnetohydrodynamics), thermal radiation and viscous dissipation in these systems further adds complexity and relevance, particularly in optimizing thermal systems. This aims to summarize the key contributions and findings in the literature regarding the effects of thermal radiation and viscous dissipation on MHD heat and mass transfer mixed convective nanofluid flow over a stretching wedge-shaped surface.

The non-linear partial differential equations, governing the flow field under consideration have been transformed by a similarity transformation into a system of non-linear ordinary differential equations (ODEs) and then solved numerically by Chebyshev collocation with the aid of mathematical software Mathematica 11.

The results reveal significant influences of thermal radiation and viscous dissipation on magnetohydrodynamics mixed convection heat and mass transfer of nanofluid flow. Resulting non- dimensional velocity, temperature and concentration profiles are then presented graphically for different valued of the parameters.

Keywords: Thermal Radiation, Viscous Dissipation, Magnetohydrodynamics, Mixed Convective Flow and Stretching Wedge-Shaped Surface

Introduction

The study of heat and mass transfer in nanofluid flow has gained significant attention due to its applications in various engineering and industrial processes. The incorporation of magnetic fields (Magneto-hydrodynamics), thermal radiation, and viscous dissipation in these systems further adds complexity and relevance, particularly in optimizing thermal systems. This aims to summarize the key contributions and findings in the literature regarding the effects of thermal radiation and viscous dissipation on MHD heat and mass transfer mixed convective nanofluid flow over a stretching wedge-shaped surface.

The conversion of kinetic energy into thermal energy due to viscous forces, affects the temperature distribution within the fluid. Gebhart (1962) [20] introduced the concept of viscous dissipation in convective heat transfer. Later studies, such as those by Aydin and Kaya (2009) [6], have focused on its effects in different flow configurations, highlighting the importance of considering viscous dissipation in high-viscosity or high-velocity flows.

Thermal radiation becomes significant in high-temperature processes, impacting the thermal boundary layer and overall heat transfer. Cogley *et al*. (1968) [11] developed a method for handling radiative heat transfer in optically thin fluids, which has been adapted for various applications.

Crane (1970) [12] initiated this field by examining flow over a linearly stretching sheet. Extending this to wedge-shaped surfaces, Vajravelu and Rollins analyzed the flow and heat transfer characteristics, providing insights into the complex boundary layer behavior.

Nanofluids, which are fluids containing nanometer-sized particles, exhibit enhanced thermal properties compared to conventional fluids. Choi and Eastman (1995) [14] pioneered this field, highlighting the potential of nanofluids in improving thermal conductivity. Subsequent studies have explored various nanoparticle materials, such as metallic and oxide nanoparticles, and their impact on heat transfer efficiency.

Choi and Eastman (1995) [13] revolutionized the field of heat transfer. Their pioneering work demonstrated the enhanced thermal properties of fluids containing nanometer-sized particles, laying the groundwork for extensive research on nanofluids. This reveals that thermal radiation, viscous dissipation, and MHD significantly impact the heat and mass transfer in nanofluid flows over stretching wedge-shaped surfaces. The integration of these factors in theoretical and numerical models is crucial for accurately predicting and optimizing the performance of thermal systems. Future research should focus on experimental validation and the development of more comprehensive models to account for additional effects such as variable properties and non-Newtonian behavior.

Model governing free convection boundary-layer flow over an isothermal inclined plate embedded in a thermally stratified porous medium in the presence of a non-uniform transverse magnetic field. Hossain *et al.* (1999) investigated the impact of radiation on boundary layer flow, revealing its importance in enhancing or suppressing heat transfer.

The influence of magnetic fields on fluid flow, known as MHD, is critical in many applications, including cooling of electronic devices, nuclear reactors, and metallurgical processes. This study on MHD flow and heat transfer of nanofluids over a stretching wedge, incorporating thermal radiation and viscous dissipation effects, extended the understanding of these interactions to more complex geometries. Their findings indicated that thermal radiation and viscous dissipation significantly influence the temperature distributions and heat transfer rates, with MHD effects further modifying the flow behavior.

Mahapatra and Gupta (2002) [34] studied heat and mass transfer in mixed convective flow over a stretching sheet, incorporating the effects of thermal radiation and viscous dissipation. Their work, though initially focused on conventional fluids, has been extended by subsequent researchers to nanofluids, providing a foundational understanding of mixed convection in complex geometries. Ghaly and Elbarbary (2002) [21] and Takhar *et al.* (1999) [23] studied the MHD BL flow of gas under diverse situations with radiation effect on the heat transfer rate.

Buongiorno (2006) [7] reviewed on convective transport in nanofluids addressed the fundamental mechanisms, such as Brownian motion and thermophoresis, which affect the heat and mass transfer properties of nanofluids. His work provided a theoretical framework for understanding nanofluid behavior in convective flows. Buongiorno has reported in his works that the absolute velocity of nanoparticles is equal to the velocity of the base fluid and the slip velocity. In his study, he has considered seven slip mechanisms, such as inertia, Brownian diffusion, thermophoresis, diffusionphoresis, Magnus effect, fluid drainage, and gravity settling. He has concluded that among them the Brownian motion and thermophoresis is important in the absence of turbulent effects and proposed a model by considering the effects of Brownian motion and thermophoresis. In this regard, several authors, have used the Buongiorno model to study the heat and mass transfer flow of nanofluids under different conditions and their diverse applications. Electrically conducting magnetohydrodynamics (MHD) fluid flow investigation throughout heated wedge has remarkable applications in the area of MHD power generators, nuclear reactors cooling, plasma studies, and boundary layer control in aerodynamics. The magnetic nanofluid represents precedence in MHD fluid flow and an external source control the heat transfer rate it is relevant to diverse fields of thermal engineering in aerospace and electronic device as well as biomedical science, healthcare devices, and microscope design, etc. The magnetic nanofluid is the colloidal postponement of the carrier liquid and magnetic nanoparticles.

Tiwari and Das (2007) [45] developed a mathematical model for studying heat transfer enhancement in a nanofluid-filled enclosure under different convective conditions. Their model has been widely used to predict heat transfer performance in various geometries and flow configurations, providing insights into the impact of nanoparticles on mixed convective flows. Wang (2007) [47] worked on mixed convection in nanofluid flow over stretching surface provided critical insights into the boundary layer behavior in such systems. He investigated how the stretching velocity and nanoparticle concentration affect the heat and mass transfer characteristics, contributing to the optimization of thermal systems using nanofluids. The study of fluid flow over stretching surfaces is relevant in materials processing and polymer extrusion.

