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On the vibration of moving distributed masses of cantilever shaped-orthotropic rectangular plate resting on constant elastic pasternak foundation

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Abstract
This research work investigates the vibration of moving distributed masses of cantilever shaped-orthotropic rectangular plate resting on constant elastic Pasternak foundation. The governing equation is a fourth order partial differential equation with variable and singular co-efficients. The solutions to the problem are obtained by transforming the fourth order partial differential equation for the problem to a set of coupled second order ordinary differential equations using the technique of Shadnam *et al.* [18] which are then simplified using modified asymptotic method of Struble. The closed form solution is analyzed, resonance conditions are obtained and the results are presented in plotted curves for both cases of moving distributed mass and moving distributed force.

Keywords: Bi-parametric foundation, orthotropic, foundation modulus, critical speed, resonance, modified frequency

1. Introduction
A plate is a flat structural element for which the thickness is small compared with the surface dimensions or a plate is a structural element which is thin and flat. By "thin", it means that the plate's transverse dimension, or thickness, is small in comparison with the length and width dimensions. That is, the plate thickness is small compared to the other dimensions. A mathematical expression of this idea is:

$$\frac{T_h}{L} \ll 1 \quad (1.1)$$

Where $'T_h'$ represents the plate's thickness, and 'L' represents the length or width dimension.
The problems connected with the analysis of the dynamic interaction of thin bodies (rods, beams, plates, and shells) with other bodies have widespread application in various fields of science and technology. There are some bridges that are cantilever in nature. One of them shown below is the Quebec Bridge, the world's largest cantilever road bridge which has a span length of 549m.

Fig 1: The Quebec bridge: The World's Largest Cantilever Road Bridge

The physical phenomena involved in the impact include structural responses, contact effects, wave propagation as well as vibration. These problems are topical ones just as from the viewpoint of fundamental research in applied mechanics, so also with respect to their applications. Since these problems belong to the problems of dynamic contact interaction, their solution is connected with severe mathematical and calculation difficulties. To overcome this impediment, a rich variety of approaches and methods have been suggested, what is embodied in a great quantity of articles and reviews in [1, 2].

Very few works have reported on the impact response of anisotropic and composite plates subjected to an initial uniaxially tensile preloading [3-5], as well as biaxial preloading [6-10], in so doing only rectangular plates are considered as the targets.Analytical investigation of the low-velocity impact response of circular orthotropic and transversely isotropic plates possessing curvilinear anisotropy under compressive preloding has been carried out recently by Rossikhin and Shitikova in [11]. Szekrenyes [12] investigated the interface fracture in orthotropic composite plates using second order shear deformation theory. Kadari [13] analyzed buckling in orthotropic nanoscale plates resting on elastic foundations. Hu and Yao [14] studied the vibration solutions of rectangular orthotropic plates by symplectic geometry method. In the same vien, Alshaya, Hunt and Rowlands [15] investigated stresses and strains in thick perforated orthotropic plates. Gbadeyan and Dada [16] found the natural frequency of rectangular plates traversed by moving concentrated masses. Awodola [17] studied the effect of plate parameters on the vibrations under moving masses of elastically supported plate resting on bi-parametric foundation with stiffness variation. Awodola and Adeoye [19] investigated the behavior of simply supported orthotropic rectangular plate by applying the technique of variable separable. Adeoye and Awodola [20] studied the dynamic behavior of orthotropic rectangular plate with clamped-clamped boundary conditions by making use of the technique of Shadnam. Due to inability of researchers to solve orthotropic plates problems by analytical methods, this work aims at solving the governing equation by analytical solution and also considers the effect of the flexural rigidities in both x and y directions.

2. Governing Equation

The dynamic transverse displacement $V(x, y, t)$ of orthotropic rectangular plates when it is resting on a bi-parametric elastic foundation and traversed by distributed mass M_r moving with constant velocity c_r along a straight line parallel to the x-axis issuing from point y=s on the y-axis with flexural rigidities D_x and D_y is governed by the fourth order partial differential equation given as.

$$D_x \frac{\partial^4}{\partial x^4} W(x,y,t) + 2B \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) + D_y \frac{\partial^4}{\partial y^4} W(x,y,t) + \mu \frac{\partial^2}{\partial t^2} W(x,y,t) - \rho h R_0 [\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)] + K_0 W(x,y,t) - G_0 [\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] - \sum_{r=1}^{N} [M_r g H(x-ct) H(y-s) - M_r (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c_r \frac{\partial^2}{\partial x \partial t} W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x-c_r t) H(y-s) W(x,y,t)] = 0 \quad (2.1)$$

Where D_x and D_y are the flexural rigidities of the plate along x and y axes respectively.

$$D_x = \frac{E_x h^3}{12(1-\nu_x \nu_y)}, D_y = \frac{E_y h^3}{12(1-\nu_x \nu_y)}, B = D_x D_y + \frac{G_o h^3}{6}$$

E_x and E_y are the Young's moduli along x and y axes respectively, G_o is the rigidity modulus, ν_x and ν_y are Poisson's ratios for the material such that $E_x \nu_y = E_y \nu_x$, ρ is the mass density per unit volume of the plate, h is the plate thickness, t is the time, x and y are the spatial coordinates in x and y directions respectively, R_o is the rotatory inertia correction factor, K_o is the foundation constant and g is the acceleration due to gravity, H(.) is the Heaviside function.
Rewriting equation (2.1), one obtains.

$$\mu \frac{\partial^2}{\partial t^2} W(x,y,t) + \mu \omega_n^2 W(x,y,t) = \rho h R_0 [\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)] - 2B \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) - D_x \frac{\partial^4}{\partial x^4} W(x,y,t) - D_y \frac{\partial^4}{\partial y^4} W(x,y,t) - K_0 W(x,y,t) + \mu \omega_n^2 W(x,y,t) + G_0 [\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] + \sum_{r=1}^{N} [M_r g H(x-c_r t) H(y-s) - M_r (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c \frac{\partial^2}{\partial x \partial t} W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x-c_r t) H(y-s) W(x,y,t)] \quad (2.2)$$

Which can be expressed further as

$$\frac{\partial^2}{\partial t^2} W(x,y,t) + \omega_n^2 W(x,y,t) = R_0 [\frac{\partial^4}{\partial x^2 \partial t^2} W(x,y,t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x,y,t)] - \frac{2B}{\mu} \frac{\partial^4}{\partial x^2 \partial y^2} W(x,y,t) - \frac{D_x}{\mu} \frac{\partial^4}{\partial x^4} W(x,y,t) - \frac{D_y}{\mu} \frac{\partial^4}{\partial y^4} W(x,y,t) + [\omega_n^2 - \frac{K_0}{\mu}] W(x,y,t) + \frac{G_0}{\mu} [\frac{\partial^2}{\partial x^2} W(x,y,t) + \frac{\partial^2}{\partial y^2} W(x,y,t)] + \sum_{r=1}^{N} [\frac{M_r g \mu H(x-c_r t)}{H} (y-s) - \frac{M_r}{\mu} (\frac{\partial^2}{\partial t^2} W(x,y,t) + 2c \frac{\partial^2}{\partial x \partial t} \frac{\partial^2}{\partial x \partial t} W(x,y,t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x,y,t)) H(x-c_r t) H(y-s) W(x,y,t)] \quad (2.3)$$

