International Journal of Multidisciplinary Research and Growth Evaluation

International Journal of Multidisciplinary Research and Growth Evaluation
ISSN: 2582-7138
Received: 02-08-2021; Accepted: 18-08-2021
www.allmultidisciplinaryjournal.com
Volume 2; Issue 5; September-October 2021; Page No. 125-134

New transformation 'AMK-Transformation' to solve ordinary linear differential equation of moment Pareto distribution

Muhammad Kashif [1], Khurrem Shehzad [2], Ali Moazzam [3], Ammara Ayoub [4], Maryam Bibi [5]
[1-4] Department of Mathematics, University of Agriculture, Faisalabad, Pakistan
[5] Department of Mathematics, University of Lahore, Pakistan

Corresponding Author: **Ali Moazzam**

Abstract
Integral transformation is important in mathematical computations since it simplifies a difficult problem. It's simple to decouple a given function's properties from its parent function. In this paper, we describe the AMK-transform, a new integral transformation for solving ordinary linear differential equations with trigonometry as variable value coefficients; as well the ordinary linear differential equation of the moment Pareto distribution, also (L.O.D.E) of its hazard and survival functions will also be solved here.

Keywords: AMK-transform, Trigonometric coefficients, Inverse of AMK-transform, new type of ordinary linear differential equations, AZ-Equation, PDF Pareto distribution.

Introduction

The integral transformation is important for complex equations since it simplifies a difficult problem. It's modest to decouple a given function's characteristics from its parent function. Following are the assignments of integrating. In this work, we familiarize the AMK-transform, a novel integral transform to solving ordinary differential equations with trigonometry as varying value factors.

Laplace transform [1] is one of the famous techniques to find solution of the ordinary linear differential equation with constant coefficient with initial conditions and can be expressed as

$$\delta_0 \frac{d^n y(\varsigma)}{d\varsigma^n} + \delta_1 \frac{d^{n-1} y(\varsigma)}{d\varsigma^{n-1}} + \delta_2 \frac{d^{n-2} y(\varsigma)}{d\varsigma^{n-2}} + \ldots + \delta_{n-1} \frac{dy(\varsigma)}{d\varsigma} + \delta_n y(\varsigma) = \Psi(\varsigma) \qquad (1)$$

Where $\delta_0, \delta_1, \delta_2, \ldots, \delta_n$ are constant. In this paper we are going to apply a new transformation to solve the linear differential equation (L.O.D.E) with sine coefficient, which have general form

$$\delta_0 (sin\varsigma)^n \frac{d^n y(sin\varsigma)}{d\varsigma^n} + \delta_1 (sin\varsigma)^{n-1} \frac{d^{n-1} y(sin\varsigma)}{d\varsigma^{n-1}} + \delta_2 (sin\varsigma)^{n-2} \frac{d^{n-2} y(sin\varsigma)}{d\varsigma^{n-2}} + \ldots + \delta_n y(sin\varsigma) = \Psi(sin\varsigma) \qquad (2)$$

The above equation is known as **AZ (Ali and Zafar) equation**. The transformation is introduced for some functions.

In economics, the Pareto distribution is a normally used distribution. It was first used by Pareto [2] to explain the distribution of income between persons. It's the applicable model for conditions where the 80–20 rule applies, i.e. when 80% of the effect creates from 20% of the origins. Definitely, a small fraction of society's wealth is used or possessed by a limited number of folks.

PDF of moment Pareto distribution is $\Psi(x) = (\beta-1)\alpha^{(\beta-1)} x^{-\beta}, \alpha > 0, \alpha < x < \infty, \beta > 0.$ where α and β are parameters and it is represented as $x \sim$ Pareto (α, β). [3]

The Laplace transformation is one of the most essential transformations for solving the L.O.D.E. with constant coefficients and certain beginning conditions and also be applicable in various (PDF) of different distributions. Here's a new transformation is applied to solve L.O.D.E of Pareto distribution in the form of Euler's equation and can be expressed as

$AMK\left\{\Psi\left(sin\varsigma\right)\right\} = \int_0^{\frac{\pi}{2}} \left(sin\varsigma\right)^{\varpi} cos\varsigma\, f\left(sin\varsigma\right) d\varsigma = \Psi^*\left(\varpi\right),$ here ϖ is constant and integral is convergent in given interval.