Kuznetsov and Nield (2010) [29] explored natural convective boundary-layer flow of nanofluids past a vertical plate. Their study incorporated the effects of Brownian motion and thermophoresis, key factors in nanofluid heat and mass transfer. Their findings highlighted the significant influence of nanoparticles on convective heat and mass transfer rates. Similarly, Makinde and Aziz (2011) [11] studied the combined effects of radiation and viscous dissipation on boundary layer flow over a stretching sheet in nanofluids, demonstrating significant alterations in heat transfer rates. Recent studies have integrated these aspects, examining the combined effects of MHD, thermal radiation, and viscous dissipation on nanofluid flow over stretching surfaces.

The investigation on various viscosities of nanofluids and developing the model on thermal conductivity by numerous researchers over the past decade. Zhang *et al.* (2014) [48] investigated the MHD flow and heat transfer of nanofluids over a stretching wedge, incorporating thermal radiation and viscous dissipation effects. Their findings indicated that both thermal radiation and viscous dissipation significantly influence the temperature profiles and heat transfer rates, with MHD effects further modifying the flow behavior.

Mahapatra *et al.* (2013) [35] examined the influence of thermal radiation and heat generation on an unsteady two-dimensional natural convection flow in an inclined enclosure heated from one side and cooled from the adjacent side under the influence of a magnetic field using staggered grid finite-difference technique has been studied. The governing equations have been solved numerically for streamlines, isotherms, local Nusselt numbers and the average Nusselt number for various values of thermal radiation and heat generation parameters by considering three different inclination angles and magnetic field directions, keeping the aspect ratio fixed. It is found that magnetic field suppresses the convection flow and its direction influences the flow pattern which results in the appearance of inner loop and multiple eddies.

Chamkha and Rashad (2014) [10] discussed the convective nanofluid flow considering heat transfer with entropy generation. Siva and Shamshuddin (2015) [42] explained the heat and mass transfer on the MHD flow of micropolar fluid in the presence of viscous dissipation and chemical reaction. The governing non-linear partial differential equations are transferred into a system of non-linear ordinary differential equations using similarity transformations and then solved numerically using the finite element method. The numerical results are compared and found to be in good agreement with previous results as special case of (viscous dissipation) the present investigation. The influence of various flow parameters on the fluid has been discussed and explained graphically.

Siva and Srinivasa (2015) [43] investigated soret effect on unsteady MHD free convective flow past a semi-infinite vertical plate in the presence of viscous dissipation. The governing equations are solved numerically by finite element method, computations are performed for a large range of governing equations. The effects of the flow parameters on the velocity, temperature and concentration are shown graphically and the numerical value of the parameters are shown in tables. Through their research on heat and mass transfer in MHD fluid flow over a stretching sheet with velocity and thermal slip conditions, Fatunmbi and Adeniyan (2018) [18] explained the influences of temperature dependent viscosity, thermal radiation and non-uniform heat generation/absorption and chemical reaction of a general order are examined on the fluid flow.

Recent investigations of boundary layer flow and heat transfer have included the consideration of nanofluids in order to enhance the thermal conductivity, for example Khan and Pop (2010) [25] investigated boundary layer flow of a nanofluid over a stretching sheet, a work which was then extended by Makinde and Olanrewaju (2010) [32] who considered the effects of convective surface boundary conditions. Gbadeyan *et al.* (2017) [19] extended the work of Makinde and Aziz by including the effects of a magnetic field and thermal radiation. Mabood *et al.* (2015) [30] provided a numerical study of MHD boundary layer flow with viscous dissipation over a nonlinearly stretching sheet and compared the results with those of previous work\

Krishna *et al.* (2018) [26] observed the Hall effects on MHD flows considering porous medium in different Plate and channels. Discussed the MHD mixed convective fluid flow with heat transfer through porous medium with the magnetic field effect. Pal and Roy (2017) [38] examined the thermal radiation effect in nanofluid flow considering the Brownian motion and slip velocity over a stretching space.

Have investigated the MHD dominated Cu-water nanofluid flow in a porous enclosure and found that for all values of heat generation parameter inside the porous cavity the entropy generation is decreasing for increasing the volume fraction and miniature volume fraction of nanofluid has a significant consequence on the thermal rendering. In a high-temperature operating system, radiation has a great impact on the heat and mass transfer of the fluid flows, Many processes are carried out at a high temperature in the fluid of engineering. In such circumstances, the investigation of radiative heat transfer plays a vital role in the choice of suitable equipment. An instance of such fields as atomic and nuclear power plants, artificial satellites, gas turbines, the aircraft industry, missile manufacturing, wind turbine, etc.

Presented the fluid flow and heat transfer analysis due to various conditions. Reddy *et al.* (2017) [40] presented MHD boundary layer nanofluid flow with heat and mass transfer characteristics considering chemical reaction. Veera *et al.* (2019) [46] explained the heat and mass transfer on magnetohydrodynamic chemically reacting flow of a micropolar fluid through a porous medium with Hall Effect. Governing equation are solved using perturbation method, the effects of various parameters were discussed on velocity, micro-rotation, concentration and temperature with boundary layer conditions. An increase in Hall current and radiation parameter augments momentum and thermal boundary layer thickness and decelerates micro-rotational velocity.

Dogonchi *et al.* (2018) [16] and investigated natural convective nanofluid flow with thermal characteristics through a porous and magnetic field. Krishna *et al.* (2021) [27] explored the Hall effects on MHD convective flows of a second grade rotating fluid in porous medium considering heat generation or absorption. Dogonchi *et al.* (2018) [17] discussed natural convection due to the generating heat and magnetic field with the shape factor of nanoparticles.

Akindele *et al.* (2021)(a) [2] examined the study of squeezing flow cannot be over emphasized due to its numerous applications in manufacturing industries, food processing, lubricating system, power transmission and medical equipment are few of such processes.. The governing partial differential equations were transformed to ordinary differential equations in terms of suitable similarity variables together with the initial and boundary conditions. The resulting equations were then solved using Newton's finite difference technique with the aid of MAPLE 18.0 software. The effect of radiation, magnetic parameter, temperature dependent heat source parameter, surface dependent heat source parameter and other associated physical parameters on the flow system were reported.

Akindele *et al.* (2021)(b) [3] studied the rapid development of modern nanotechnology has brought about the importance of particles of nanometer-size (normally less than 100 nm) as compared to those of macro meter-size. The Nano-fluid model was used to account for the effects of Brownian and Thermophoresis motion on the flow system. The resulting nonlinear system of equations is then solved using Newton's Finite difference technique by MAPLE 18.0 software. The numerical solution obtained showed the effects of the associated physical parameters on the velocities, temperature, pressure and the concentration profiles presented graphically.