Where ω_n^2 is the natural frequencies? $n = 1,2,3, \ldots$

The initial conditions, without any loss of generality, is taken as

$$W(x,y,t) = 0 = \frac{\partial}{\partial t}W(x,y,t) \quad (2.4)$$

3. Analytical Approximate Solution

In order to solve equation (2.3), one applies technique of Shadnam *et al.* [18] which requires that the deflection of the plates be in series form as

$$W(x,y,t) = \sum_{n=1}^{N} \Phi_n(x,y)Q_n(t) \quad (3.1)$$

Where $\Phi_n(x,y) = \Phi_{ni}(x)\Phi_{nj}(y)$ and

$$\Phi_{ni}(x) = \sin\frac{\nu_{ni}}{L_x}x + A_{ni}\cos\frac{\nu_{ni}}{L_x}x + B_{ni}\sinh\frac{\nu_{ni}}{L_x}x + C_{ni}\cosh\frac{\nu_{ni}}{L_x}x$$
$$\Phi_{ni}(y) = \sin\frac{\nu_{nj}}{L_y}y + A_{nj}\cos\frac{\nu_{nj}}{L_y}y + B_{nj}\sinh\frac{\nu_{nj}}{L_y}y + C_{nj}\cosh\frac{\nu_{nj}}{L_y}y \quad (3.2)$$

The right hand side of equation (2.3) written in the form of series takes the form

$$\sum_{n=1}^{\infty} R_0[\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t) + \frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)] - \frac{2B}{\mu}\frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t) - \frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}W(x,y,t)$$
$$-\frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}W(x,y,t + [\omega_n^2 - \frac{K_0}{\mu}]W(x,y,t) + \frac{G_0}{\mu}[\frac{\partial^2}{\partial x^2}W(x,y,t) + \frac{\partial^2}{\partial y^2}W(x,y,t)] + \quad (3.3)$$
$$\sum_{r=1}^{N} [\frac{M_r g\mu H(x-c_r t)}{H}(y-s) - \frac{M_r}{\mu}(\frac{\partial^2}{\partial t^2}W(x,y,t) + 2c_r\frac{\partial^2}{\partial x\partial t}W(x,y,t) + c_r^2\frac{\partial^2}{\partial x^2}$$
$$W(x,y,t))H(x-c_r t)H(y-s)W(x,y,t)] = \sum_{n=1}^{N} \Phi_n(x,y)\sigma_n(t)$$

On multiplying both sides of equation (3.4) by $\Phi_m(x,y)$, integrating on area A of the plate and considering the orthogonality of $\Phi_m(x,y)$, one obtains

$$\sigma_n(t) = \frac{1}{\Delta}\sum_{n=1}^{\infty}\int_A [R_0(\frac{\partial^4}{\partial x^2\partial t^2}W(x,y,t) + \frac{\partial^4}{\partial y^2\partial t^2}W(x,y,t)) - \frac{2B}{\mu}\frac{\partial^4}{\partial x^2\partial y^2}W(x,y,t) -$$
$$\frac{D_x}{\mu}\frac{\partial^4}{\partial x^4}W(x,y,t) - \frac{D_y}{\mu}\frac{\partial^4}{\partial y^4}W(x,y,t + (\omega_n^2 - \frac{K_0}{\mu})W(x,y,t) + \frac{G_0}{\mu}(\frac{\partial^2}{\partial x^2}W(x,y,t) \quad (3.4)$$
$$+\frac{\partial^2}{\partial y^2}W(x,y,t)) + \sum_{r=1}^{N} [\frac{M_r g}{\mu}H(x-c_r t)H(y-s) - \frac{M_r}{\mu}(\frac{\partial^2}{\partial t^2}W(x,y,t) + 2c_r\frac{\partial^2}{\partial x\partial t}$$
$$W(x,y,t) + c_r^2\frac{\partial^2}{\partial x^2}W(x,y,t))H(x-c_r t)H(y-s)W(x,y,t)]\Phi_m(x,y)dA$$

And zero when $n \neq m$

Where

$$\Delta = \int_A \Phi_n^2(x,y)dA \quad (3.5)$$

Making use of equation (3.4) and taking into account equation (3.1), equation (3.3) can be written as

$$\Phi_n(x,y)[\ddot{Q}_n(t) + \omega_n^2 Q_n(t)] = \frac{\Phi_n(x,y)}{\Delta}\sum_{q=1}^{\infty}\int_A [R_0(\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)\ddot{Q}_q(t) +$$
$$\frac{\partial^2\Phi_q(x,y)}{\partial y^2}\Phi_m(x,y)\ddot{Q}_q(t)) - \frac{2B}{\mu}\frac{\partial^2\Phi_q(x,y)}{\partial x^2\partial y^2}\Phi_m(x,y)Q_q(t) - \frac{D_x}{\mu}\frac{\partial^4\Phi_q(x,y)}{\partial x^4}\Phi_m(x,y)$$
$$Q_q(t) - \frac{D_y}{\mu}\frac{\partial^4\Phi_q(x,y)}{\partial y^4}\Phi_m(x,y)Q_q(t) + (\omega_n^2 - \frac{K_0}{\mu})\Phi_q(x,y)\Phi_m(x,y)Q_q(t) + \frac{G_0}{\mu}(\frac{\partial^2\Phi_q(x,y)}{\partial x^2}$$
$$\Phi_m(x,y)Q_q(t) + \frac{\partial^2\Phi_q(x,y)}{\partial y^2}\Phi_m(x,y)Q_q(t)) + \sum_{r=1}^{N}(\frac{M_r g\mu\Phi_m(x,y)H(x-c_r t)}{H}(y-s) - \frac{M_r}{\mu}$$
$$(\Psi_q(x,y)\Phi_m(x,y)\ddot{Q}_q(t) + 2c_r\frac{\partial\Phi_q(x,y)}{\partial x}\Phi_m(x,y)\dot{Q}_q(t) + c_r^2\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t))H(x-c_r t)H(y-s))]dA \quad (3.6)$$

On further simplification of equation (3.6), one obtains

$$\ddot{Q}_n(t) + \omega_n^2 Q_n(t) = \frac{1}{\Delta}\sum_{q=1}^{\infty}\int_A [R_0(\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)\ddot{Q}_q(t) + \frac{\partial^2\Phi_q(x,y)}{\partial y^2}\Phi_m(x,y)\ddot{Q}_q(t)$$
$$) - \frac{2B}{\mu}\frac{\partial^2\Phi_q(x,y)}{\partial x^2\partial y^2}\Phi_m(x,y)Q_q(t) - \frac{D_x}{\mu}\frac{\partial^4\Phi_q(x,y)}{\partial x^4}\Phi_m(x,y)Q_q(t) - \frac{D_y}{\mu}\frac{\partial^4\Phi_q(x,y)}{\partial y^4}\Phi_m(x,y)$$
$$Q_q(t) + (\omega_n^2 - \frac{K_0}{\mu})\Phi_q(x,y)\Phi_m(x,y)Q_q(t) + \frac{G_0}{\mu}(\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t) + \frac{\partial^2\Phi_q(x,y)}{\partial y^2}$$
$$\Phi_m(x,y)Q_q(t)) + \sum_{r=1}^{N}(\frac{M_r g}{\mu}\Phi_m(x,y)H(x-c_r t)H(y-s) - \frac{M_r}{\mu}(\Phi_q(x,y)\Phi_m(x,y)\ddot{Q}_q(t)$$
$$+2c_r\frac{\partial\Phi_q(x,y)}{\partial x}\Phi_m(x,y)\dot{Q}_q(t) + c_r^2\frac{\partial^2\Phi_q(x,y)}{\partial x^2}\Phi_m(x,y)Q_q(t))H(x-c_r t)H(y-s))]dA \quad (3.7)$$

