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Stability Analysis of a Mass-Spring System under the Influence of an Electric Field Using Transfer Function

Dawood Mohammadi [1*], **Ghulam Hazrat Aimal Rasa** [2]
[1-2] Department of Mathematics, Kabul Education University, Kabul 1001, Afghanistan

* Corresponding Author: **Dawood Mohammadi**

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Abstract
This research investigates the dynamic stability of a mass-spring system under the influence of an electric field using the analytical transfer function method. The mass-spring system, as a fundamental model in mechanical vibrations, exhibits altered stability criteria when subjected to electrostatic forces. By linearizing the nonlinear governing differential equation of the system, an effective stiffness term incorporating the electric field's influence is derived. The system's transfer function is obtained via Laplace transform, enabling stability analysis in the frequency domain through pole placement.
In the non-oscillatory stable state and the damped oscillatory state under weak electric fields, the system remains stable, and the poles are located in the left half of the complex plane. In the marginally stable state at a critical voltage, one of the poles reaches the origin, indicating neutral stability. The instability of the system is such that under strong electric fields, a positive real pole appears, leading to instability.
The results highlight the interplay between mechanical damping and electrostatic forces, offering insights for designing microelectromechanical systems where electric fields are utilized for actuation. This study bridges classical mechanics and control theory, providing a framework for stability analysis in electromechanical systems.

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1. Introduction

Mass-spring dynamical systems serve as fundamental models in mechanical engineering and physics, playing a crucial role in understanding the behavior of more complex systems [5]. These systems are widely used in industrial applications, including automotive engineering, aerospace systems, and Micro-Electro-Mechanical Systems (MEMS) [6]. However, when they are subjected to external fields such as electric fields, their dynamic behavior can change significantly [7].
Recent research has demonstrated that applying an electric field can alter the natural frequency of the system and under certain conditions, lead to instability [6]. On the other hand, the transfer function method serves as a powerful analytical tool, enabling high-precision frequency-domain analysis of system responses [9]. This method is not only theoretically significant but also highly applicable in practical scenarios, such as active control systems [10]. Previous studies have primarily focused on analyzing mass-spring systems under normal conditions, but the influence of electric fields on their stability and the use of transfer functions for analysis remain relatively novel research areas.
The primary objective of this study is to analyze the stability of mass-spring systems under the influence of an electric field using transfer functions. By combining advanced analytical methods and numerical simulations, this research seeks to answer the following questions:

- How can the stability of a mass-spring system under an electric field be analyzed using transfer functions?

- How is the transfer function of the system under an electric field derived?
- What factors can lead to instability in such systems?

This research provides a comprehensive analysis of the dynamic stability of mass-spring systems under electric fields using the analytical transfer function method. By developing an accurate and linearized model, the study examines the interaction between electrostatic forces and mechanical system parameters. The findings don't only offer a deeper understanding of the dynamic behavior of these systems in electromechanical environments but also propose practical solutions for optimizing the design of MEMS, micro-electromechanical sensors, and vibration control systems. The results of this study can assist engineers in predicting and preventing unwanted instabilities in sensitive systems.

This research consists of six main sections: introduction, literature review, fundamental concepts, research findings, discussion, and conclusion. By analyzing pole locations in the complex plane, the stability conditions of the system under various scenarios are investigated. Finally, the obtained results are discussed and summarized to provide practical strategies for controlling the stability of such systems.

2. Literature Review

The mass-spring system, as one of the most fundamental dynamic models in mechanical engineering and physics, has been studied since the beginning. However, investigating the effect of external fields such as electric fields on the behavior of this system is a relatively newer topic that has attracted researchers' attention in recent years. Below, the historical background of this subject is presented with references to relevant articles and books:

If the force moving an object along a closed path (back and forth) performs no network on the object, that force is conservative. Another way to identify a conservative force is that the work is done by the force along different paths which identical starting and ending points must be equal. The restoring force (spring force) is an example of conservative forces [11].

