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Linear transformation and operators

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Abstract
In this paper we discuss here linear Transformation and operators and illustrate the definition with an example. Let V *and* W be a vector space over the field F let T and U be the linear transformation from V into W. The function $(T + U)$ defined by.

$$(T + U)(V) = TV + UV$$

Is a linear transformation from V into W let SEF the function (ST) defined by

$$(ST)\ (V) = \ S(TV)$$

Is also linear transformation from V to W the set of all linear transformation from V into W[, together in the addition and scalar transformation defined a some is a vector space over the field.

Proof:

Let T *and* U are the linear transformation from V into W then

$$(T + U)(SV + W) = T(SV + W) + U(SV + W)$$
$$= \ S(TV) + TW \ + S(UV) + UW)$$
$$= \ S(TV + UV) + (TW + UW)$$
$$= \ S(T + U)V + (T + U)W$$

Which shows that $(T + U)$ is a linear transformation. Similarly we have

$$(rT)\ (SV + W) = r\ (T(SV + W)\ )$$

$$= r(S(TV) + \ T(W)\ )$$
$$= rS(TV) + r\ (TW)$$
$$= S(r\ (TV)\ ) + rT(W)$$
$$= S(\ (rT)V) + \ (rT)W$$

Which shows that (rT) is a linear transformation.

Keywords: transformation, Linear, function, algebra

Introduction

In linear algebra we say that a transformation between two vectors is a that assigns a vector is one space to another space. In this paper we mention these theories we verify the linear properties using.

Properties of matrix and matrix scalar multiplication

Example (1)

Let T: $M_{\text{3x3}} \rightarrow$ be a transformation such that $T(A) = \text{rank}(A)$

Show that T is not linear.

Proof:
To show that T is not linear then we use two matrices say A & B such that

$$T(A + B) \neq T(A) + T(B)$$

Observe that rank of two matricies is 3 then.

$$T(A) + \ T(B) = \text{Rank}\ (A)\ + \text{Rank}\ (B) = 6$$

While $T(A+B) =$ Rank $(A+B) \leq 3$ clearly $T(A+B) \neq T(A) + T(B)$
We can find out let.

$$A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \text{ and } A = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$$

Then $T(A) = 3$ and $T(B) = 3$

$$T(A+B) = T \begin{vmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 1$$

$$T = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix}$$

Then

$$1 = T(A+B) \neq T(A) + T(B) = 6$$

Example 2:- if W1 and W2 is a subspace of vector space V(F) with W1+W2 is again a subspace of V(F) and also W1+W2 is.

$$w1 + w2 = L(w1Uw2)$$

Proof:- let $\gamma 1, \gamma 2 \in w1 + w2$ such that

$$\gamma 1 = \propto 1 + \beta 1 \,;\, \propto 1 \propto 2 \in w1$$
$$\gamma 2 = \gamma 2 + \beta 2 \,;\, \beta 1, \beta 2 \in w2$$
$$\text{a}\, \gamma 1 + \gamma 2 = a(\propto 1 + \propto 2) + (a\, \beta 1 + \beta 2) \in w1 + w2$$

Now $\propto \in w1, \beta \in w2$

$$\gamma \in w1 + w2$$
$$\Rightarrow \gamma = \propto + \beta$$
$$\propto . \beta \in w1\, U\, w2$$
$$\gamma = 1. \propto + 1. \beta \in L(w1\, U\, w2)$$
$$\Rightarrow w1 + w2 \leq L(w1 + w2) - - - - (1)$$
$$Now\ \gamma \in L(w1\, U\, w2)$$
$$\Rightarrow \gamma = \Sigma\, \text{ai} \propto \text{i} + \text{bj}\, \beta\text{j} \in \text{w1} + \text{w1}$$
$$\Rightarrow L(w1\, U\, w2) \leq w1 + w2 - - - - (2)$$

From 1 and 2

$$w1 + w2 = L(w1 + w2)$$

Example 3:- Consider linear transformation T: M_2 (R)→M_2(R)

Such that $T(A) = PAQ;\ when\ P = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\ \&\ Q = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

Then find Rank and nullity of transformation.
Proof:- $T(A) = PAQ$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a + c & b + d \\ 2a + 2c & 2b + 2d \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2a + 2c + b + d & 2a + 2c + b + d \\ 4a + 4c + 2b + 2d & 4a + 4c + 2b + 2d \end{bmatrix}$$

$$\Rightarrow T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = 0$$

$\Rightarrow 2a + 2c + b + d = 0$

$N(T) = \{\begin{bmatrix} a & b \\ c & d \end{bmatrix} | \, 2a + 2c + b + d = 0$

dim $N(T) = 4 - 3 = 3$ = nullity (T)
dim $M2(R) = 4$
Nullity$(T) = 3$
R and (T) = 1

Example 4: Consider the vector space V of semi-infinite real sequence R where $V = (v1, v2, v3, \ldots \ldots \ldots \ldots \ldots) \in V$ with $Vn \in R$ for $n \in N$ let L: $V \to V$ be the left shift linear transformation defined by.

$Lv = (v2, v3, v3, \ldots \ldots \ldots \ldots)$

and R : $V \to V$ be the right shift linear transformation defined by.