The system in equation (3.7) is a set of coupled ordinary differential equations
Using the Fourier series representation, the Heaviside functions take the form

$$H(x - c_r t) = \frac{1}{4} + \frac{1}{\pi}\sum_{r=1}^{N} \frac{\sin(2n+1)\pi(x-c_r t)}{2n+1}, 0 < x < 1 \quad (3.8)$$

$$H(y - s) = \frac{1}{4} + \frac{1}{\pi}\sum_{r=1}^{N} \frac{\sin(2n+1)\pi(y-s)}{2n+1}, 0 < y < 1 \quad (3.9)$$

On putting equations (3.8) and (3.9) into equation (3.7) and simplifying one obtains

$$\ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\Delta}\sum_{q=1}^{\infty} [R_0 T_0 \ddot{Q}_q(t) - \frac{2B}{\mu} T_1 Q_q(t) - \frac{D_x}{\mu} T_2 Q_q(t) - \frac{D_y}{\mu} T_3 Q_q(t) + (\omega_n^2 - \frac{K_0}{\mu}) T_4 Q_q(t) + \frac{G_0}{\mu} T_5 Q_q(t) - \sum_{r=1}^{N} \frac{M_r}{\mu}((T_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi c_r t}{2j+1})(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi c_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t) + 2c_r t(T_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi c_r t}{2j+1})(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\sin(2k+1)\pi s}{2k+1}))\dot{Q}_q(t) + c_r^2(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi c_r t}{2j+1})(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi c_r t}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1})) Q_q(t))] = \sum_{q=1}^{\infty}\sum_{r=1}^{N} \frac{M_r g}{\mu\Delta}\Phi_m(ct)\Phi_m(s) \quad (3.10)$$

Which is the transformed equation governing the problem of an orthotropic rectangular plate resting on bi-parametric elastic foundation?
Where

$$T_0 = \int_A [\frac{\partial^2}{\partial x^2}\Phi_q(x,y)\Phi_m(x,y) + \frac{\partial^2}{\partial y^2}\Phi_q(x,y)\Phi_m(x,y)]dA \quad (3.11)$$

$$T_1 = \int_A \frac{\partial^2}{\partial x^2}[\frac{\partial^2}{\partial x^2}\Phi_q(x,y)]\Phi_m(x,y)dA \quad (3.12)$$

$$T_2 = \int_A \frac{\partial^4}{\partial x^4}[\Phi_q(x,y)]\Phi_m(x,y)dA \quad (3.13)$$

$$T_3 = \int_A \frac{\partial^4}{\partial y^4}[\Phi_q(x,y)]\Phi_m(x,y)dA \quad (3.14)$$

$$T_4 = \int_A \Phi_q(x,y)\Phi_m(x,y)dA \quad (3.15)$$

$$T_5 = \int_A [\frac{\partial^2}{\partial x^2}\Phi_q(x,y) + \frac{\partial^2}{\partial y^2}\Phi_q(x,y)]\Phi_m(x,y)dA \quad (3.16)$$

$$T_6 = \frac{1}{16}\int_A \Phi_q(x,y)\Phi_m(x,y)dA \quad (3.17)$$

$$E_1^* = \int_A \Phi_q(x,y)\Phi_m(x,y)\sin(2j+1)\pi x dA \quad (3.18)$$

$$E_2^* = \int_A \Phi_q(x,y)\Phi_m(x,y)\cos(2j+1)\pi x dA \quad (3.19)$$

$$E_3^* = \int_A \Phi_q(x,y)\Phi_m(x,y)\sin(2k+1)\pi y dA \quad (3.20)$$

$$E_4^* = \int_A \Phi_q(x,y)\Phi_m(x,y)\cos(2k+1)\pi y dA \quad (3.21)$$

$$E_5^* = E_1^*, E_6^* = E_2^*, E_7^* = E_3^*, E_8^* = E_4^* \quad (3.22)$$

$$T_7 = \frac{1}{16}\int_A \frac{\partial}{\partial x}\Phi_q(x,y)\Phi_m(x,y)dA \quad (3.23)$$

$$E_9^* = \int_A \frac{\partial}{\partial x}(\Phi_q(x,y))\Phi_m(x,y)\sin(2j+1)\pi x dA \quad (3.24)$$

$$E_{10}^* = \int_A \frac{\partial}{\partial x}(\Phi_q(x,y))\Phi_m(x,y)\cos(2j+1)\pi x dA \quad (3.25)$$

$$E_{11}^* = \int_A \frac{\partial}{\partial x}(\Phi_q(x,y))\Phi_m(x,y)\sin(2k+1)\pi y dA \quad (3.26)$$

$$E_{12}^* = \int_A \frac{\partial}{\partial x}\Phi_q(x,y)\Phi_m(x,y)\cos(2k+1)\pi y dA \quad (3.27)$$

$$E_{13}^* = E_9^*, E_{14}^* = E_{10}^*, E_{15}^* = E_{11}^*, E_{16}^* = E_{12}^* \quad (3.28)$$

$$T_8 = \frac{1}{16}\int_A \frac{\partial^2}{\partial x^2}(\Phi_q(x,y))\Phi_m(x,y)dA \quad (3.29)$$

$$E_{17}^* = \int_A \frac{\partial^2}{\partial x^2}(\Phi_q(x,y))\Phi_m(x,y)\sin(2j+1)\pi x dA \quad (3.30)$$

$$E_{18}^* = \int_A \frac{\partial^2}{\partial x^2}(\Phi_q(x,y))\Phi_m(x,y)\cos(2j+1)\pi x dA \quad (3.31)$$

$$E_{19}^{*}=\int_{A}\Phi_{q}(x,y)\Phi_{m}(x,y)\sin(2k+1)\pi y dA \quad (3.32)$$
$$E_{20}^{*}=\int_{A}\frac{\partial^{2}}{\partial x^{2}}(\Phi_{q}(x,y))\Phi_{m}(x,y)\cos(2k+1)\pi y dA \quad (3.33)$$
$$E_{21}^{*}=E_{17}^{*},E_{22}^{*}=E_{18}^{*},E_{23}^{*}=E_{19}^{*},E_{24}^{*}=E_{20}^{*} \quad (3.34)$$

$\Phi_m(x,y)$ is assumed to be the products of functions $\Phi_{pm}(x)\Phi_{bm}(y)$ which are the beam functions in the directions of x and y axes respectively. That is

$$\Phi_m(x,y)=\Phi_{pm}(x)\Phi_{bm}(y) \quad (3.35)$$

Where

$$\Phi_{pm}(x)=\sin\lambda_{pm}x+A_{pm}\cos\lambda_{pm}x+B_{pm}\sinh\lambda_{pm}x+C_{pm}\cosh\lambda_{pm}x \quad (3.36)$$
$$\Phi_{bm}(y)=\sin\lambda_{bm}+A_{bm}\cos\lambda_{bm}y+B_{bm}\sinh\lambda_{bm}y+C_{bm}\cosh\lambda_{bm}y$$

Where $A_{pm},B_{pm},C_{pm},A_{bm},B_{bm}$ and C_{bm} are constants determined by the boundary conditions. And Φ_{pm} and Φ_{bm} are called the mode frequencies

Where

$$\lambda_{pm}=\frac{\xi_{pm}}{L_x},\lambda_{bm}=\frac{\xi_{bm}}{L_y} \quad (3.37)$$