Akindele *et al.* (2021)(c) [4] investigated the unsteady convective flow of nanofluid using Blasius-Rayleigh-Stokes variable with slip effect. The non-Newtonian fluid problem is examined mathematically via a system of governing partial differential equations to determine the effect of moving slot parameter, thermal Grashof number and mass Grashof number on the flow. A suitable similarity variable is employed to transform these equations into the corresponding ordinary differential equations and then solved using the method of Newton's Finite differences adopted from MAPLE 18.0 software. From the graphical representations derived, an overview of the effects of some physical parameters on the velocity, temperature and the concentration profiles is then presented.

Obalalu *et al.* (2021) [2] analyses the simulation of non-Darcian Casson flow subject to a second-order velocity slip and heat transfer due to nanofluid over a permeable stretching surface is exemplified numerically. The model, which is governed by the system of PDEs, accomplishes the Chebyshev collocation Method (CCM). It is vital to remark that the account for the second-order slip velocity in the boundary conditions decreases the velocity component. In addition, the role of (Prandtl-number = 7 water) on the Skin friction coefficient becomes more significant when the Variable viscosity increases compared to the (Prandtl-number = 0.71 air). Also, non-Newtonian Casson fluid parameter show a solid characteristic when yield stress is more than the shear stress. Consequently, those parameters contribute to the cooling plate.

Kumar *et al.* (2021) [28] studied MHD Casson Fluid Flow along MHD Casson flow fluid over an inclined plate with aligned magnetic, Hall current and thermal radiation in the presence of Chemical reaction is examined. The governing equations are solved using perturbation method. The effects of various physical parameters like as chemical reaction parameter, radiation parameter, Casson parameter, Schmidt number, Grashof number, modified Grashof number, Prandtl number, magnetic parameter, inclined angle, Hall parameter and Aligned parameter are discussed for velocity, temperature and concentration. The skin-friction, Nusselt number and Sherwood number are also obtained and are shown in tabular form.

The research on heat and mass transfer in mixed convective nanofluid flows has been extensive, with significant contributions from various researchers. Pioneering works by Choi and Eastman, foundational studies by Buongiorno, and comprehensive analyses by Kuznetsov and Nield, Tiwari and Das, and Wang have advanced our understanding of nanofluid behavior. Studies by Chamkha, Makinde, Aziz, Khan, and Pop have further elucidated the combined effects of magnetic fields, thermal radiation, and viscous dissipation on nanofluid flows. Experimental research by Eastman, Das, and others has provided critical validation for theoretical models, driving the field forward. Reviews by Wong and Leon and Kakaç have summarized these advancements and identified future research directions, ensuring continuous progress in optimizing thermal systems using nanofluids.

Recent investigations highlight how viscous dissipation influences the thermal performance of nanofluids in MHD flows. For instance, a study on hybrid nanofluid MHD motion over a stretching surface revealed that viscous dissipation alters the heat rates, especially when combined with factors like thermal radiation and internal heat sources. In 2023, a study analysed the viscous dissipation and radiation effects in MHD flow of Casson fluids past a moving wedge. The researchers found that viscous dissipation significantly affects temperature profiles and skin friction under varying magnetic and thermal boundary conditions. Another study by Jayanthi S. and Niranjan H. (2023) [24] focused on MHD nanofluids flow along a vertical surface under the effect of Joule heating, viscous dissipation and activation energy. This research demonstrated how viscous dissipation interacts with thermal radiation and chemical reactions, influencing heat and mass transfer in engineering applications.

Saeed *et al.* (2022) [41] examined the magnetohydrodynamic Casson fluid flow through an extending surface embedded in a porous medium. The analytical solution of the modeled problem is utilized with the help of homotopy analysis method (HAM). The convergence region of the applied technique is portrayed graphically. The results show that the augmented Darcy number, Casson and magnetic parameters have declined the velocity profile of the Casson fluid flow.

The heat transfer characteristics of a nanofluid depend on the size, volume fraction, shape and thermal properties of nanoparticles as well as the base fluid properties. Generally speaking, numerical simulation of the velocity field, the temperature field and the heat transfer rate of nanofluids can be performed using either a single-phase approach or a two-phase approach. The former assumes that the base fluid and the nanoparticles are in thermal equilibrium, and thus, move with the same velocity. The latter considers the notion that slip velocity between the base fluid and the nanoparticles may not be zero, in which case relevant slip mechanisms between solid and liquid phases such as Brownian diffusion and thermophoresis may be included in the model.

Investigated the Magnetohydrodynamic mixed convection heat and mass transfer of nanofluid flow over a stretching wedge-shaped surface with the effect of thermophoresis and brownian motion. The nature of the MHD BL nanofluid flow along a stretching wedge-shaped surface with radiation, heat source, and chemical reaction parameters was numerically investigated. The fundamental equations was transformed into ordinary differential equations (ODEs) by the usual transformation and the numerical solutions are found by employing Runge-Kutta fourth-order method by exploiting symbolic software MATLAB via the shooting method. It was established that Brownian motion and thermophoresis parameters play a crucial role in the variation of temperature and concentration profiles and also in the development of thermal and concentration boundary layers.

Oyinkansola *et al.* (2023) [37] the magnetohydrodynamic (MHD) chemically reactive of Casson non-Newtonian nanofluid flow on a two-dimensional incompressible steady from stretched sheet in a porous quiescent medium with buoyancy effect is investigated numerically. Additional effects included in the originality of the model are the applied magnetic field and solar radiation effect. The Chebyshev collocation method (CCM) was used to solve the ordinary differential equations (ODEs) with MATHEMATICA 11.3 software. The numerical outcomes demonstrate the effect of different physical parameters of the fluid, and it was observed that the velocity profile increased as the thermal and mass Grashof number increased due to an increase in buoyant force caused by heat transferred from the vertical plate to the fluid but decreased as the Casson parameter increased due to a decrease in its yield stress, porosity, and magnetic parameter. Also Analyses reveal that the thermal profile reduce with an increase in variable thermal conductivity parameter.

Areo *et al.* (2024) [1] studied on the influence of radiating fluid, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. The radiative and viscous dissipation effects on a steady two-dimensional magnetohydrodynamics free convection flow along stretching sheet with heat generation is analyzed. The nonlinear partial differential equations governing the flow field under consideration was transformed by a similarity transformation into a system of nonlinear ordinary differential equations and then solved numerically by shooting iteration with the aid of mathematical software Maple 18. It was drawn that radiating fluid can increase the rate of temperature because with increase in the radiating fluid the temperature tends to increase.

Investigated the effect of a Riga plate on the flow characteristics of a casson hybrid nanofluid through a stretching cylindrical embedded in a porous medium in the presence of an exponential heat source and thermal radiation. The governing partial differential equations are converted into a system of nonlinear ordinary differential equations using similarity variables and and the heat transfer phenomena, fluid flow dynamics and nanoparticle behavior in the base fluid were captured through numerical analysis. The Runge Kunta Order 4 with shooting technique was used to obtain numerical solution. The result showed that the Riga plate affected the flow patterns.