Main Result

Definition 1[4]

Let Ψ is a function described on $(a,b),$ then the integral transform for function Ψ that is $\Psi^*\left(\varpi\right)$ described as $\Psi^*\left(\varpi\right) = \int_a^b k\left(\varpi,\varsigma\right)\Psi\left(\varsigma\right) d\varsigma,$ Where k is a fix kernel of the transformation with two variable, $a,b \in R,$ the above integral is convergent.

Definition (2) [4]

Ali Moazzam and Kashif-Transformation (AMK-transformation)

AMK-transform of $\Psi\left(sin\varsigma\right)$ and $\varsigma \in \left[0,\frac{\pi}{2}\right]$ is described as $AMK\left\{\Psi\left(sin\varsigma\right)\right\} = \int_0^{\frac{\pi}{2}} \left(sin\varsigma\right)^{\varpi} cos\varsigma\Psi\left(sin\varsigma\right) d\varsigma = \Psi^*\left(\varpi\right),$ here ϖ is constant and the integral is convergent in interval $\left[0,\frac{\pi}{2}\right]$.

Property (1) (Linear Property) [4]

$AMK\left\{\alpha\Psi\left(sin\varsigma\right) \pm \beta g\left(sin\varsigma\right)\right\} = \alpha AMK\left\{\Psi\left(sin\varsigma\right)\right\} \pm \beta AMK\left\{g\left(sin\varsigma\right)\right\},$ where α,β are constants.

Theorem (1) [4]

If $\Psi\left(\varpi\right)$ is defined as $AMK\left\{\Psi(sin\varsigma)\right\} = \Psi^*(\varpi)$ and a is a constant then $AMK\left\{\left(sin\varsigma\right)^a \Psi\left(sin\varsigma\right)\right\} = \Psi^*\left(\varpi + a\right).$

Definition (2) [4]

Let $\Psi\left(sin\varsigma\right)$ be a function and $AMK\left\{\Psi\left(sin\varsigma\right)\right\} = \Psi^*\left(\varpi\right)$ then $\Psi\left(sin\varsigma\right)$ is said to be the inverse of $\Psi^*\left(\varpi\right)$ and defined as $AMK^{-1}\left\{\Psi^*\left(\varpi\right)\right\} = \Psi\left(sin\varsigma\right)$.

Theorem (2) [4]

Let $\Psi^*(\varpi) = AMK\{\Psi(sin\varsigma)\}$ and a be a constant then $AMK^{-1}\{\Psi^*(\varpi + a)\} = \left(sin\varsigma\right)^a AMK^{-1}\{\Psi^*(\varpi)\}$

Transformation for some functions [4]

We are going to find the AMK-transformation for some functions, like logarithmic, fix function, trigonometric function and other functions.

Table 1: Transformation for some functions

Function on $\Psi(sin\varsigma)$	$AMK\left\{\Psi\left(sin\varsigma\right)\right\} = \int_0^{\frac{\pi}{2}} \left(sin\varsigma\right)^{\varpi} \cos\varsigma\Psi\left(sin\varsigma\right) d\varsigma = \Psi\left(\omega\right)$	**Region of convergence**
$\Psi\left(sin\varsigma\right) = 1$	$\frac{1}{\varpi + 1}$	$\varpi > -1$
$\Psi\left(sin\varsigma\right) = k$	$\frac{k}{\varpi + 1}$	$\varpi > -1$
$\Psi\left(sin\varsigma\right) = sin^n\varsigma$	$\frac{1}{\varpi + \left(n+1\right)}$	$\varpi > -(n+1)$
$\Psi\left(sin\varsigma\right) = cos\left(alnsin\varsigma\right)$	$\frac{\varpi + 1}{\left(\varpi + 1\right)^2 + \left(a\right)^2}$	$\varpi > -1$