Considering a unit mass, equation (3.10) can be rewritten as

$$\ddot{Q}_n(t)+\omega_n^2Q_n(t)-\frac{1}{\Delta}\sum_{q=1}^{\infty}[R_0T_0\ddot{Q}_q(t)-\frac{2B}{\mu}T_1Q_q(t)-\frac{D_x}{\mu}T_2Q_q(t)-\frac{D_y}{\mu}T_3Q_q(t)$$
$$+(\omega_n^2-\frac{K_0}{\mu}T_4)Q_q(t)+\frac{G_0}{\mu}T_5Q_q(t)-\varpi\varphi((T_6+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}-$$
$$\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_5^*$$
$$\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_8^*$$
$$\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t)+2c(T_7+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1}$$
$$)(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{j=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}-$$
$$\sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}))\dot{Q}_q(t)$$
$$+c^2(T_8+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{19}^* \quad (3.38)$$
$$\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{20}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{21}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{22}^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}))Q_q(t))]$$
$$=\sum_{q=1}^{\infty}\frac{Mg}{\mu\Delta}\Phi_m(ct)\Phi_m(s)$$

Equation (3.38) is the fundamental equation of the problem. Where

$$\varpi=\frac{M}{\mu\varphi},\varphi=L_xL_y \quad (3.39)$$
$$\Phi_m(ct)=\sin\alpha_m(t)+A_m\cos\alpha_m(t)+B_m\sinh\alpha_m(t)+C_m\cosh\alpha_m(t) \quad (3.40)$$
$$\Phi_m(s)=\sin\lambda_m+A_m\cos\lambda_m+B_m\sinh\lambda_m+C_m\cosh\lambda_m \quad (3.41)$$
$$\alpha_m=\frac{\Gamma_m c}{L_x},\lambda_m=\frac{\Gamma_m s}{L_y} \quad (3.42)$$

3.1 Orthotropic Rectangular Plate Traversed by a Moving Force

In moving force, we account for only the load being transferred to the structure. In this case, the inertia effect is negligible. Setting $\varpi=0$ in the fundamental equation (3.38), one obtains.

$$\ddot{Q}_n(t)+(1-\frac{T_4}{\mu\Delta})\omega_n^2Q_n(t)-\frac{1}{\mu\Delta}[\mu R_0T_0\ddot{Q}_n(t)-2BT_1Q_n(t)-D_xT_2Q_n(t)-D_yT_3$$
$$Q_n(t)-K_0T_4Q_n(t)+G_0T_5Q_n(t)+\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2Q_q(t) \quad (3.43)$$
$$-D_yT_3Q_q(t)+(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))]=\frac{Mg}{\mu\Delta}\Phi_m(ct)\Phi_m(s)$$

Which can further be simplified as

$$\ddot{Q}_n(t)+\xi_n^2Q_n(t)-\gamma[\mu R_0T_0\ddot{Q}_n(t)-2BT_1Q_n(t)-D_xT_2Q_n(t)-D_yT_3Q_n(t)-K_0T_4Q_n(t)$$

$$+G_0T_5Q_n(t)+\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2Q_q(t)-D_yT_3Q_q(t)+(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))]=\gamma Mg\Phi_m(ct)\Phi_m(s) \quad (3.44)$$

Where $\xi_n^2=(1-\frac{T_4}{\mu\Delta})\omega_n^2$

Expanding and re-arranging equation (3.44), one obtains.

$$[1-\gamma\mu R_0T_0]\ddot{Q}_n(t)+(\xi_n^2-\gamma J_6)Q_n(t)-\gamma\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2\,Q_q(t)-D_yT_3Q_q(t)+(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))=\gamma Mg\Phi_m(ct)\Phi_m(s) \quad (3.45)$$

Simplifying further, one obtains

$$\ddot{Q}_n(t)+\frac{(\xi_n^2-\gamma J_6)}{[1-\gamma\mu R_0T_0]}Q_n(t)+\frac{\gamma}{[1-\gamma\mu R_0T_0]}\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2\,Q_q(t)-D_yT_3Q_q(t)+(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))=\frac{\gamma Mg}{[1-\gamma\mu R_0T_0]}\Phi_m(ct)\Phi_m(s) \quad (3.46)$$

Where

$$\gamma=\frac{1}{\mu\Delta}, J_6=-2BT_1-D_xT_2-D_yT_3-K_0T_4+G_0T_5 \quad (3.47)$$

For any arbitrary ratioγ, defined as

$\gamma^*=\frac{\gamma}{1+\gamma}$, one obtains

$\gamma=\frac{\gamma^*}{1-\gamma^*}=\gamma^*+o(\gamma^{*2})+\ldots$

For only$o(\gamma^*)$, one obtains

$\gamma=\gamma^*$

On application of binomial expansion,

$$\frac{1}{1-\gamma^*\mu R_0T_0}=1+\gamma^*\mu R_0T_0+o(\gamma^{*2})+\ldots \quad (3.48)$$

On putting equation (3.48) into equation (3.46), one obtains

$$\ddot{Q}_n(t)+(\xi_n^2-\gamma^*J_6)(1+\gamma^*\mu R_0T_0+o(\gamma^{*2})+\ldots)Q_n(t)+\gamma^*(1+\gamma^*\mu R_0T_0+o(\gamma^{*2})+\ldots)\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2Q_q(t)-D_yT_3Q_q(t)+(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))=\gamma^*aMg(1+\gamma^*\mu R_0T_0+o(\gamma^{*2})+\ldots)\Phi_m(ct)\Phi_m(s) \quad (3.49)$$

Retaining only $o(\gamma^*)$, equation (3.49) becomes

$$\ddot{Q}_n(t)+[\xi_n^2(1+\gamma^*\mu R_0T_0)-\gamma^*J_6]Q_n(t)+\gamma^*\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2Q_q(t)-D_yT_3Q_q(t)+(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))=\gamma^*Mg\Phi_m(ct)\Phi_m(s) \quad (3.50)$$

Which is simplified further as

$$\ddot{Q}_n(t)+\xi_n^2Q_n(t)+\gamma^*\sum_{q=1,q\neq n}^{\infty}(\mu R_0T_0\ddot{Q}_q(t)-2BT_1Q_q(t)-D_xT_2Q_q(t)-D_yT_3Q_q(t)\,(\mu\omega_q^2-K_0T_4)Q_q(t)+G_0T_5Q_q(t))=\gamma^*Mg\Phi_m(ct)\Phi_m(s) \quad (3.51)$$

Where

$$J_7=[\xi_n^2(1+\gamma^*\mu R_0T_0)-\gamma^*J_6] \quad (3.52)$$

Using Struble's technique, one obtains

$$\xi_{nn}=\xi_n-(\frac{\xi_n^2-J_7}{2\xi_n}) \quad (3.53)$$

Which is the modified frequency for moving force problem.

Using equation (3.53), the homogeneous part of equation (3.51) can be written as

$$\ddot{Q}_n(t)+\xi_{nn}^2Q_n(t)=0 \quad (3.54)$$

Hence, the entire equation (3.51) gives

$$\ddot{Q}_n(t)+\xi_{nn}^2Q_n(t)=\gamma^*Mg\Phi_m(ct)\Phi_m(s) \quad (3.55)$$

On solving equation (3.55) one obtains

$$Q_n(t)=\frac{Mg\gamma^*\Phi_m(s)}{\xi_{nn}(\alpha_m^4-\xi_{nn}^4)}[(\alpha_m^2+\xi_{nn}^2)(\alpha_m\sin\xi_{nn}t-\xi_{nn}\sin\alpha_m t)-A_m\xi_{nn}(\alpha_m^2+\xi_{nn}^2)(\cos\alpha_m t-\cos\xi_{nn}t)-B_m(\alpha_m^2-\xi_{nn}^2)(\alpha_m\sin\xi_{nn}t-\xi_{nn}\sinh\alpha_m t)+C_m\xi_{nn}(\alpha_m^2-\xi_{nn}^2)(\cosh\alpha_m t-\cos\xi_{nn}t)] \quad (3.56)$$