In this research work, the impact of thermal radiation and viscous dissipation on magnetohydrodynamics (MHD) heat and mass transfer mixed convective nanofluid flow over a stretching wedge-shaped surface.

Mathematical Formulation

In this study, steady two dimensional mixed convective magnetohydrodynamic (MHD) fluid of nanofluid over a stretching wedge-shaped with chemical reaction of radiating effects. The fluid is viscous, incompressible, and electrically conducting. The coordinates xy are taken along and vertical to the wedge surface, respectively. The velocity o the stretching wedge is taken into account $U_w(x)=ax^m$ where a>0 while $U(x)=bx^m$ indicating the free stream velocity outside the boundary layer of the nanofluid. A magnetic effect B_0 is used normally to the flow of the nanofluid.

The governing boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU(x)}{dx}v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho f}(u-U)-\left(\frac{\upsilon}{k_1}\right)U+\beta_T g(T-T_\infty)+\beta_c^* g(C-C_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\upsilon}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 -\frac{1}{(\rho C_P)_f}\frac{\partial q_r}{\partial y}+\frac{Q_0}{(\rho C_P)_f}(T-T_\infty) \tag{3}$$

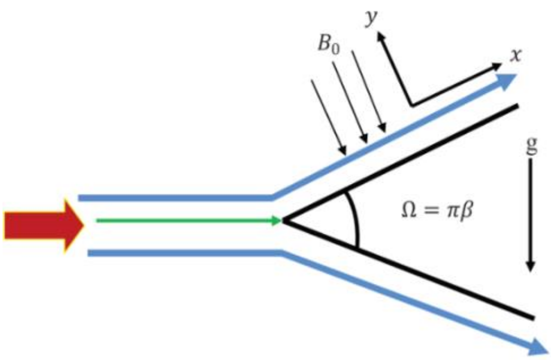
$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-K^*(C-C_\infty) \tag{4}$$

The appropriate boundary conditions are

At y=0, $u=U_w=ax^m$, v=0, $-k\frac{\partial T}{\partial y}=h_f(T_f-T)$, $D_B\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\frac{\partial T}{\partial y}=0$, and at $y\to\infty,\ u\to U=bx^m,\ T\to T_\infty,\ C\to C_\infty$ (5)

The radiative heat flux can be taken as

$$q_r=\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$



Wedge flow coordinate system. (MHD mixed convection heat and mass transfer of nanofluid flow over a stretching wedge-shaped surface).

Where T^4 is the linear sum of the temperature. Using Taylor series expansion, T^4 can be expanded T_∞ as:

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+\frac{12T_\infty^2}{2}(T-T_\infty)^2+\frac{24T_\infty}{6}(T-T_\infty)^3 \tag{7}$$

The temperature differences are taken to be very small, so neglecting the higher-degree terms involving $(T - T_\infty)$

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty) \tag{8}$$

Putting (8) into (6)

$$q_r=-\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y} \tag{9}$$

Substitute (9) into (3)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{\upsilon}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 -\frac{1}{(\rho C_P)_f}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{(\rho C_P)_f}(T-T_\infty) \tag{10}$$

Simplification of Partial Differential Equations

In other to convert the set of PDEs i.e ((2) (4) and (10)) into a set of dimensionless ODEs, we introduce the following usual transformation:

$$\psi=\left(\frac{2Ux\upsilon}{1+m}\right)^{\frac{1}{2}}f, \tag{11.1}$$

$$\eta=\left[\frac{(1+m)U}{2x\upsilon}\right]^{\frac{1}{2}}y, \tag{11.2}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}, \tag{11.3}$$

$$\phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty}, \tag{11.4}$$

$$u=Uf'(\eta) \tag{11.5}$$

$$v = \frac{yU}{2x} f'(\eta) - \left(\frac{U\upsilon}{2x(1+m)} \right)^{\frac{1}{2}} f(\eta) \qquad (11.6)$$

Where ψ is the stream function, θ and φ are the non-dimensional temperature and concentration parameters. Produces the following differential equations.

$$f'''(\eta) + \frac{f''(\eta) f(\eta)}{1+m} + 2Gr\theta + 2Gm\phi + M(1 - f'(\eta)) + \beta - \beta KP_0 \qquad (12)$$

$$\theta''\left(1 + \frac{4}{3}R\right) + \Pr\left(\frac{1}{1+m} f\theta' + Nb\theta'\phi' + Nt\theta'^2 + Q\theta \right) + \Pr Ecf''^2(\eta) = 0 \qquad (13)$$

$$\phi''(\eta) + \frac{Nt}{Nb}\theta''(\eta) + Sc(f\phi' - K\phi) \qquad (14)$$

With the boundary conditions

f(0)=0, f(0)=A, $\theta'(0) = -Bi(1-\theta(0)),\ Nb\phi'(0) + Nt\theta'(0) = 0,$ at η=0, $f' \to 1,\ \theta \to 0,$
$\phi \to 0,$ at $\eta \to \infty$ (15)

Where:

$f(\eta) = f,$
$\theta(\eta) = \theta,$
$\phi(\eta) = \phi,$

$\beta = \frac{2m}{1+m}$ Is the Gradient of hartree pressure?

$M = \frac{2x\sigma B_0^2}{\rho_f U(1+m)}$ Is the magnetic field parameter

$Gr = \frac{\beta_T g(T_f - T_\infty)x}{U^2(1+m)}$ Is the thermal Grashof number

$Gm = \frac{\beta_c^* g(C_f - C_\infty)}{U^2(1+m)}$ Is the mass Grashof number

$\Pr = \frac{\upsilon}{\alpha}$ Is the Parndtl number

$R = \frac{4T_\infty^3\sigma^*}{kK^*}$ Is the radiation parameter

$Nb = \frac{\tau D_B(C_f - C_\infty)}{\upsilon}$ Is the Brownian motion parameter

$Nt = \frac{\tau D_T(T_f - T_\infty)}{\upsilon T_\infty}$ Is the Thermophoresis parameter

$Q = \frac{2xQ_0}{(\rho C_p)_f U(1+m)}$ Is the heat generation parameter

$Sc = Le \times \Pr = \frac{\upsilon}{D_B}$ Is the Schmidt number

$Bi = \frac{h_f}{k}\sqrt{\frac{2x\upsilon}{(m+1)U}}$ is the Biot number

$A = \frac{U_w}{U_\infty} = \frac{a}{b}$ is the dimensionless parameter

$Ec = \frac{U^2}{C_p(T_f - T_\infty)}$ is the Eckert number

Results and Discussion

The solution of heat and mass transfer flow of magnetohydrodynamic, in the presence of viscous dissipation and thermal radiation over a stretching wedge-shaped surface of a mixed convective fluid flow give (12-14) along with the boundary conditions. The physical properties were investigated examining the effect of magnetic parameter M, Gradient of Hartree pressure β, Radiation R, Prandtl number Pr, Brownian motion Nb, Thermophoresis Nt, viscous dissipation, Schmidt parameter on Velocity, Concentration and Temperature profile.