$\Psi(sin\varsigma)=sin(alnsin\varsigma)$	$\dfrac{a}{(\varpi+1)^2+(a)^2}$	$\varpi>-1$
$\Psi(sin\varsigma)=cosh(aln(sin\varsigma))$	$\dfrac{\varpi+1}{(\varpi+1)^2-(a)^2}$	$\varpi>-1$
$\Psi(sin\varsigma)=sinh(a(sin\varsigma))$	$\dfrac{-a}{(\varpi+1)^2-(a)^2}$	$\varpi>-1$
$\Psi(sin\varsigma)=[ln(sin\varsigma)]^n$	$\dfrac{n!(-1)^n}{(1+\varpi)^{1+n}}$	$\varpi>-1$

Theorem (3) [4]

If the $\Psi(sin\varsigma)$ is described for $\xi>0$ and its derivatives $\frac{d(sin\varsigma)}{d\varsigma},\frac{d^2(sin\varsigma)}{d\varsigma^2},\frac{d^3(sin\varsigma)}{d\varsigma^3},...,\frac{d^n(sin\varsigma)}{d\varsigma^n}$ are exit then $AMK\left[(sin\varsigma)^n y^{(n)}(sin\varsigma)\right]$

$$=y^{(n-1)}(1)+(-1)^n(n+\varpi)y^{(n-2)}(1)+(-1)^{n-1}(n+\varpi)((n-1)+\varpi)y^{(n-3)}(1)+...+(\varpi+n)((n-1)+\varpi)+...+(\varpi+2)y(1)-(\varpi+n)!.\Psi^*(\varpi).$$

AMK-Transformation on Moment Pareto Distribution [3]

PDF of distribution, moment Pareto is $\Psi(x)=(\beta-1)\alpha^{(\beta-1)}x^{-\beta},\alpha>0,\alpha<x<\infty,\beta>0.$

AMK-Transformation on First Order O.D.E Moment Pareto Distribution

The first order differential equation of moment Pareto distribution is given as

$x\Psi'(x)+\beta\Psi(x)=0$ and $\Psi(1)=(\beta-1)\alpha^{(\beta-1)},\Psi'(1)=-\beta(\beta-1)\alpha^{\beta-1}$, $\Psi''(1)=\beta(\beta+1)(\beta-1)\alpha^{(\beta-1)}$

Sol.

Substitute $x=sin\varsigma$

so equation will become

$sin\varsigma\Psi'(sin\varsigma)+\beta\Psi(sin\varsigma)=0$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma\Psi'(sin\varsigma)\}+\beta AMK\{\Psi(sin\varsigma)\}=0$$

$$\Psi(1)-(\varpi+1)\Psi^*(\varpi)+\beta\Psi^*(\varpi)=0$$

$$(\varpi+1-\beta)\Psi^*(\varpi)=\Psi(1)$$

$$\Psi^*(\varpi)=\frac{\Psi(1)}{(\varpi+1-\beta)}$$

by applying inverse of transform

$$\Psi(sin\varsigma)=(\beta-1)\alpha^{(\beta-1)}(sin\varsigma)^{-\beta}$$

As $x=sin\varsigma$ so

$$\Psi(x)=(\beta-1)\alpha^{(\beta-1)}(x)^{-\beta}$$

which is the required PDF.

AMK-Transformation on Second Order O.D.E Moment Pareto Distribution

The differential equation of second order of PDF moment Pareto is given as

$x\Psi''(x)+(\beta+1)\Psi'(x)=0$ and $\Psi(1)=(\beta-1)\alpha^{(\beta-1)},\Psi'(1)=-\beta(\beta-1)\alpha^{\beta-1}$, $\Psi''(1)=\beta(\beta-1)(\beta+1)\alpha^{(\beta-1)}$

Sol.