The spring force law is named after the 17th-century British physicist Hooke's law. Hooke was the first scientist to discover this law [1]. In 1687, Newton introduced the laws of motion and gravitation [12]. In the 19th century, the development of differential equations governing mass-spring systems was carried out by mathematicians such as Lagrange and Rayleigh. These studies helped understand the behavior of free and forced vibrations in mechanical systems [13]. In the early 20th century, with the advancement of classical physics and electromagnetism, the effect of external fields (such as electric and magnetic fields) on mechanical systems became important. Initial studies in this field were conducted by Maxwell and Hertz [14]. The effect of electric fields on mechanical systems was investigated in fields such as MEMS and nanosystems. These studies showed that electrostatic forces can significantly change the dynamic behavior of systems [15]. Recent research in the field of dynamical systems under electric fields has shown that these fields can significantly affect system stability [6]. Studies on the effect of electric fields on MEMS have demonstrated that understanding the dynamic stability of systems is essential for ensuring their proper performance [16]. Electromechanical coupling effects in MEMS can lead to unexpected instabilities in frequency ranges that conventional stability analysis methods cannot predict [18]. The modified transfer function method with high accuracy can be used to analyze the stability of systems under external fields, although its efficiency in completely nonlinear conditions requires further investigation [28]. Stability analysis of mass-spring systems under friction effects was studied by the prominent scientist Lyapunov, showing that even with nonlinear friction, the system eventually reaches a stable state, and they provided a practical computational method for examining this stability [17]. A study on the application of analytical methods in electromechanical system dynamics analyzed the effect of electric fields on mechanical systems and showed that this analysis could help improve system design and control [3]. The motion analysis of a mass-spring oscillator under external force was studied, indicating that the presence of mass in the spring leads to a reduction in the system's natural frequency. This reduction is significant in real systems and should not be ignored, and its stable state is such that the system oscillates at the driving force frequency [29].

Analytical and numerical methods for investigating the stability of dynamical systems under external fields have been developed [9]. The equation of nonlinear mass-spring-damper system considering wind force effects was modeled, which is used in various space structures, aircraft wings, and all high-altitude structures [2]. The use of transfer functions as a powerful analytical tool for studying the stability of dynamical systems under external fields has expanded [10]. A general analysis of oscillatory systems and methods for solving equations of motion has been conducted, showing that mass-spring systems exhibit stable behavior under normal conditions but may become unstable under external forces [5, 19]. The mass-spring system with external driving force has been investigated, analyzing interesting physical phenomena such as resonance and beats [20]. The dynamics of mass-spring-damper systems in ships under external forces such as waves have been studied, focusing on resonance, added mass, and hydrodynamic damping. The results show that by controlling the impact frequency and optimizing damping and stiffness coefficients, dangerous phenomena such as resonant oscillations can be prevented [21]. A study on the effect of dry friction (Coulomb) on the dynamic behavior of damped mass-spring systems under harmonic excitation shows that the presence of modal damping along with dry friction leads to a reduction in resonant peak amplitudes in continuous slip conditions [22]. The effect of external forces on dynamical systems has been studied, showing that external forces can lead to more complex oscillations and even system instability [3, 23].

A proposed control model for electromechanical mass-spring-damper systems using the backstepping technique compared to conventional PID (Proportional-Integral-Derivative) controllers achieves the best performance of control systems and shows that the backstepping control technique provides better performance with a more stable control system, especially with increasing selected mechanical load [24]. The study of mass-spring systems under pendulum motion and perpendicular mass in static and dynamic conditions using MEMS technology was conducted, showing that pendulum oscillations and instabilities in electrostatic structures supported by microsprings affect the reliability of electromechanical systems [25].

According to previous research, mass-spring systems are recognized as fundamental models in dynamic analysis. However, when these systems are under the influence of electric fields, their dynamic behavior can change significantly. Using transfer functions as an analytical tool enables comprehensive study of system behavior in the frequency domain. This analysis is not only theoretically important but also has practical applications such as in active control systems. The historical background of the subject shows that mass-spring systems have been extensively studied, but the effect of electric fields on the stability of these systems and the use of transfer functions for their analysis is a relatively new topic. The present research, by focusing on these aspects, helps fill research gaps and provides innovative perspectives in this field.