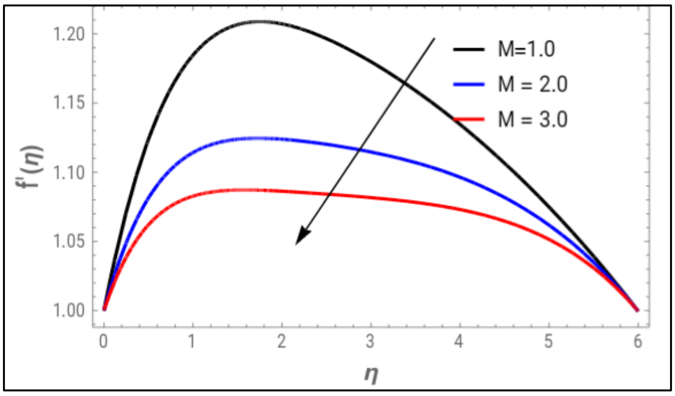
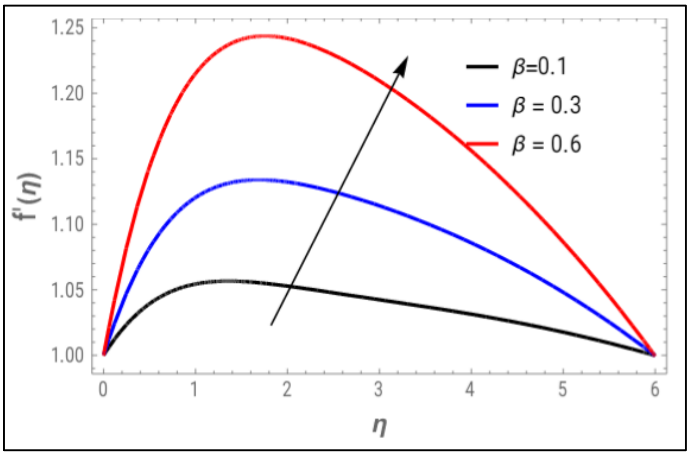
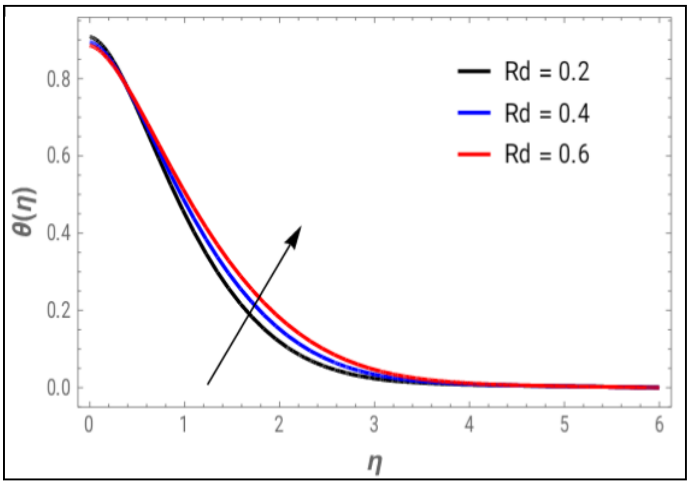
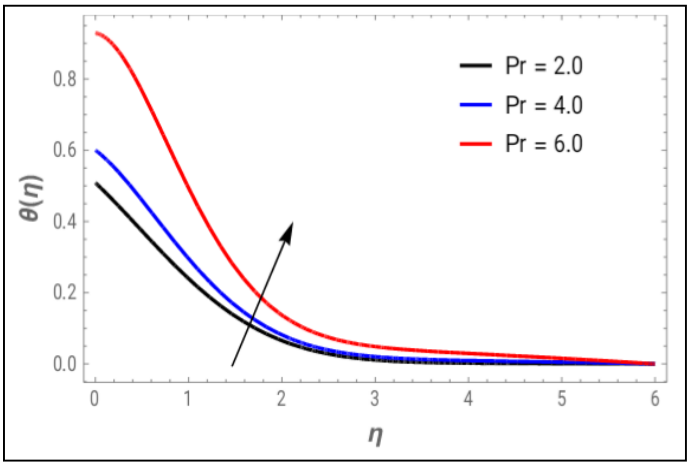
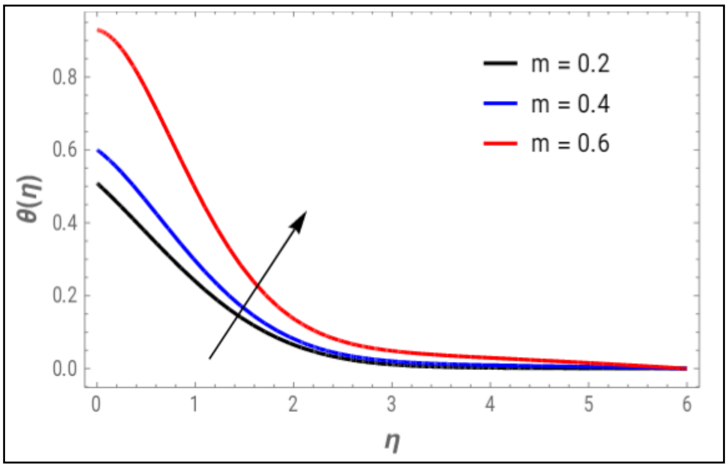