Which on inversion yields

$$W(x,y,t)=\sum_{pm=1}^{\infty}\sum_{qm=1}^{\infty}\frac{Mg\gamma^*\Phi_m(s)}{\xi_{nn}(\alpha_m^4-\xi_{nn}^4)}[(\alpha_m^2+\xi_{nn}^2)(\alpha_m\sin\xi_{nn}t-\xi_{nn}\sin\alpha_m t)-A_m\xi_{nn}(\alpha_m^2+\xi_{nn}^2)(\cos\alpha_m t-\cos\xi_{nn}t)-B_m(\alpha_m^2-\xi_{nn}^2)(\alpha_m\sin\xi_{nn}t-\xi_{nn}\sinh\alpha_m t)+C_m\xi_{nn}(\alpha_m^2-\xi_{nn}^2)(\cosh\alpha_m t-\cos\xi_{nn}t)](\sin\frac{\xi_{pm}}{L_x}x+A_{pm}\cos\frac{\xi_{pm}}{L_x}x+B_{pm}\sinh\frac{\xi_{pm}}{L_x}x+C_{pm}\cosh\frac{\xi_{pm}}{L_x}x)(\sin\frac{\xi_{qm}}{L_y}y+A_{qm}\cos\frac{\xi_{qm}}{L_y}y+B_{qm}\sinh\frac{\xi_{qm}}{L_y}y+C_{qm}\cosh\frac{\xi_{qm}}{L_y}y) \quad (3.57)$$

Which is the transverse displacement response to a moving force.

3.2 Orthotropic Rectangular Plate Traversed by a Moving Mass

In moving mass problem, the moving load is assumed rigid, and the weight and as well as inertia forces are transferred to the moving load. That is the inertia effect is not negligible. Thus $\varpi\neq 0$ and so it is required to solve the entire equation (3.38).To solve the equation, one employs analytical approximate method. This method is known as an approximate analytical method of Struble. The homogeneous part of equation (3.38) shall be replaced by a free system operator defined by the modified frequency ξ_{nn}. Thus, the entire equation becomes.

$$\ddot{Q}_n(t)+\xi_{nn}^2Q_n(t)+\varpi\varphi^*\sum_{q=1}^{\infty}[(T_6+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t)+2c(T_7+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{j=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1}))\dot{Q}_q(t)+c^2(T_8+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_{17}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{18}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{19}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{20}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{21}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{22}^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{23}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{24}^*\frac{\sin(2k+1)\pi s}{2k+1}))Q_q(t)]=\sum_{q=1}^{\infty}\frac{Mg}{\mu\Delta}\Phi_m(ct)\Phi_m(s) \quad (3.58)$$

Where $\varphi^*=\frac{1}{\mu\varepsilon^*}$

On expanding equation (3.58), one obtains

$$\ddot{Q}_n(t)+\xi_{nn}^2Q_n(t)+\varpi\varphi^*[(T_6+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_1^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_2^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_3^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_4^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_5^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_6^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_7^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_8^*\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_n(t)+2c(T_7+\frac{1}{\pi^2}(\sum_{j=1}^{\infty}E_9^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{10}^*\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty}E_{11}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{j=1}^{\infty}E_{12}^*\frac{\sin(2k+1)\pi s}{2k+1})+\frac{1}{4\pi}(\sum_{j=1}^{\infty}E_{13}^*\frac{\cos(2j+1)\pi ct}{2j+1}-\sum_{j=1}^{\infty}E_{14}^*\frac{\sin(2j+1)\pi ct}{2j+1})+\frac{1}{4\pi}(\sum_{k=1}^{\infty}E_{15}^*\frac{\cos(2k+1)\pi s}{2k+1}-\sum_{k=1}^{\infty}E_{16}^*\frac{\sin(2k+1)\pi s}{2k+1})$$

$$))\dot{Q}_n(t) + c^2(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})$$
$$(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1}$$

$$))Q_n(t)] + \varpi\varphi^* \sum_{q=1,q\neq n}^{\infty} [(T_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^*$$
$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^*$$
$$\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t) + 2c(T_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1}$$
$$)(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1}$$
$$))\dot{Q}_q(t) + c^2(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})$$

$$(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \quad (3.59)$$
$$))Q_q(t)] = Mg\varphi\Phi_m(ct)\Phi_m(s)$$

On rearranging and simplifying equation (3.59), one obtains

$$(1 + \varpi\varphi^*(T_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_3^*$$
$$\frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} -$$
$$\sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1})$$
$$))\ddot{Q}_n(t) + 2c\varpi\varphi^*(T_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1}$$
$$)(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1}$$

$$))\dot{Q}_n(t) + (\xi_{nn}^2 + \varpi\varphi^* c^2(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})$$
$$(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1}$$
$$)))Q_n(t) + \varpi\varphi^* \sum_{q=1,q\neq n}^{\infty} [(T_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^*$$
$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^*$$

$$\frac{\sin(2k+1)\pi s}{2k+1}))\ddot{Q}_q(t) + 2c(T_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1}$$
$$)(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1}$$
$$))\dot{Q}_q(t) + c^2(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})$$

$$(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1}$$
$$-\sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \quad (3.60)$$
$$))Q_q(t)] = Mg\varphi\Phi_m(ct)\Phi_m(s)$$

On further simplifications and re-arrangement, one obtains

$$\ddot{Q}_n(t) + 2c\varpi\varphi^*(T_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{11}^*$$
$$\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^*$$

$$\frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi ct}{2k+1}))\dot{Q}_n(t) +$$
$$(\xi_{nn}^2(1 - \varpi\varphi^*(T_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1})$$
$$(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} -$$
$$\sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi ct}{2k+1})))) +$$
$$c^2\varpi\varphi^*(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{19}^*$$
$$\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))Q_n(t)$$
$$+\varpi\varphi^* \sum_{q=1,q\neq n}^{\infty} [(T_6 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_3^*$$
$$\frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^*$$
$$\frac{\sin(2j+1)\pi ct}{2j+1}) + (\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi ct}{2k+1}))\ddot{Q}_q(t) +$$
$$2c(T_7 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi ct}{2k+1}$$

$$-\sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1})$$
$$+(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi ct}{2k+1}))\dot{Q}_q(t) + c^2(T_8 + \frac{1}{\pi^2}(\sum_{j=1}^{\infty} E_{17}^*$$
$$\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1})(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \quad (3.61)$$
$$\frac{\sin(2k+1)\pi ct}{2k+1}) + \frac{1}{4\pi}(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1}) + \frac{1}{4\pi}$$
$$(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))Q_q(t)] = Mg\varphi\Phi_m(ct)\Phi_m(s)$$

Applying modified asymptotic method of Struble, equation (3.61) can be rewritten as for homogeneous case

$$\ddot{Q}_n(t) + \vartheta_n^2 Q_n(t) = 0 \quad (3.62)$$

Hence, the entire equation becomes

$$\ddot{Q}_n(t) + \vartheta_n^2 Q_n(t) = Mg\varphi\Phi_m(ct)\Phi_m(s) \quad (3.62)$$

Where

$$\vartheta_n = \xi_{nn} - \frac{1}{2\xi_{nn}}(\xi_{nn}^2\varpi\varphi^*(T_6 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1})) \quad (3.63)$$
$$-c^2\varpi\varphi^*(T_8 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1})))$$

Which is the modified frequency representing the frequency of the free system.
Taking into account equation (3.64), equation (3.61) can be rewritten as

$$\ddot{Q}_n(t) + \vartheta_n^2 Q_n(t) = Mg\varphi\Phi_m(s)[\sin\alpha_m(t) + A_m\cos\alpha_m t + B_m\sinh\alpha_m t + C_m\cosh\alpha_m t] \quad (3.64)$$