3. Fundamental Concepts

Hooke's law is one of the fundamental principles of physics and mechanics that describes the behavior of linear elastic materials under external forces. This law states that the restoring force of an elastic object (such as a spring) is proportional to its deformation and is applied in the opposite direction of the deformation, where the negative sign indicates that the force direction is opposite to the displacement. This law is formulated as equation (1) [26].

$$F = -kx \tag{1}$$

Where F is the restoring force is measured in newtons, k is the spring constant or stiffness coefficient in newtons per meter, and x is the displacement from equilibrium position in meters. Newton's second law, known as the fundamental law of classical dynamics, describes the relationship between the force acting on an object, its mass, and the resulting acceleration (the acceleration of an object is proportional to the net force acting on it and is in the direction of that force, and is inversely proportional to the object's mass). This law is formulated as follows [27].

$$\sum F = m.a \tag{2}$$

Where: $\sum F$is the vector sum of all forces acting on the object, m is the mass of the object, and $a$is the acceleration of the object. In this section, we discuss linear systems. A linear system is a system for which the superposition principle holds true, meaning the response resulting from the simultaneous application of multiple inputs equals the sum of the responses resulting from each individual input. Mathematically, this system characteristic can be expressed as follows [9].

$$\begin{cases} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \\ \vdots \\ x_n(t) \to y_n(t) \end{cases} \Rightarrow a_1x_1(t) + a_2x_2(t)+\ldots+a_nx_n(t) \to a_1y_1(t) + a_2y_2(t)+\ldots+a_ny_n(t) \tag{3}$$

Furthermore, a time-invariant system conceptually means that the system's behavior and characteristics remain constant over time. Mathematically, this characteristic can be expressed as follows.

$$x(t) \to y(t) \longrightarrow x(t-t_0) \to y(t-t_0) \tag{4}$$

Furthermore, a Linear Time-Invariant System (LTI System) is a system that possesses both properties of linearity and time-invariance, and is called a linear time-invariant system.

The transfer function is also called the transmission function. The transfer function of a time-invariant system is the ratio of the Laplace transform of the system's output to the Laplace transform of its input. Therefore, if we denote the system's transfer function by$G(s)$, we have:

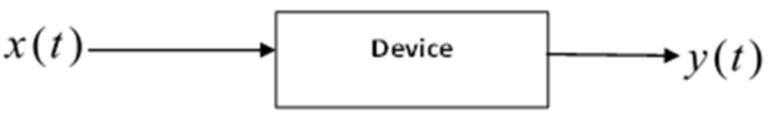

Fig 1: Shows a system with input $x(t)$ and output$y(t)$

$$G(s) = \frac{\ell[y(t)]}{\ell[x(t)]} = \frac{Y(s)}{X(s)}$$

Where $y(t)$ and $x(t)$ represent the system output and input [4].

When describing a dynamical system with a differential equation, the variable appearing on the left side of the differential equation$y(t)$ represents the system output, while the variable on the right side $x(t)$ represents the system input. Frequency in the transfer function is a crucial concept in linear dynamical systems, signifying the rate of change in input and output signals. Higher frequencies correspond to faster rates of change. Using the Laplace transform, we can convert time-domain representations to frequency-domain representations of input and output. This essentially transforms differential equations into algebraic equations that are simpler to analyze. The Laplace transform of a time-domain function $f(t)$ is defined as follows:

$$F(s) = l[F(t)] = \int_0^{\infty} e^{-st} f(t)dt \quad (5)$$

Where the parameters $s = \sigma + i\omega$ is a complex frequency variable.

We now begin our discussion on the mass-spring system model under the influence of an electric field. The mass-spring system is a linear dynamical system described by the following differential equation:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0 \quad (6)$$

Equation (6) is a homogeneous second-order linear differential equation and is not subjected to any external force. If subjected to an external force F_{ext}, its equation takes the following form [12]:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_{ext} \quad (7)$$

We now examine Equation (7) under the influence of an electric field. When an electric field is applied to the system, an electrostatic force F_{ele} acts on the mass [8], which we express as follows:
The electrostatic force between two parallel plate electrodes is derived from electrostatic theory and the calculation of the system's potential energy. Consider two parallel plates with area A and initial separation d, where one plate is fixed and the other is movable with displacement x (thus the new inter-plate separation becomes $(d - x)$), and a voltage v is applied across the plates. We have the capacitance of the parallel-plate capacitor is defined as follows:

$$C(x) = \frac{\varepsilon_0 A}{d-x} \quad (8)$$

Where ε_0 is the vacuum permittivity and $A$is the area of each plate.