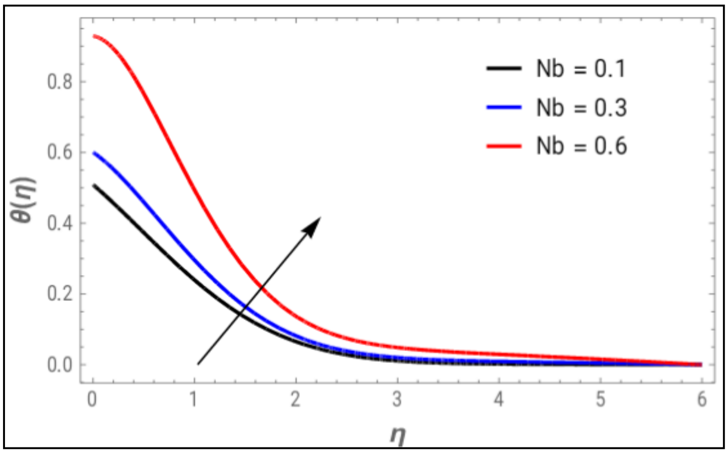
Fig 1: Graph of M against velocity

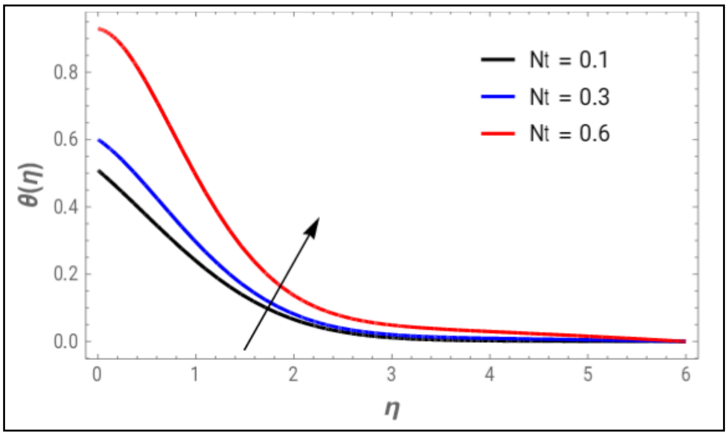

f'(η)
η
β=0.1
β = 0.3
β = 0.6

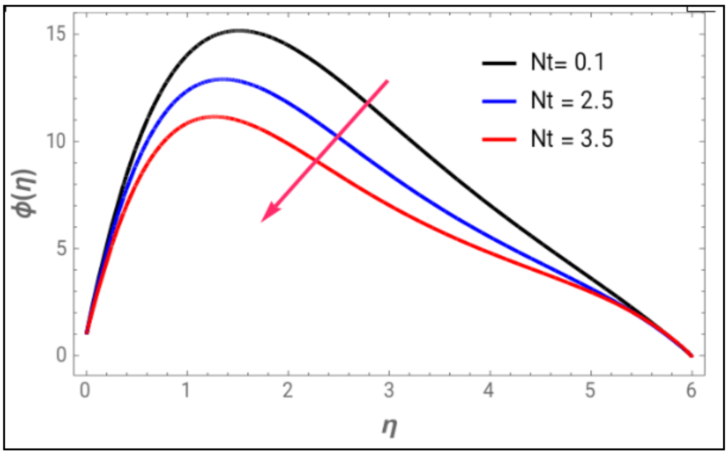

θ(η)
η
Rd = 0.2
Rd = 0.4
Rd = 0.6

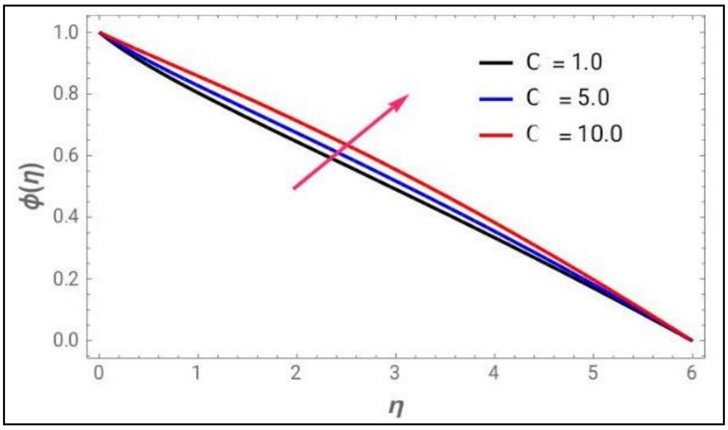

θ(η)
η
Pr = 2.0
Pr = 4.0
Pr = 6.0

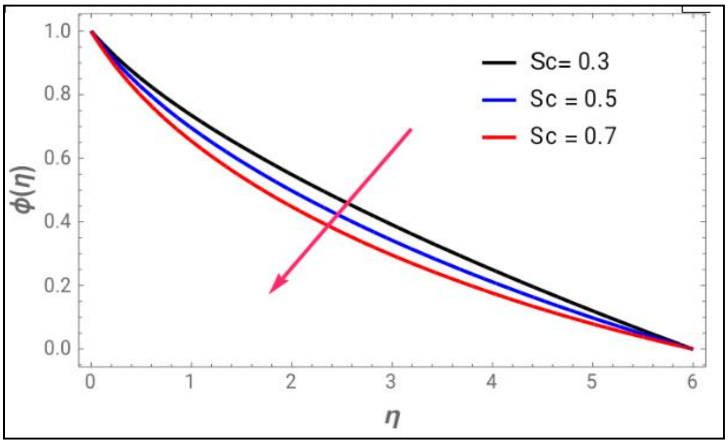

θ(η)
η
m = 0.2
m = 0.4
m = 0.6


Nb = 0.1
Nb = 0.3
Nb = 0.6
θ(η)
η


Nt = 0.1
Nt = 0.3
Nt = 0.6
θ(η)
η


Nt= 0.1
Nt = 2.5
Nt = 3.5
ϕ(η)
η


C = 1.0
C = 5.0
C = 10.0
ϕ(η)
η



Discussion of Results

These graphs illustrate the relationships between various parameters and their impacts on velocity, temperature and concentration profiles in different physical systems. The result is applicable in heat transfer and mass transport in various engineering fields (aerospace, chemical, biomedical, etc.), environmental studies (pollutant transport, climate modeling) and industrial processes (chemical reactions, material processing).

The effect of magnetic parameter M on the velocity profile as observed in figure 1 is increasing and the velocity profile is decreasing. This shows that the relationship between magnetic parameter and velocity appears to be inversely proportional. The magnetic parameter represents the strength of magnetic field, in magnetohydrodynamics flows, the magnetic field induces a Lorentz force, the force opposes the fluid flow which leads to a decrease in velocity. In figure 2, the effect of gradient of Hartree pressure β on the velocity profile. As β increases, the velocity profile increase, the relation between β and velocity appears to be directly proportional. Increased β accelerates blood flow. Higher β overcome vascular resistance, enhancing fluid motion, Velocity increase indicates improved blood circulation and perfusion.

With increasing values Rd the temperature profile is increasing in the range of $0 \le \eta \le 0.95$ in figure 3. The link between radiation and temperature appears to be directly proportional. Radiation energy absorption heats the material or fluid, increased radiation enhances heat transfer, raising temperatures. Temperature profile changes indicate variations in thermal gradients. Figure 4 illustrate the variation of the Prandtl number Pr on the temperature profile. As Pr increases, temperature profile increases, the connection between Pr and temperature appears to be directly proportional. Increase Prandtl number reduces thermal diffusivity, Thicker thermal boundary layers lead to higher temperatures and temperature profile changes indicate variations in heat transfer rates.