Solving equation (3.65), one obtains

$$Q_n(t) = \frac{Mg\varphi\Phi_m(s)}{\vartheta_n(\alpha_m^4 - \vartheta_n^4)}[(\alpha_m^2 + \vartheta_n^2)(\alpha_m\sin\vartheta_n t - \vartheta_n\sin\alpha_m t) - A_m\vartheta_n(\alpha_m^2 + \vartheta_n^2)(\cos\alpha_m t$$
$$-\cos\vartheta_n t) - B_m(\alpha_m^2 - \vartheta_n^2)(\alpha_m\sin\vartheta_n t - \vartheta_n\sinh\alpha_m t) + C_m\vartheta_n(\alpha_m^2 - \vartheta_n^2)(\cosh\alpha_m t \quad (3.65)$$
$$-\cos\vartheta_n t)]$$

Which on inversion yields

$$W(x,y,t) = \sum_{pm=1}^{\infty}\sum_{qm=1}^{\infty} \frac{Mg\varphi\Phi_m(s)}{\vartheta_n(\alpha_m^4 - \vartheta_n^4)}[(\alpha_m^2 + \vartheta_n^2)(\alpha_m\sin\vartheta_n t - \vartheta_n\sin\alpha_m t) - A_m\vartheta_n(\alpha_m^2$$
$$+\vartheta_n^2)(\cos\alpha_m t - \cos\vartheta_n t) - B_m(\alpha_m^2 - \vartheta_n^2)(\alpha_m\sin\vartheta_n t - \vartheta_n\sinh\alpha_m t) + C_m\vartheta_n(\alpha_m^2 - \vartheta_n^2) \quad (3.66)$$
$$(\cosh\alpha_m t - \cos\vartheta_n t)](\sin\frac{\xi_{pm}}{L_x}x + A_{pm}\cos\frac{\xi_{pm}}{L_x}x + B_{pm}\sinh\frac{\xi_{pm}}{L_x}x + C_{pm}\cosh\frac{\xi_{pm}}{L_x}x)$$

Which is the transverse displacement response to a moving mass of a rectangular plate.

4. Illustrative Examples

4.1 Orthotropic Rectangular Plate with Cantilever End Conditions

For an orthotropic rectangular plate with free right hand and clamped left hand ends is considered. The boundary conditions are given by

$$W(0,y,t) = 0 = \frac{\partial W(L_x,y,t)}{\partial x}\ \frac{\partial^2 W(0,y,t)}{\partial^2 x} = 0 = \frac{\partial^3 W(L_x,y,t)}{\partial^3 x} \quad (4.1)$$

$$W(x,0,t) = 0 = \frac{\partial W(x,L_y,t)}{\partial y}\ \frac{\partial^2 W(x,0,t)}{\partial^2 y} = 0 = \frac{\partial^3 W(x,L_y,t)}{\partial^3 y} \quad (4.2)$$

Thus, for the normal modes

$$\xi_{pm}(0) = 0 = \frac{\partial \xi_{pm}(0)}{\partial x}\ \frac{\partial^2 \xi_{pm}(0)}{\partial^2 x} = 0 = \frac{\partial^3 \xi_{pm}(L_x)}{\partial^3 x} = 0 \quad (4.3)$$

$$\xi_{qm}(0) = 0 = \frac{\partial \xi_{qm}(0)}{\partial y}\ \frac{\partial^2 \xi_{qm}(0)}{\partial^2 y} = 0 = \frac{\partial^3 \xi_{qm}(L_y)}{\partial^3 y} = 0 \quad (4.4)$$

For simplicity, our initial conditions are of the form

$$V(x,y,0) = 0 = \frac{\partial V(x,y,o)}{\partial t} \quad (4.5)$$

Using the boundary conditions in equations (4.1) to (4.4) and the initial conditions given by equation (4.5), it can be shown that

$$A_{pm} = \frac{\sin\xi_{pm} - \sinh\xi_{pm}}{\cos\xi_{pm} - \cosh\xi_{pm}} = \frac{\cos\xi_{pm} - \cosh\xi_{pm}}{\sinh\xi_{pm} + \sin\xi_{pm}} \quad (4.6)$$

$$A_{qm} = \frac{\sin\xi_{qm} - \sinh\xi_{qm}}{\cos\xi_{qm} - \cosh\xi_{qm}} = \frac{\cos\xi_{qm} - \cosh\xi_{qm}}{\sinh\xi_{qm} + \sin\xi_{qm}} \quad (4.7)$$

In the same vein, we have

$$A_m = \frac{\sin\xi_m - \sinh\xi_m}{\cos\xi_m - \cosh\xi_m} = \frac{\cos\xi_m - \cosh\xi_m}{\sinh\xi_m + \sin\xi_m} \quad (4.8)$$

$$B_{pm} = -1, B_{qm} = -1, \Rightarrow B_m = -1 \quad (4.9)$$

$$C_{pm} = -A_{pm}, C_{qm} = -A_{qm}, \Rightarrow C_m = -A_m \quad (4.10)$$

And from equation (4.8), one obtains

$$\cos\xi_m \cosh\xi_m = -1 \quad (4.11)$$

Which is termed the frequency equation for the dynamical problem, such that

$$\xi_1 = 1.875, \xi_2 = 4.694, \xi_3 = 7.855 \quad (4.12)$$

On using equations (4.8), (4.9), (4.10) and (4.12) in equations (3.57) and (3.66), one obtains the displacement response to a moving force and a moving mass of cantilever orthotropic rectangular plate resting on bi-parametric condition respectively.

5. Discussion of the Analytical Solutions

For this undammed system, it is desirable to examine the phenomenon of resonance. From equation (3.57), it is explicitly shown that the cantilever orthotropic rectangular plate resting on constant elastic foundation and traverse by moving distributed force with uniform speed reaches a state of resonance whenever

$$\varpi_n = \tau_n \quad (5.1)$$

while equation (3.66) shows that the same cantilever orthotropic rectangular plate resting on constant elastic foundation and traverse by moving distributed force with uniform speed reaches a state of resonance when

$$\varpi_n = \vartheta_n \quad (5.2)$$

Where

$$\vartheta_n = \tau_n - \frac{1}{2\tau_n}(\phi\eta(P_6 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} F_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} F_8^* \frac{\sin(2k+1)\pi s}{2k+1})) - c^2\phi\eta(P_8 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} F_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} F_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1}))) \quad (5.3)$$

Comparing equations (5.1) and (5.2), one obtains

$$\vartheta_n = \tau_n[1 - \frac{1}{2\tau_n^2}(\phi\eta(P_6 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} F_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} F_8^* \frac{\sin(2k+1)\pi s}{2k+1})) - c^2\phi\eta(P_8 + \frac{1}{4\pi}(\sum_{k=1}^{\infty} F_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} F_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1})))] = \tau_n \quad (5.4)$$

6. Graphs of the Numerical Solutions

To illustrate the analysis presented in this work, orthotropic rectangular plate is taken to be of length $L_y = 0.923m$, breadth $L_x = 0.432m$ the load velocity c=0.8123 m/s and $s = 0.4m$.The results are presented on the various graphs below for the simply supported boundary conditions.

6.1 Graphs for Cantilever End Conditions

Figures 6.1 and 6.2 display the effect of foundation modulus (K_o) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of foundation modulus (K_o) increases.