The electrostatic potential energy in a capacitor is stored as electric field energy between its plates, and the stored energy in the capacitor is given by:

$$U(x) = \frac{1}{2}C(x)v^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d-x}v^2 \quad (9)$$

The electrostatic force is the negative derivative of the potential energy with respect to displacement x (principle of energy conservation).

$$F_{elc} = -\frac{dU}{dx} \quad (10)$$

By differentiating with respect to $U(x)$, we have:

$$F_{elc} = -\frac{d}{dx}\left(\frac{1}{2}\frac{\varepsilon_0 A}{d-x}v^2\right) = \frac{\varepsilon_0 A v^2}{2(d-x)^2} \quad (11)$$

Therefore, the mathematical model of the mass-spring system under the influence of an electric field is as follows:

$$m\ddot{x} + c\dot{x} + kx = \frac{\varepsilon_0 A v^2}{2(d-x)^2} \quad (12)$$

Equation (12) is a second-order differential equation and becomes nonlinear due to $(d - x)^2$the term in the denominator. To analyze the stability of this equation using the transfer function method, (12) must be converted to a linear differential equation. Using Taylor series expansion, its linearized equation is as follows:

$$m\ddot{x} + c\dot{x} + \left(k - \frac{\varepsilon_0 A v^2}{d^3}\right)x = \frac{\varepsilon_0 A v^2}{2d^2} \quad (13)$$

Equation (13) is a second-order linear differential equation, where m represents the mass, c the damping, $k_{eff} = k - \frac{\varepsilon_0 A v^2}{d^3}$the effective stiffness with respect to displacement x, and $F_{ext} = \frac{\varepsilon_0 A v^2}{2d^2}$the constant external force acting on the system. After substitution, equation (13) can be written in the following form:

$$m\ddot{x} + c\dot{x} + F_{eff}x = F_{ext} \quad (14)$$

We now focus on deriving the transfer function of the mathematical model of the mass-spring system. To obtain the transfer function, we apply the Laplace transform to both sides of equation (14), yielding:

$$\ell\{m\ddot{x} + c\dot{x} + F_{eff}x\} = \ell\{F_{ext}\} \tag{15}$$

$$ms^2X(s) + csX(s) + k_{eff}X(s) = F_{ext}(s) \tag{16}$$

$$X(s)[ms^2 + cs + k_{eff}] = F_{ext}(s) \tag{17}$$

$$H(s) = \frac{X(s)}{F_{ext}(s)} = \frac{1}{ms^2+cs+k_{eff}} \tag{18}$$

$$H(s) = \frac{1}{ms^2+cs+(k-\frac{\varepsilon_0 Av^2}{d^3})} \tag{19}$$

Equation (19) represents the transfer function of the mass-spring system model under the influence of an electric field.
In this section, we analyze the stability of the transfer function of the mathematical model of the mass-spring system under the influence of an electric field. The system stability is determined using the transfer function by analyzing the characteristic equation. To evaluate the stability of (19), we examine the denominator of the transfer function through root locus analysis:

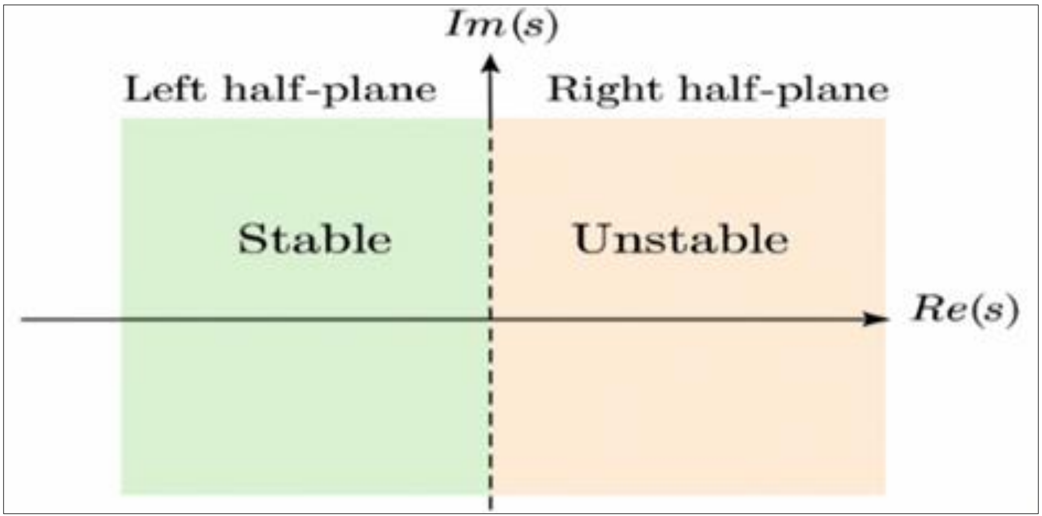

Fig 2: Shows the complex plane, illustrating stability and instability using the roots of the transfer function [9, 19].

$$ms^2 + cs + (k - \frac{\varepsilon_0 Av^2}{d^3}) = 0 \tag{20}$$

Or

$$ms^2 + cs + k_{eff} = 0 \tag{21}$$

Equation (21) represents the characteristic equation of Equation (14). The roots of this equation (system poles) are as follows:

$$s = \frac{-c \pm \sqrt{c^2-4mk_{eff}}}{2m}$$

For stability, all coefficients of the characteristic equation must be positive.