Figure 5 illustrate that the temperature profile is mascularly increasing for the rising values of m. It is also observed that for large values of m, the increasing rate of temperature is perceptible. The relationship between magnetic parameter and temperature appears to be directly proportional. Lorentz forces generate heat through electrical resistance, reduced fluid flow decreases convective heat transfer and increased viscous heating contributes to temperature rise. The effect of the parameter Nb on temperature profile elucidate figure 6. It is noticed that the temperature profile enhances in the range $0 \le \eta \le 0.95$ with enhancement of Nb, the relation between Brownian motion and temperature appears to be directly proportional. Brownian motion increases particle kinetic energy, collisions between particles transfer energy, raising temperatures enhanced thermal diffusion increases temperature gradients.

Figure 7 and 8 display the temperature and concentration profile for the effect of Nt. It is evident that the flow steams become warmed using thermophoresis and consequently exasperate nanoparticle deprivation, which emphasizes the fluid velocity and increases the nanoparticle's movement. For the increasing effect Nt, the temperature profile increases and away from the wedge surface, the temperature profile becomes static $\eta \to \infty$ which is depicted in Figure 7. This means that for the effect of the thermophoresis parameter the thermal BL is thicker in the surrounding area of the wedge surface and away from the wedge surface the BL thickness is consistent. Therefore because of the collision between elevated thermal energy and low-grade energy particles the heat transfer rate increases. From figure 8 It is found that in the case of concentration profile, Nt drives particles away from hotter regions, Particles accumulate in colder areas. Increased thermophoresis strengthens particle migration.

Figure 9 demonstrate the slight increases in concentration profile with the effect of viscous dissipation. As viscous dissipation increases, concentration profile increases, the connection between viscous dissipation and concentration appears to be directly proportional. Viscous forces slow down fluid flow, allowing particles to accumulate, energy conversion increases temperature, influencing particle transport. Viscous dissipation, convection and reaction rates. In addition, Figure 10 depict a slight decrease in concentration profile with the effect of Schmidt parameter. The relation between Sc and concentration appears to be inversely proportional. Higher Sc indicates slower diffusion relative to momentum transport, Thicker concentration boundary layers reduce mass transfer rates. Increased Schmidt weakens the impact of diffusion on concentration profiles.

Conclusion

A numerical investigation of the MHD boundary layer of nanofluid flow over a stretching wedge-shaped surface is performed taking into account the influence thermal radiation and viscous dissipation. The effects of important physical parameters on the velocity, energy and concentration profiles are discussed with help of graphical presentation.

The outcomes are summarized as

(i) The velocity profile increases for the increasing values of the stretching parameter, while the opposite trend is observed for the effect of the wedge angle parameter within the boundary layer region as well as the momentum boundary layer thickness increases and decreases respectively.

(ii) The concentration profile is increasing near the wedge surface, on the other hand away from the wedge surface, the concentration profile is decreasing and finally, it will static with $\eta \to \infty$, the effect of the stretching parameter.

References

1. Aworinde A, Okunea SO, Olorunsanjo A, Areo AO, Adegbite P, Akande MO, Akindele AO. The effect of a Riga plate on *Casson hybrid nanofluid* flow along a stretching cylinder with an exponential heat source and thermal radiation. Journal of Science and Technology. 2024;16(1). ISSN 2229-8460, e-ISSN: 2600-79245.
2. Akindele AO, Ogunsola AW, Obalalu AM, Adegbite P, Ajala AO, Alao S. MHD flow of nanofluid with non-uniform heat source or sink in the presence of chemical reaction and activation energy. Journal of Advance Research in Applied Science; 2021:8(5). ISSN: 2208-2352.
3. Akindele AO, Ogunsola AW. A study of non-isothermal permeable flow of nanofluid in a stretchable rotating disk system. Journal of Mathematics and Computer Science. 2021;11(2):1486-98. ISSN: 1927-5307.
4. Akindele AO, Ajala OA, Adegbite P, Ogunsola AW. Convective flow of nanofluids using Blasius-Reyleigh-Stoke variable with slip effect. IOSR Journal of Mathematics. 2021;17(3):1-8. e-ISSN: 2278-5728, p-ISSN: 2319-765X.
5. Areo AO, Olajide AO, Aworinde A, Odebiyi OA, Adetunde IA. On the influence of radiating fluid, heat generation, and viscous dissipation on MHD free convection flow along a stretching sheet. Paripex-Indian Journal of Research; 2024:13(7). ISSN 2250-1991.
6. Aydin O, Kaya A. Viscous dissipation effects on heat transfer in a moving plate in a parallel stream. International Journal of Thermal Sciences; 2009:48(5).
7. Buongiorno J. Convective transport in nanofluids. Journal of Heat Transfer; 2006:128(3).
8. Chamkha AJ. Hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium. International Journal of Engineering Science. 1997;35(10-11):975-86.
9. Chamkha AJ, Rashad AM, Armaghani T, Mansour MA. Journal of Thermal Analysis. 2018;132:1291.
10. Chamkha AJ, Rashad AM. The Canadian Journal of Chemical Engineering. 2014;92:758.
11. Cogley AC, Vincenti WG, Giles GC. Differential approximation for radiative transfer in a gray medium with conduction and convection. Journal of Heat Transfer; 1968:90(1).
12. Crane LJ. Flow past a stretching plate. Journal of Applied Mathematics and Physics (ZAMP); 1970:21(4).
13. Choi SUS, Eastman JA. Effect of velocity and thermal slip conditions with radiation on heat transfer flow of ferrofluids. Journal of Applied Mathematics and Physics; 1995:7(6).
14. Choi US, Eastman JA. Enhanced heat transfer using nanofluids. Journal of Argonne National Laboratory Report; c1995. ANL/MSD/CP-84938.
15. Chamkha AJ. Hydromagnetic flow and heat transfer over a continuous moving surface. Journal of Numerical Methods for Heat and Fluid Flow; 1996:6(3).
16. Dogonchi AS, Chamkha AJ, Seyyedi SM, Davood DD. Radiative nanofluid flow and heat transfer between parallel disks with penetrable and stretchable walls considering Cattaneo–Christov heat flux model. Heat Transfer-Asian Research; 2018:47(5).
17. Dogonchi AS, Chamkha AJ, Ganji DD. Magnetohydrodynamic nanofluid natural convection in a cavity under thermal radiation and shape factor of nanoparticles impacts. Journal of Applied Sciences; 2018:8(12).
18. Fatunmbi EO, Adeniyan A. MHD stagnation point-flow of micropolar fluids past a permeable stretching plate in porous media with thermal radiation, chemical reaction, and viscous dissipation. Journal of Advances in Mathematics and Computer Science. 2018;26(1):1-19. ISSN 2456-9968.
19. Gbadeyan JA, Oyekunle TL, Abubakar JU. Thermal radiation and chemical reaction effects on free convective heat and mass transfer flow through an irregular channel. FUW Trends in Science Technology Journal. 2017;2(1B):462-77. e-ISSN: 24085162; p-ISSN: 20485170.
20. Gebhart B. Viscous dissipation in potential flow. Journal of Fluid Mechanics; 1962:5(9).
21. Ghaly AA, Elbarbary EME. MHD boundary layer flow over a continuously moving plate. Journal of Applied Mathematics and Computation; 2002:131(2-3).
22. Hani U, Mohammad A, Mohammad Shah A. Magnetohydrodynamic mixed convection heat and mass transfer of nanofluid flow over a stretching wedge-shaped surface with the effect of thermophoresis and Brownian motion. Journal of Nanofluids. 2023;12:1590-604.
23. Hossain MA, Takhar HS, Chamkha AJ. Radiation effect on mixed convection along a vertical plate with uniform surface temperature. Journal of Heat and Mass Transfer; 1999:35(4).
24. Jayanthi S, Niranjan H. Effects of Joule heating, viscous dissipation, and activation energy on nanofluid flow induced by MHD on a vertical surface. Symmetry. 2023;15:314. doi:10.3390/sym15020314.
25. Khan WA, Pop I. Boundary-layer flow of a nanofluid past a stretching sheet. International Journal of Heat and Mass Transfer. 2010;53(11-12):2477–83.
26. Krishna MV, Bharathi K, Chamkha AJ. Hall effects on MHD peristaltic flow of Jeffrey fluid through a porous medium in a vertical stratum. Interface Phenomena and Heat Transfer. 2018;6(3):253-68. doi:10.1615/InterfacPhenomHeatTransfer.2019030215.
27. Krishna VM, Ahammad AN, Chamkha AJ. Case Studies in Thermal Engineering. 2021;27:101229.
28. Kumar KK, Baby Rani CH, Papa Rao AV. MHD Casson fluid flow with an inclined plate in the presence of Hall and aligned magnetic effects. Journal of Fluid Mechanics and Thermodynamics; 2021:17(1).
29. Kuznetsov AV, Nield DA. Natural convective boundary layer flow of a nanofluid past a vertical plate. Journal of Thermal Sciences; 2010:49(2).
30. Mabood F, Khan WA, Ismail AIM. MHD boundary layer flow and heat transfer of nanofluids over a