Substitute $x = sin\varsigma$

So equation will become

$$sin\varsigma\Psi''(sin\varsigma) + (\beta+1) f'(sin\varsigma) = 0$$

multiply by $sin\varsigma$ on both sides

$$sin^2\varsigma\Psi''(sin\varsigma) + (\beta+1)(sin\varsigma)\Psi'(sin\varsigma) = 0$$

by applying AMK-Transform and using theorem 3,

$$\{AMKsin\varsigma^2\Psi''(sin\varsigma)\} + (\beta+1)AMK\{(sin\varsigma)\Psi'(sin\varsigma)\} = 0$$

$$\Psi'(1) - (\varpi+2)\Psi(1) + (\varpi+2)(\varpi+1)\Psi^*(\varpi) + (\beta+1)[\Psi(1) - (\varpi+1)\Psi^*(\varpi)] = 0$$

$$-\beta(\beta-1)\alpha^{(\beta-1)} - (\varpi+2)[(\beta-1)\alpha^{(\beta-1)}] + (\varpi+2)(\varpi+1)\Psi^*(\varpi) + (\beta-1)(\beta+1)\alpha^{(\beta-1)} - (\beta+1)(\varpi+1)\Psi^*(\varpi) = 0$$

$$\Psi^*(\varpi) = \frac{\beta(\beta-1)\alpha^{(\beta-1)}}{(\varpi+1)} + \frac{(\varpi+2)(\beta-1)\alpha^{(\beta-1)}}{(\varpi+1)} - \frac{(\beta+1)(\beta-1)\alpha^{(\beta-1)}}{\varpi+1}$$

$$\Psi^*(\varpi) = (\beta-1)\alpha^{(\beta-1)} \frac{(\varpi+1)}{(\varpi+1)(\varpi+(1-\beta))}$$

by applying inverse of transform

$$\Psi(sin\varsigma) = (\beta-1)\alpha^{\beta-1}(sin\varsigma)^{-\beta}$$

As $x = sin\varsigma$ so

$$\Psi(x) = (\beta-1)\alpha^{\beta-1}(x)^{-\beta}$$

which is the required PDF.

AMK-Transformation on Third Order O.D.E Moment Pareto Distribution

The differential equation of third order of moment Pareto PDF is given as

$$xf\Psi'''(x) + (\beta+2)\Psi''(x) = 0$$ and $\Psi(1) = (\beta-1)\alpha^{(\beta-1)}, \Psi'(1) = -\beta(\beta-1)\alpha^{\beta-1}$, $\Psi''(1) = \beta(\beta-1)(\beta+1)\alpha^{(\beta-1)}$

Sol.

Substitute $x = sin\varsigma$

so equation will become

$$sin\varsigma\Psi'''(sin\varsigma) + (\beta+2)\Psi''(sin\varsigma) = 0$$

multiply by $sin\varsigma^2$ on both sides

$$sin\varsigma^3\Psi'''(sin\varsigma) + (\beta+2)(sin\varsigma)^2\Psi''(sin\varsigma) = 0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma^3\Psi'''(sin\varsigma)\} + (\beta+2)AMK\{(sin\varsigma)^2\Psi''(sin\varsigma)\} = 0$$

$$\Psi''(1) - (\varpi+3)\Psi'(1) + (\varpi+2)(\varpi+1)\Psi(1) - (\varpi+3)(\varpi+2)(\varpi+1)\Psi^*(\varpi) + (\beta+2)[\Psi'(1) - (\varpi+2)\Psi(1) + (\varpi+2)(\varpi+1)\Psi^*(\varpi)] = 0$$

$$\Psi''(1)-(\varpi+3)\Psi'(1)+(\varpi+3)(\varpi+2)\Psi(1)+(\beta+2)\Psi'(1)-(\beta+2)(\varpi+2)\Psi(1)$$

$$=(\varpi+3)(\varpi+2)(\varpi+1)\Psi^*(\varpi)-(\beta+2)(\varpi+2)(\varpi+1)\Psi^*(\varpi)$$