- $m \succ 0$ is always positive because it represents mass.
- $c \succ 0$ is always positive because it is the damping coefficient.
- $k_{eff} = k - \frac{\varepsilon_0 Av^2}{d^3} \succ 0$

Analysis of casesk_{eff}:

1. If $k_{eff} \succ 0$, then $k \succ \frac{\varepsilon_0 A v^2}{d^3}$.

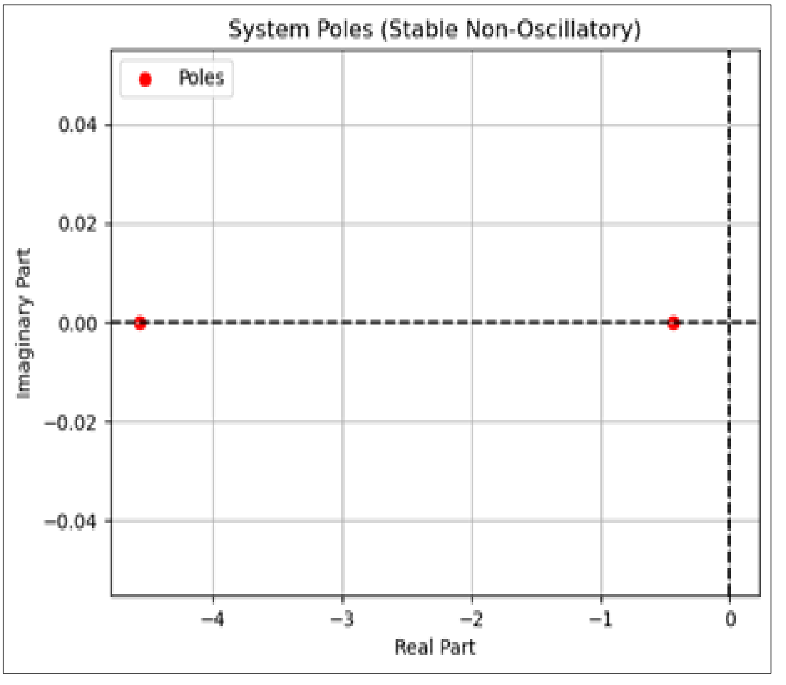

Fig 3: Stable system with non-oscillatory response [Researcher]

- If $c^2 - 4mk_{eff} \geq 0$, The roots are real and both negative. In this case, the system is stable with a non-oscillatory response (meaning the system gradually returns to equilibrium after applying an external force or voltage change, which occurs due to strong damping in the system). The points (poles) lie on the real axis to the left of the imaginary axis (left half-plane).

- If $c^2 - 4mk_{eff} \prec 0$, It has complex roots:

$$s = -\frac{c}{2m} \pm i\frac{\sqrt{c^2 - 4mk_{eff}}}{2m}$$

It is observed that the real part is negative; therefore, in this case, the system is stable with a damped oscillatory response (meaning after applying a disturbance such as a voltage change, it gradually returns to equilibrium with decaying oscillations, which occurs due to limited damping in the system, and their energy dissipates over time). The points lie in the upper and lower left half-plane, indicating damped oscillations.