nonlinear stretching sheet. Journal of Magnetism and Magnetic Materials. 2015;374:569-76.
31. Makinde OD, Aziz A. Mixed convection from a convectively heated vertical plate to a fluid with internal heat generation. Journal of Heat Transfer. 2011;133(12):122501.
32. Makinde OD, Olanrewaju PO. Buoyancy effects on the thermal boundary layer over a vertical plate with a convective surface boundary condition. Journal of Fluids Engineering. 2010;132(4):044502.
33. Menni Y, Chamkha AJ, Azzi A, Zidani C. International Journal of Fluid Mechanics. 2017;47:23.
34. Mahapatra TR, Gupta AS. Mixed convection heat and mass transfer from a vertical plate in a porous medium. Journal of Numerical Methods for Heat and Fluid Flow; 2002:12(6).
35. Mahapatra TR, Dulal P, Sabyasachi M. Mixed convection flow in an inclined enclosure under a magnetic field with thermal radiation and heat generation. International Communications in Heat and Mass Transfer. 2013;41:47-56.
36. Obalalu AM, Ajala OA, Abdulrazaq R, Akindele OA. The influence of variable electrical conductivity on non-Darcian Casson nanofluid flow with first- and second-order slip conditions. Journal of Partial Differential Equations in Applied Mathematics. 2021;4:100084.
37. Oyinkansola SM, Obalalu AM, Ajala OA, Toyibat B, Ogunsola A, Akindele AO. Thermal radiation effect on Non-Newtonian Casson fluid through a porous material over a magnetic field with buoyancy. International Journal of Thermofluid Science and Technology. 2023;10(1):100103. doi:10.36963/IJTST.2023100103.
38. Pal D, Roy N. Influence of Brownian motion and thermal radiation on heat transfer of a nanofluid over a stretching sheet with slip velocity. International Journal of Applied Computational Mathematics. 2017;3:3355-77. doi:10.1007/s40819-016-0303-3.
39. Ogunsola AW, Akindele AO. MTH 509: Fluid Dynamics II; c2024.
40. Reddy PS, Sreedevi P, Chamkha AJ. MHD boundary layer flow, heat, and mass transfer analysis over a rotating disk through a porous medium saturated by Cu-water and Ag-water nanofluids with chemical reaction. International Journal of Thermal Sciences. 2017;307:46-55.
41. Saeed A, Algehyne EA, Aldhabani MS, Abdullah D, Kumam P, Kumam W. Mixed convective flow of a magnetohydrodynamic Casson fluid through a permeable stretching sheet with first-order chemical reaction. PLOS One; c2022. doi:10.1371/journal.pone.0265238.
42. Siva RS, Shamshuddin MD. Heat and mass transfer on the MHD flow of micropolar fluid in the presence of viscous dissipation and chemical reaction. Procedia Engineering. 2015;127:885-92.
43. Siva RS, Srinivasa RR. Soret effect on unsteady MHD free convective flow past a semi-infinite vertical plate in the presence of viscous dissipation. International Journal for Computational Methods in Engineering Science and Mechanics; 2015:16(2).
44. Takhar HS, Chamkha AJ, Nath G. MHD boundary layer flow past a continuously moving plate. Journal of Numerical Methods for Heat and Fluid Flow; 2002:12(6).
45. Tiwari SK, Das MK. Heat transfer augmentation in a porous medium by using internal heat generation. International Journal of Numerical Methods for Heat and Fluid Flow; 2007:17(6).
46. Veera MK, Reddy MG, Chamkha AJ. Heat and mass transfer on MHD rotating flow of second-grade fluid past an infinite vertical plate embedded in a uniform porous medium with Hall effects. Applied Mathematics and Scientific Computing. 2019;417-27.
47. Wang XQ. Mixed convective heat transfer of nanofluids in a vertical channel. International Journal of Thermal Sciences; 2007:46(8).
48. Zhang Z, Jian L, Wang F, Zhang L, Fang X. Thermodynamic properties and thermal stability of ionic liquid-based nanofluids containing graphene as advanced heat transfer fluids for medium-to-high-temperature applications. International Journal of Thermal Sciences. 2014;63:519-23.