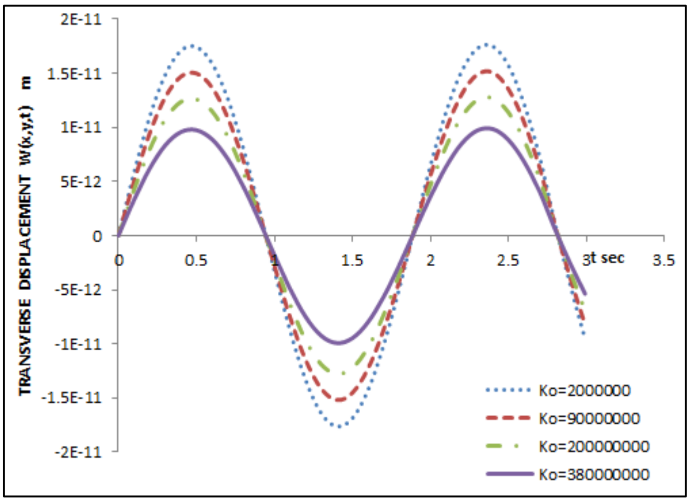

Fig 1: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Force

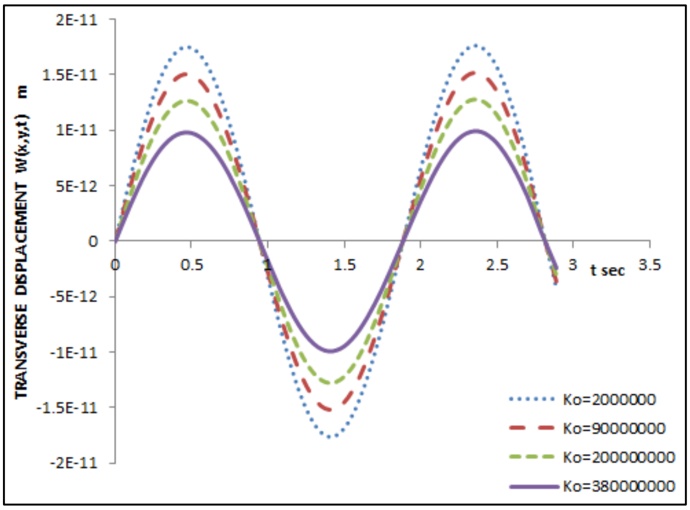

Fig 2: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Mass

Figures 6.3 and 6.4 display the effect of shear modulus (G_o) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of shear modulus (G_o) increases.

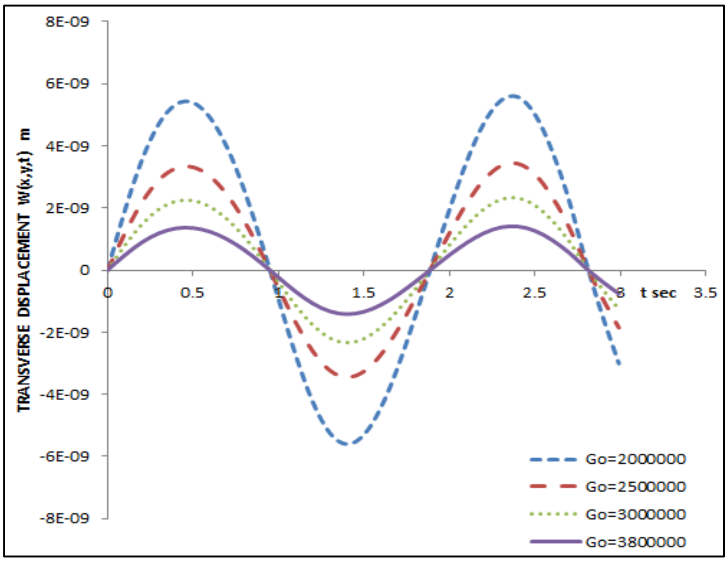

Fig 3: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Force

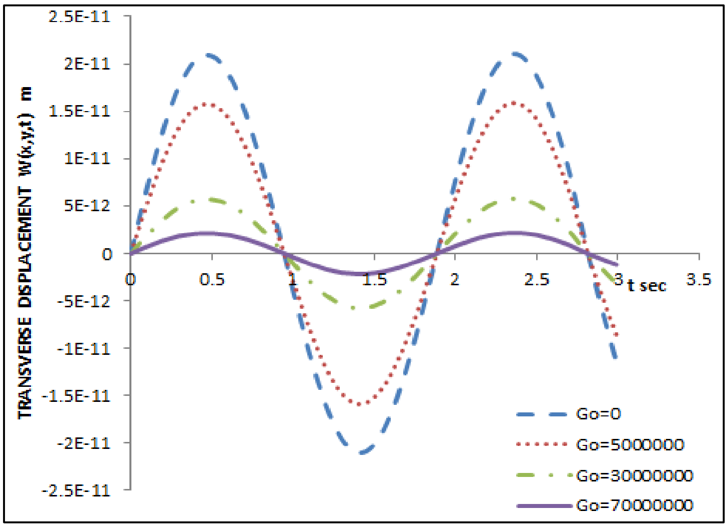

Fig 4: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Mass

Figures 6.5 and 6.6 display the effect of flexural rigidity of the plate along x-axis (D_x) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity (D_x) increases.

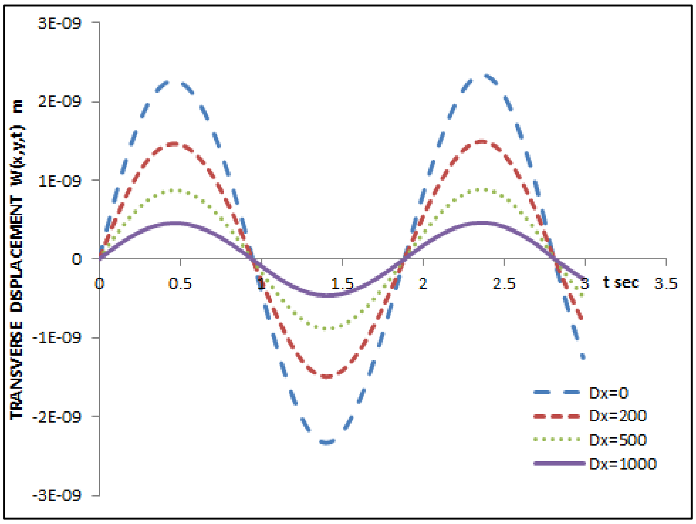

Fig 4: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Force

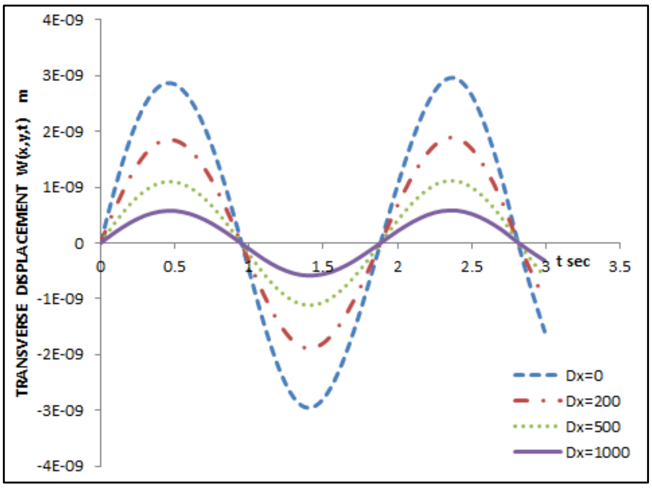

Fig 5: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Mass

Figures 6.7 and 6.8 display the effect of flexural rigidity of the plate along y-axis (D_y) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity (D_y) increases.