$RV = (0, V1, V2V \ldots \ldots \ldots \ldots ..$

Notice that L is onto but not one to one and R is one to one but not on to therefore neither transformation is in veritable.
Operator norms:- Intuitively, the operators norm is the largest factor by which a linear transformation can increase the length of vector, this provide a simple woist case charteization of any linear transformation.
Definition :- Let V & W be two nor med vector space and let T : V → W be a linear transformation the inducted operator norm of T is defined to.

$$\|T\| = \frac{Sup}{v \in V - \{50\}} \left\| \frac{TV}{V} \right\| = \frac{sup}{v \in V} \|V\| = 1 \|TV\|$$

This norm also has a new property that follows easily from this definition. The induced operator norm is called sub multiplicative because $\|TV\| \leq \|TV\| \, \|V\|$. From this, it is easy to see that it also provides a sub multiplicative norm for the algebra of linear operator in that.

$\|UTV\| \leq \|U\| \, \|TV\| \leq \|U\| \, \|T\| \, \|V\|$

A common question the operator norm is, "How do i know the two ex presence give the same result". To see this in unite.

$$\underset{v \in V - \{0\}}{Sup} \left\| \frac{TV}{V} \right\| = \underset{v \in V - \{0\}}{Sup} = \left\| T \frac{v}{\| V \|} \right\| = \underset{u \in V \, \| U \| = 1}{Sup} \| Tu \|$$

Example:- (1) Find $\| 2x + 5 \|$ if $2x + 5 \in p1(R)$ & inner product is

$$< f, g > = \int_0^1 f(x) \, g(x) \, dx$$

Sol:-

$\|2x + 5\| = \sqrt{< 2x + 5, 2x + 5 >}$

$= \sqrt{\int_0^1 (2x + 5)^2 \, dx}$

$\frac{\sqrt{7^3 - 5^3}}{6}$

Example (2) Consider $\begin{bmatrix} 1 & 1+i \\ 1-i & 3i \end{bmatrix} \in M_2(C)$

Find $\left\| \begin{matrix} 1 & 1+i \\ 1+i & 3i \end{matrix} \right\|$

Sol:- $\left\| \begin{matrix} 1 & 1+i \\ 1+i & 31 \end{matrix} \right\| = \sqrt{tr\left\{ \begin{pmatrix} 1 & 1+i \\ 1+i & 3 \end{pmatrix} \begin{pmatrix} 1 & 1-i \\ 1-i & 3 \end{pmatrix} \right\}}$

$$= \sqrt{1^2 + (1^2+1^2) + (1^2 + 1^2) + 3^2}$$

$$= \sqrt{1 + 2 + 2 + 3}$$

$$= \sqrt{14}$$

Example (3)
A linear transformation T: V→ W is bounded if and only if it is continues.
Proof:- Suppose that T is bounded there M such that $\|TV\| \leq M\,\|V\|$ for all $V \in V$ let$V_1 + V_2$,............ be a convergent sequence in v_1 then.

$$\|Tvi - Tvj\| = \|T(Vi - Vj)\| \leq M\,\|Vi - Vj\|$$

This implies that TV_1, TV_2 ....is a convergent sequence in W and T is continues. Conversely assume T is continuous and notice that TO =0 therefore, for any $\in > 0$, there is a $\partial > 0$ such that $\|TV\| < \in$ for all $\|V\| < S$ since the norm of $u = \frac{Sv}{2\|V\|}$ is equal to $S/_2$ we get.

$$\|TV\| = \left\|T\frac{Sv}{2\|V\|}\right\| \frac{2\|V\|}{\partial} < \frac{2\in}{\partial}\,\|V\|$$

The value $M = \frac{2\epsilon}{\partial}$ senses as an upper bound on $\|T\|$.

Then by showing that linear that formation over spaces are continues one concludes that they are also bounded this is accomplished in the following theorem.

Example (4)
Let $11 - 11$ be a sub multiplicative operator and $T: V \rightarrow V$ be the linear operator with $\|T\| < 1$. then, $(I - T)^{-1}$ exists and

$$(\mathrm{I} - \mathrm{T})^{-2} = \sum_{\mathrm{i}=0}^{00} \mathrm{Ti}$$

Proof, First, we observe that the sequence.

$$An = \sum_{i=0}^{1} Ti$$

Is can cling this follows from the fact that, for $m < n$ hence.

$$\|An - Am\| = \| \sum_{i=m}^{n-1} Ti \| \leq \sum_{i=m}^{n-1} \|T\|\, i$$

$$= \frac{\|T\|m\ \|T\|n}{1 - \|T\|} \leq \frac{\|\ T\ \|\ m}{1 - \|\ T\ \|}$$

Since this goes to zero as $\rightarrow \infty$, we see that the limiton $n \rightarrow \infty$ An exists.
Next, we observe that

$$(I - T)\,(I + T + T^2 + \ldots\ldots..+T^{4-1}) = \mathrm{I} - \mathrm{T}^4$$

sin $\|T\| < 1$, we have lim $_{K}\rightarrow \infty\ T^{K} = 0$ because $\|Tk\| \leq \|T\|^k \rightarrow 0$ Taking the limit $n \rightarrow a$ of both sides we get.

$$(I - T)\sum_{i=o}^{a} T = \lim_{n\to\infty} (I - T^n) = I$$

Like, reversing the order multiplication results in the same result. This shows that $\sum_{i=o}^{\infty} t^{\,i}$ must be the increase of $I = T$.
If one only needs to support that $I - T$ is non-singular, then proof by contradiction is some what simple. Suppose $I - T$ is singular, that there exists a non-zero vector V such that $(I - T)V = 0$. But this implies that
$\|V\| = \|TV\| \leq \|T\|\,\|V\|. Since\ \|V\| \neq 0$, this gives the contradiction $\|T\| \geq 1$ and implies that $I - T$ is non-singular.

Conclusion

Linear transformation and operations are useful because the presence the stature of a vector spaces. So many qualitative assessments of vectors spaces that Is the domain of linear transformation may under certain conditions atomically hold the image of the linear transformation.

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