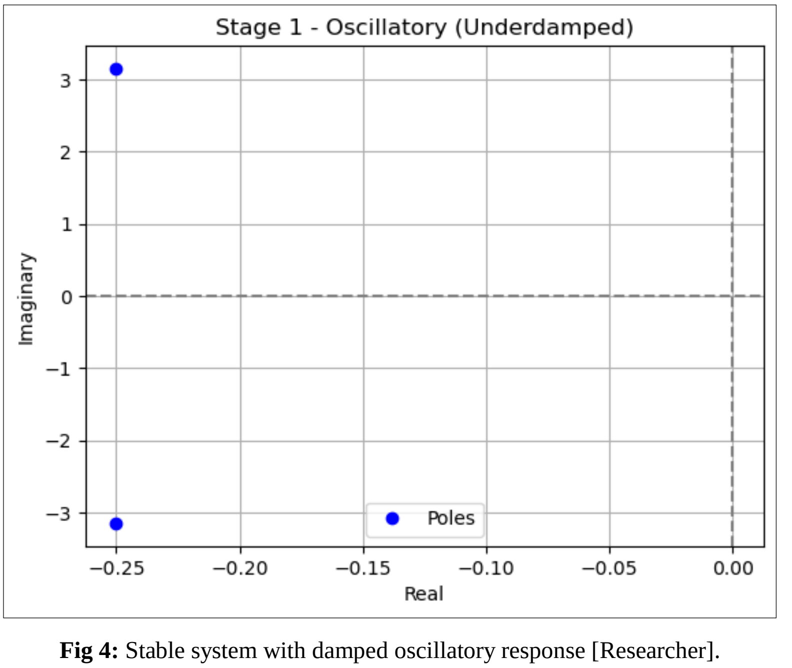

Fig 4: Stable system with damped oscillatory response [Researcher].

2. If $k_{eff} = 0$, then $k = \frac{\varepsilon_0 A v^2}{d^3}$.

In this case, $s = \frac{-c \pm \sqrt{c^2}}{2m}$ One root is zero located at the origin on the horizontal axis and the other is $\frac{-c}{m}$ located in the left half-plane on the real axis. In this case, the system is at the stability boundary or exhibits semi-stability (the zero root indicates no return to the initial equilibrium).

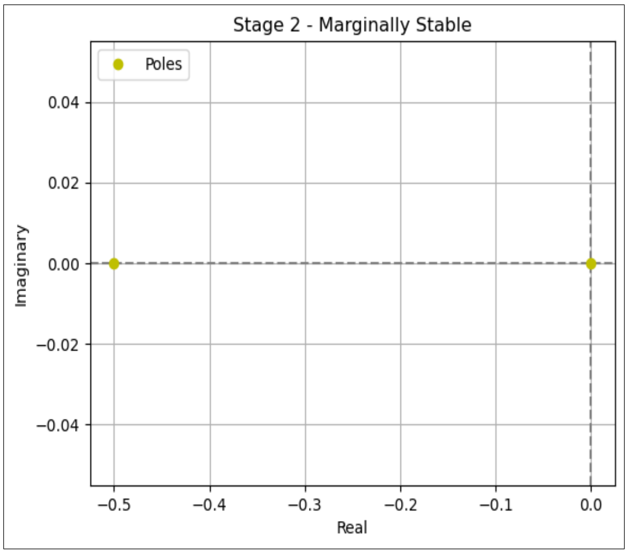

Fig 5: System at the stability boundary or semi-stability [Researcher].

3. If $k_{eff} \prec 0$, then $k \prec \frac{\varepsilon_0 A v^2}{d^3}$ meaning the electric field is so strong that it produces a negative effect, and the system no longer tends to return to equilibrium. Since $k_{eff} \prec 0$, the expression under the square root is always positive that is: $c^2 - 4mk_{eff} \succ 0$, Therefore, one root is always positive and real, and the other is negative and real. In this case, the positive root lies in the right half-plane of the real axis, and the negative root lies in the left half-plane of the real axis.

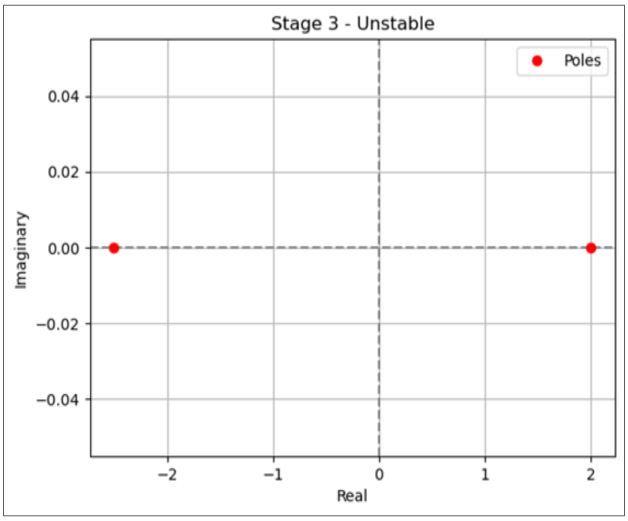

Fig 6: Unstable system with exponentially growing response [Researcher].