$$\Psi''(1)-(\varpi+3)\Psi'(1)+(\varpi+2)(\varpi+1)\Psi(1)+(\beta+2)\Psi'(1)-(\beta+2)(\varpi+2)\Psi(1)=(\varpi+3)(\varpi+2)\Psi^*(\varpi)[(\varpi+3)-\beta-2]$$

$$(\beta-1)\alpha^{\beta-1}[\beta(\beta+1)+(\varpi+3)\beta+(\varpi+3)(\varpi+2)-\beta(\beta+2)-(\beta+2)(\varpi+2)]=(\varpi+3)(\varpi+2)(\varpi+(1-\beta))\Psi^*(\varpi)$$

$$(\beta-1)\alpha^{\beta-1}(\beta^2+\beta+\varpi\beta+3\beta+\varpi^2+5\varpi+6-\beta^2-2\beta-\beta\varpi-2\beta-2\varpi-4)=(\varpi+3)(\varpi+2)(\varpi+(1-\beta))\Psi^*(\varpi)$$

$$(\beta-1)\alpha^{\beta-1}(\varpi^2+3\varpi+2)=(\varpi+3)(\varpi+2)(\varpi+(1-\beta))\Psi^*(\varpi)$$

$$\Psi^*(\varpi)=\frac{(\beta-1)\alpha^{\beta-1}(\varpi^2+3\varpi+2)}{(\varpi^2+3\varpi+2)(\varpi+(1-\beta))}$$

by applying inverse of transform

$$\Psi(sin\varsigma)=(\beta-1)\alpha^{\beta-1}(sin\varsigma)^{-\beta}$$

As $x=sin\varsigma$ so

$$\Psi(x)=(\beta-1)\alpha^{\beta-1}(x)^{-\beta}$$

which is the required PDF.

AMK Transformation on Survival Function of Moment Pareto Distribution

Survival Function of moment Pareto distribution is $\Psi(x)=\alpha^{(\beta-1)}x^{-(\beta+1)},\alpha>0,\alpha<x<\infty,\beta>0.$

AMK-Transformation on First Order O.D.E of Survival Function of Moment Pareto Distribution.

The differential equation of first order of survival function of moment Pareto is given as

$$x\Psi'(x)+(\beta+1)\Psi(x)=0 \text{ and } \Psi(1)=\alpha^{(\beta-1)},\Psi'(1)=-(\beta+1)\alpha^{\beta-1},\ \Psi''(1)=(\beta+1)(\beta+2)\alpha^{(\beta-1)}$$

Sol.

Substitute $x=sin\varsigma$

so equation will become

$$sin\varsigma\Psi'(sin\varsigma)+(\beta+1)\Psi(sin\varsigma)=0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma\Psi'(sin\varsigma)\}+(\beta+1)AMK\{\Psi(sin\varsigma)\}=0$$

$$\Psi(1)-(\varpi+1)\Psi^*(\varpi)+(\beta+1)\Psi^*(\varpi)=0$$

$$[\varpi+\{1-(\beta+1)\}]\Psi^*(\varpi)=\Psi(1)$$

$$\Psi^*(\varpi)=\frac{\Psi(1)}{(\varpi+(1-(1+\beta))}$$

by applying inverse of transform

$$\Psi(sin\varsigma)=\alpha^{(\beta-1)}(sin\varsigma)^{-(\beta+1)}$$

As $x=sin\varsigma$ so

$$\Psi(x) = \alpha^{(\beta-1)}(x)^{-(\beta+1)}$$

which is the required PDF.

AMK-Transformation on Second Order O.D.E of Survival Function of Moment Pareto Distribution

The differential equation of second order of survival function of moment Pareto is given as

$$x\Psi''(x) + (\beta+2)\Psi'(x) = 0 \text{ and } \Psi(1) = \alpha^{(\beta-1)}, \Psi'(1) = -(\beta+1)\alpha^{\beta-1}, \Psi''(1) = (\beta+1)(\beta+2)\alpha^{(\beta-1)}$$

Sol.