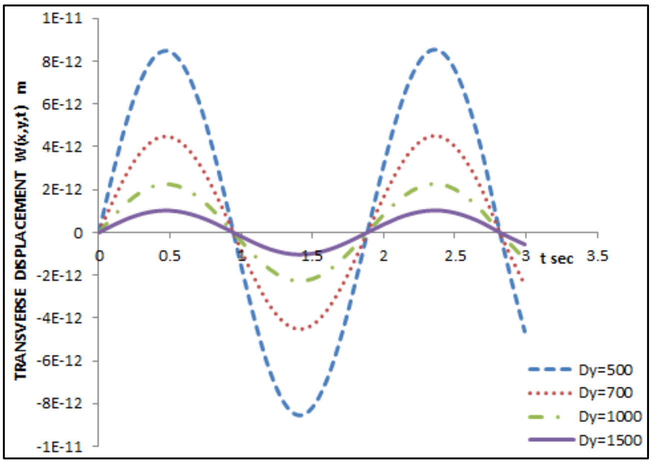

Fig 6: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Force

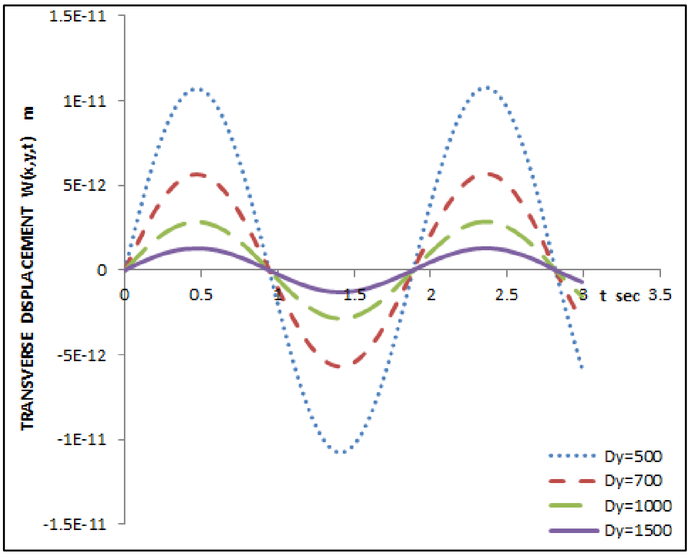

Fig 7: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Mass

Figures 6.9 and 6.10 display the effect of rotatory inertia (R_o) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the R_o increases.

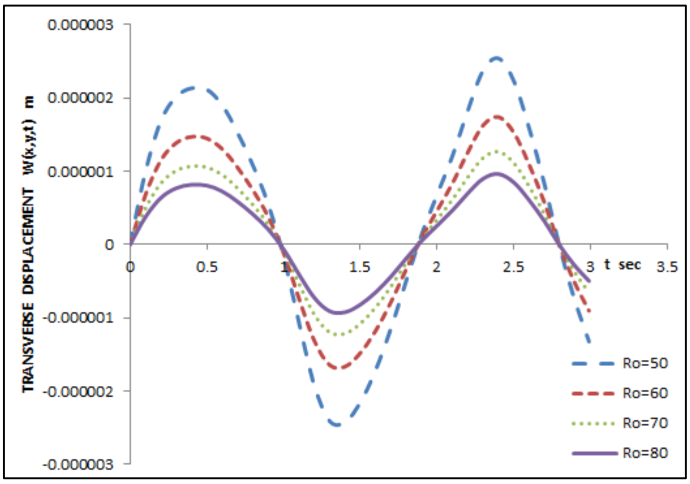

Fig 8: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Force

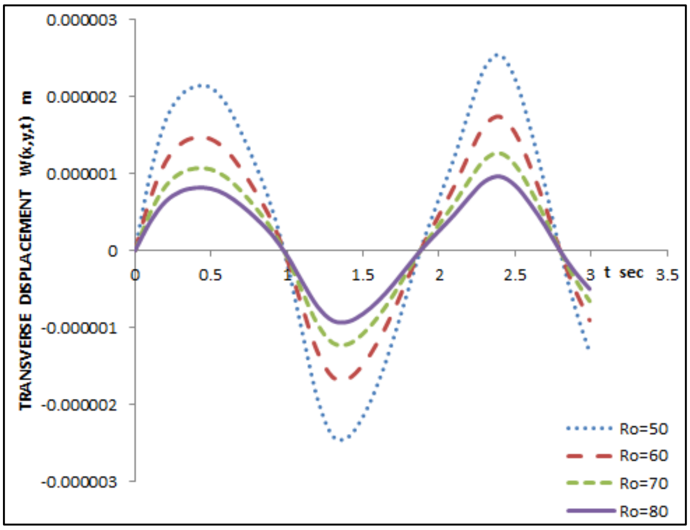

Fig 9: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Mass

Figure 6.11 displays the comparison between moving force and moving mass for fixed values of R_o, G_o, K_o D_x and Dy.

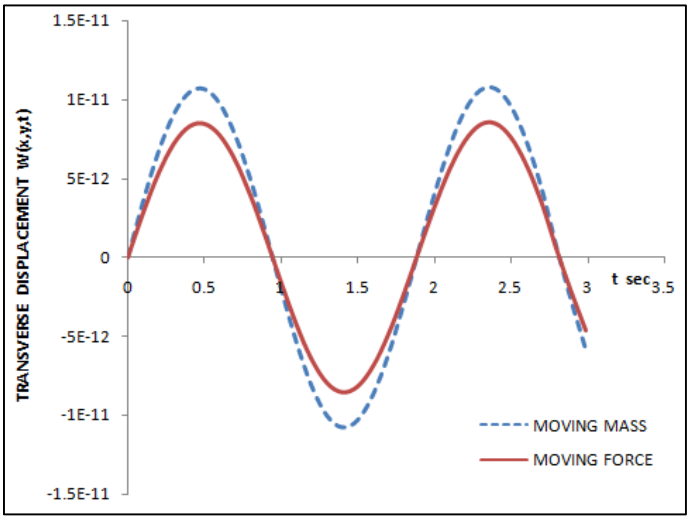

Fig 10: Displacement Profile of Comparison between Moving Force and Moving Mass

7. Conclusion

In this work, the problem concerning of vibration of moving distributed masses of cantilever shaped-orthotropic rectangular plates resting on constant elastic Pasternak foundation has been studied. The closed form solutions of the fourth order partial differential equations with variable and singular coefficients governing the orthotropic rectangular plates is obtained for both cases of moving force and moving mass using a solution technique that is based on the separation of variables which was used to remove the singularity in the governing fourth order partial differential equation which reduces it to a sequence of coupled second order differential equations. The modified asymptotic technique of Struble and Laplace transformation techniques are then employed to obtain the analytical solution to the two-dimensional dynamical problem.

The solutions are then analyzed. The analyzed results show that, for the same natural frequency and the critical speed for the moving mass problem is smaller than that of the moving force problem. Resonance is attained earlier in the moving mass system than in the moving force problem. That is to say the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

The results in plotted curves depict that as the rotatory inertia correction factor R_o increases, the amplitudes of plates decrease for both cases of moving force and moving mass problems. The flexural rigidities along both the x-axis D_x and y-axis D_y increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems. As the shear modulus G_o and foundation modulus K_o increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems.

It is shown further from the results that for fixed values of rotatory inertia correction factor, flexural rigidities along both x-axis and y-axis, shear modulus and foundation modulus, the amplitude for the moving mass problem is greater than that of the moving force problem which is an evidence that resonance is reached earlier in moving mass problem than in moving force problem of cantilever-shaped orthotropic rectangular plates resting on constant elastic Pasternak foundation.

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