Therefore, the system is unstable with an exponentially growing response (meaning the system, when subjected to the slightest disturbance such as a voltage change, instead of returning to equilibrium or maintaining a bounded response, diverges from equilibrium indefinitely at an exponential rate).

4. Main Results

The stability of the mass-spring system under the influence of an electric field has been analyzed using the transfer function method, with respect to: system state, effective stiffness, pole locations in the complex plane, system response, stability condition, and its relationship with voltage, summary of stability stages for system under Electric field as follows:

Table 1: Summary of Stability Stages for Mass-Spring System Under Electric Field Using Transfer Function

Stages	State	Effective stiffness condition k_{eff}	Pole locations in the complex plane	System response	Stability Status	Voltage dependency
Stage 1 - Part 1	Stable non-oscillatory	$k_{eff} \succ 0$ $c^2 - 4mk_{eff} \geq 0$	Two real poles, both negative on the real axis in the left half-plane	Slow return without oscillation	Stable	Low voltage, weak electric effect
Stage 1 - Part 2	Stable with damped oscillatory response	$k_{eff} \succ 0$ $c^2 - 4mk_{eff} \prec 0$	Two complex conjugate poles with negative real parts	Decaying damped oscillation	Stable	Low to medium voltage
Stage 2	Stability boundary	$k_{eff} = 0$	One pole at zero and one negative real pole	Slow damping or constant	Marginally stable	Critical voltage
Stage 3	Unstable	$k_{eff} \prec 0$	One positive real pole and one negative real pole	Exponential growth, diverges from equilibrium	Unstable	High voltage, strong electric field

5. Discussion

A review of the literature reveals that previous studies primarily focused on the dynamic analysis of mass-spring systems under normal conditions, employing numerical or state-space methods to investigate the effects of electric fields [6, 15], while the analytical frequency-domain transfer function approach received less attention. Additionally, earlier research modeled the external force on the system nonlinearly [8, 18], but did not provide quantitative stability criteria for the linearized system. Although prior studies mentioned electromechanical instability phenomena [25], they lacked a comprehensive classification of stability states based on voltage levels. Furthermore, past research either examined mechanical damping [17, 22] or studied electric field effects [6, 15] but failed to systematically analyze the interaction between these two factors. Previous studies also lacked clear analytical relationships between system parameters and stability criteria, often treating mechanical and electrical effects in isolation.

The current study addresses these gaps by introducing a novel analytical framework based on transfer functions. Unlike prior research, which predominantly relied on numerical methods [6, 15], this study derives the system's transfer function (Equation 19) and analyzes pole locations, enabling precise frequency-domain stability assessment. By linearizing the system and presenting quantitative stability criteria (Equation 13), it offers a comprehensive classification of stability states under varying voltage levels (Table 1), a contribution absents in earlier work [25]. This analytical approach provides deeper insight into the interaction between electrostatic forces and mechanical system parameters. Moreover, this study bridges prior research gaps through an analytical-numerical approach. Unlike earlier works that separately examined mechanical damping [17, 22] and electric fields [6, 15], it presents a unified model (Equation 12) and concurrently analyzes the interplay of these factors, delivering a systematic framework for stability evaluation. By deriving precise analytical relationships between design parameters and stability criteria (Equations 13–21), it rectifies shortcomings in previous research. Thus, this study establishes a comprehensive analytical framework for predicting instability conditions.

6. Conclusion

This study systematically investigates the stability of mass-spring systems under the influence of an electric field using analytical methods, particularly the transfer function approach. The most significant achievements of this research include the derivation of a linearized model that describes the effects of electric fields on mass-spring systems, stability criteria that determine stability conditions based on effective stiffness and damping coefficients, a non-oscillatory response in highly damped systems indicating strong stability where disturbance energy is rapidly absorbed by damping rather than generating oscillations, and a damped oscillatory response representing dynamic stability where the system can oscillate around the equilibrium point but these oscillations gradually fade due to damping. Additionally, a zero root indicates the stability boundary, introducing a constant component in the system response (the system does not return to its initial state), while a negative root represents damping. Future research should investigate nonlinear effects beyond small-displacement approximations and study more advanced control techniques to manage instability under strong field conditions. This work serves as a bridge between theoretical mechanics and applied engineering, providing a framework for optimizing electromechanical systems in emerging technologies.

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