Substitute $x = sin\varsigma$
so equation will become

$$sin\varsigma\Psi''(sin\varsigma) + (\beta+2)\Psi'(sin\varsigma) = 0$$

multiply by $sin\varsigma$ on both sides

$$sin\varsigma^2\Psi''(sin\varsigma) + (\beta+2)(sin\varsigma)\Psi'(sin\varsigma) = 0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma^2\Psi''(sin\varsigma)\} + (\beta+2)AMK\{(sin\varsigma)\Psi'(sin\varsigma)\} = 0$$

$$\Psi'(1) - (\varpi+2)\Psi(1) + (\varpi+2)(\varpi+1)\Psi^*(\varpi) + (\beta+2)[\Psi(1) - (\varpi+1)\Psi^*(\varpi)] = 0$$

$$-(\beta+1)\alpha^{(\beta-1)} - (\varpi+2)\alpha^{(\beta-1)} + (\varpi+2)(\varpi+1)\Psi^*(\varpi) + (\beta+2)\alpha^{(\beta-1)} - (\beta+2)(\varpi+1)\Psi^*(\varpi) = 0$$

$$[\varpi + \{1-(\beta+1)\}]\Psi^*(\varpi) = \frac{(\beta+1)\alpha^{(\beta-1)}}{(\varpi+1)} + \frac{(\varpi+2)\alpha^{(\beta-1)}}{(\varpi+1)} - \frac{(\beta+2)\alpha^{(\beta-1)}}{\varpi+1}$$

$$\Psi^*(\varpi) = \alpha^{(\beta-1)}\frac{(\varpi+1)}{(\varpi+1)(\varpi+(1-(\beta+1)))}$$

by applying inverse of transform

$$\Psi(sin\varsigma) = \alpha^{\beta-1}(sin\varsigma)^{-(\beta+1)}$$

As $x = sin\varsigma$ so

$$\Psi(x) = \alpha^{\beta-1}(x)^{-(\beta+1)}$$

which is the required survival function.

AMK-Transformation on Third Order O.D.E of Survival Function of Moment Pareto Distribution

The differential equation of third order of survival function of moment Pareto is given as

$$x\Psi'''(x) + (\beta+3)\Psi''(x) = 0 \text{ and } \Psi(1) = \alpha^{(\beta-1)}, \Psi'(1) = -(\beta+1)\alpha^{\beta-1}, \Psi''(1) = (\beta+1)(\beta+2)\alpha^{(\beta-1)}$$

Sol.

Substitute $x = sin\varsigma$
so equation will become

$$sin\varsigma\Psi'''(sin\varsigma) + (\beta+3)\Psi''(sin\varsigma) = 0$$

multiply by $sin\varsigma^2$ on both sides

$$sin\varsigma^3\Psi'''(sin\varsigma)+(\beta+3)(sin\varsigma)^2\Psi''(sin\varsigma)=0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma^3\Psi'''(sin\varsigma)\}+(\beta+3)AMK\{(sin\varsigma)^2\Psi''(sin\varsigma)\}=0$$

$$\Psi''(1)-(\varpi+3)\Psi'(1)+(\varpi+2)(\varpi+1)\Psi(1)-(\varpi+3)(\varpi+2)(\varpi+1)\Psi^*(\varpi)+(\beta+3)$$

$$[\Psi'(1)-(\varpi+2)\Psi(1)+(\varpi+2)(\varpi+1)\Psi^*(\varpi)]=0$$

$$\Psi''(1)-(\varpi+3)\Psi'(1)+(\varpi+3)(\varpi+2)\Psi(1)+(\beta+3)\Psi'(1)-(\beta+3)(\varpi+2)\Psi(1)$$

$$=(\varpi+3)(\varpi+2)(\varpi+1)\Psi^*(\varpi)-(\beta+3)(\varpi+2)(\varpi+1)\Psi^*(\varpi)$$

$$\Psi''(1)-(\varpi+3)\Psi'(1)+(\varpi+2)(\varpi+1)\Psi(1)+(\beta+3)\Psi'(1)-(\beta+3)(\varpi+2)\Psi(1)$$

$$=(\varpi+3)(\varpi+2)\Psi^*(\varpi)[(\varpi+3)-\beta-3]$$

$$\alpha^{\beta-1}[(\beta+2)(\beta+1)+(\varpi+3)(\beta+1)+(\varpi+3)(\varpi+2)-(\beta+1)(\beta+3)-(\beta+3)(\varpi+2)]$$

$$=(\varpi+3)(\varpi+2)(\varpi+(1-(\beta+1)\Psi^*(\varpi)$$

$$\alpha^{\beta-1}(\beta^2+2\beta+2+\varpi\beta+\varpi+3\beta+3+\varpi^2+5\varpi+6-\beta^2-\beta-3\beta-3-\beta\varpi-2\beta-3\varpi-6)$$

$$=(\varpi+3)(\varpi+2)(\varpi+(1-(\beta+1)))\Psi^*(\varpi)$$

$$\alpha^{\beta-1}(\varpi^2+3\varpi+2)=(\varpi+3)(\varpi+2)(\varpi+(1-(1+\beta)\Psi^*(\varpi)$$

$$\Psi^*(\varpi)=\frac{\alpha^{\beta-1}(\varpi^2+3\varpi+2)}{(\varpi^2+3\varpi+2)(\varpi+(1-(1+\beta)))}$$

by applying inverse of transform

$$\Psi(sin\varsigma)=\alpha^{\beta-1}(sin\varsigma)^{-(\beta+1)}$$

As $x=sin\varsigma$ so

$$\Psi(x)=\alpha^{\beta-1}(x)^{-(\beta+1)}$$

which is the required survival function.

AMK-Transformation on O.D.E of Hazard Function of Moment Pareto Distribution

Hazard function of moment Pareto distribution is defined as $\Psi(x)=\dfrac{\beta-1}{x},x>0;\beta>0$.

AMK-Transformation on First Order O.D.E of Hazard Function of Moment Pareto Distribution

The differential equation of first order of hazard function of moment Pareto is given as

$x\Psi'(x)+\Psi(x)=0$ and $\Psi(1)=\beta-1=\Psi''(1)$, $\Psi'(1)=-(\beta-1)$

Sol.

Substitute $x=sin\varsigma$

so equation will become

$$sin\varsigma\Psi'(sin\varsigma) + \Psi(sin\varsigma) = 0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma\Psi'(sin\varsigma)\} + AMK\{\Psi(sin\varsigma)\} = 0$$

$$\Psi(1) - (\varpi + 1)\Psi^*(\varpi) + \Psi^*(\varpi) = 0$$

$$(\varpi)\Psi^*(\varpi) = \Psi(1)$$

$$\Psi^*(\varpi) = \frac{\Psi(1)}{(\varpi + (1-1))}$$

by applying inverse of transform

$$\Psi(sin\varsigma) = \frac{\beta - 1}{sin\varsigma}$$

As $x = sin\varsigma$ so

$$\Psi(x) = \frac{\beta - 1}{x}$$

Which is the required hazard function.

AMK-Transformation on Second Order O.D.E of Hazard Function of Moment Pareto Distribution

The differential equation of second order of hazard function of moment Pareto is given as

$x\Psi''(x) + 2\Psi'(x) = 0$ and $\Psi(1) = \beta - 1 = \Psi''(1)$, $\Psi'(1) = -(\beta - 1).$

Sol.

Substitute $x = sin\varsigma$

so equation will become

$$sin\varsigma\Psi''(sin\varsigma) + 2\Psi'(sin\varsigma) = 0$$

multiply by $sin\varsigma$ on both sides

$$sin\varsigma^2\Psi''(sin\varsigma) + 2sin\varsigma\Psi'(sin\varsigma) = 0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma^2\Psi''(sin\varsigma)\} + 2AMK\{sin\varsigma\Psi'(sin\varsigma)\} = 0$$

$$\Psi'(1) - (\varpi + 2)\Psi(1) + (\varpi + 2)(\varpi + 1)\Psi^*(\varpi) + 2(\varpi + 1)\Psi(1) - 2(\varpi + 1)\Psi^*(\varpi) = 0$$

$$\Psi^*(\varpi) = \frac{(\beta - 1)(\varpi + 2) + (\beta - 1) - 2(\beta - 1)}{\varpi(\varpi + 1)}$$

$$\Psi^*(\varpi) = \frac{(\varpi - 1)(\varpi + 1)}{\varpi(\varpi + 1)}$$

$$\Psi^*(\varpi) = \frac{(\beta - 1)}{(\varpi + (1-1))}$$

by applying inverse of transform

$$\Psi(sin\varsigma) = \frac{\beta - 1}{sin\varsigma}$$

As $x = sin\varsigma$ so

$$\Psi(x) = \frac{\beta - 1}{x}$$

which is the required hazard function.

AMK-Transformation on Third Order O.D.E of Hazard Function of Moment Pareto Distribution

The differential equation of third order of hazard function of moment Pareto is given as

$$x\Psi'''(x) + 3\Psi''(x) = 0 \text{ and } \Psi(1) = \beta - 1 = \Psi''(1),\ \Psi'(1) = -(\beta - 1)$$

Sol.

Substitute $x = sin\varsigma$

so equation will become

$$sin\varsigma\Psi'''(sin\varsigma) + 3\Psi''(sin\varsigma) = 0$$

multiply by $sin^2\varsigma$ on both sides

$$sin\varsigma^3\Psi'''(sin\varsigma) + 3sin\varsigma^2\Psi''(sin\varsigma) = 0$$

by applying AMK-Transform and using theorem 3,

$$AMK\{sin\varsigma^3\Psi'''(sin\varsigma)\} + 3AMK\{sin\varsigma^2\Psi'(sin\varsigma)\} = 0$$

$$\Psi''(1) - (\varpi + 3)\Psi'(1) + (\varpi + 3)(\varpi + 2)\Psi(1) - (\varpi + 2)(\varpi + 1)(\varpi + 3)\Psi^*(\varpi) + 3\Psi'(1) - 3(\varpi + 2)\Psi(1) + (\varpi + 2)(\varpi + 1)\Psi^*(\varpi) = 0$$

$$\Psi^*(\varpi) = \frac{(\beta - 1)(\varpi^2 + 3\varpi + 2)}{\varpi(\varpi^2 + 3\varpi + 2)}$$

$$\Psi^*(\varpi) = \frac{(\beta - 1)}{\varpi}$$

$$\Psi^*(\varpi) = \frac{(\beta - 1)}{(\varpi + (1 - 1))}$$

by applying inverse of transform

$$\Psi(sin\varsigma) = \frac{\beta - 1}{sin\varsigma}$$

As $x = sin\varsigma$ so

$$\Psi(x) = \frac{\beta - 1}{x}$$

which is the required hazard function.

Conclusion

Table 1 describes the list of AMK-Transformation of certain special functions, constant, dependent function on sine trigonometric ratio and some logarithmic functions as well. The method of transforming is an easy way to find the solution of much differential and integral equation. Here's we have described the AMK-transformation of various functions and some important results for this transformation. Also the application of AMK-Transformation to solve linear ordinary differential equation (L.O.D.E) of moment Pareto distribution, also the L.O.D.E of its hazard and hazard functions.

References

1. B Martin, M Golubitsky. Differential equations and their applications. (1st edn) , Springer-Verlag, New York, 1983.
2. BC Arnold. Encyclopedia of statistical sciences. (3rd edn), John Wiley, 2008.
3. SH Bhatti, S Hussain, T Ahmad, M Aslam, M Aftab, MA Raza. Efficient estimation of Pareto model: Some modified percentile estimators. PloS one. 2018; 13:95-100.
4. A Moazzam, M Kashif, U Amjed, MI Khawar. Devolpment of a new transformation to solve a new type of ordinary linear differential equation. Bulletin of mathematics and statistics research. 2021; 9(3):56-60.